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- ✓ Design of columns

- ✓ Design of beams

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INTRODUCTION

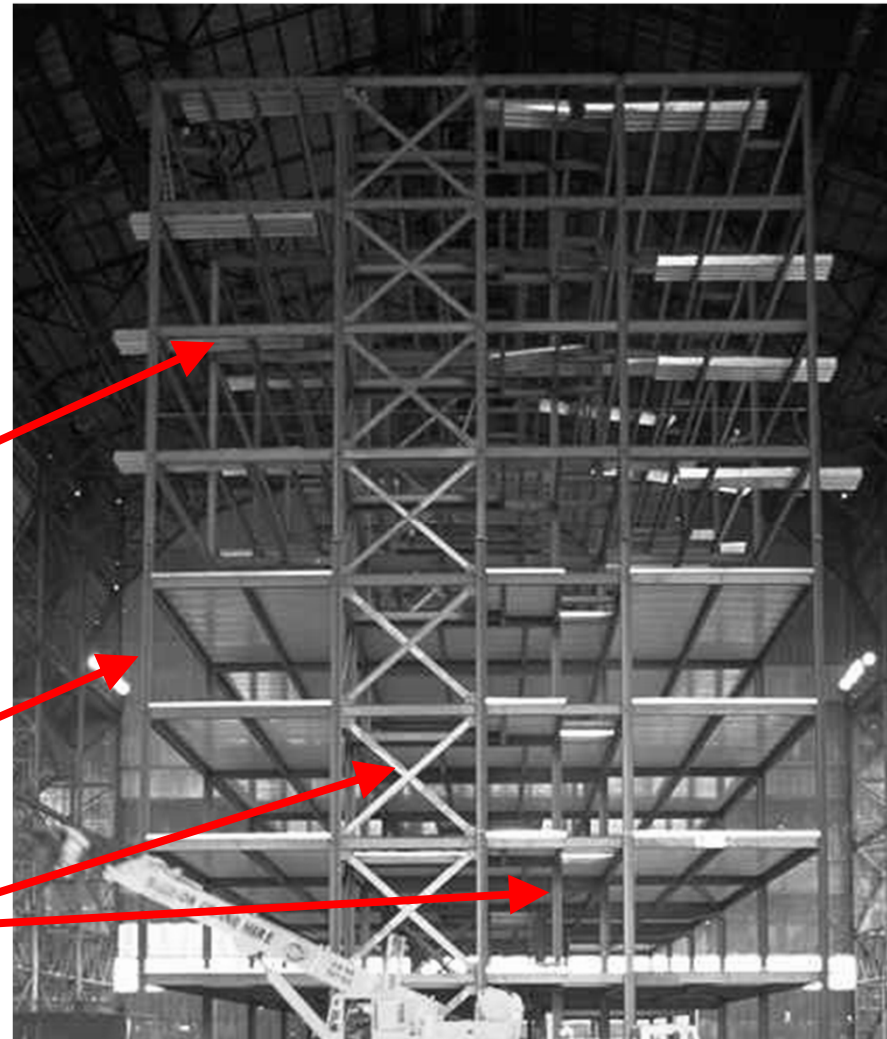**Main internal forces and combinations**

Bending+Shear

Compression+Bending+Shear

Tension/Compression

Torsion – less common

Building – master example (Cardington - UK)

INTRODUCTION

Member design:

i) resistance of cross sections;

ii) member buckling resistance.

RESISTANCE OF CROSS SECTIONS

- **Cross section classification -** Class 1; Class 2; Class 3 and Class 4.

- **Clause 6.2 of Eurocode 3, part 1.1** provides different approaches, depending of cross section shape, cross section class and type of internal forces (N, M+V, N+M+V,....):

  – **elastic criteria (clause 6.2.1(5));**

$$\left(\frac{\sigma_{x,Ed}}{f_y / \gamma_{M0}}\right)^2 + \left(\frac{\sigma_{z,Ed}}{f_y / \gamma_{M0}}\right)^2 - \left(\frac{\sigma_{x,Ed}}{f_y / \gamma_{M0}}\right)\left(\frac{\sigma_{z,Ed}}{f_y / \gamma_{M0}}\right) + 3\left(\frac{\tau_{Ed}}{f_y / \gamma_{M0}}\right)^2 \le 1$$
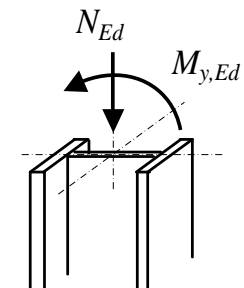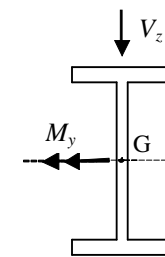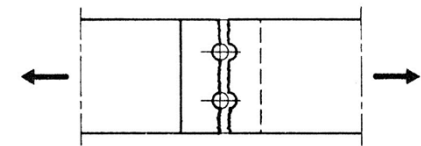
INTRODUCTION

— **linear summation of the utilization ratios — class 1/2/3 (clause 6.2.1(7));**

$$\frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed}}{M_{y,Rd}} + \frac{M_{z,Ed}}{M_{z,Rd}} \leq 1$$

— **nonlinear interaction formulas — class 1/2 (clause 6.2.1(6)).**

- **Section properties** – gross section, net section (deduction for holes) or effective section (class 4 or shear lag effects) (clause 6.2.2 of EC3-1-1).

MEMBER BUCKLING RESISTANCE

- **Buckling resistance** (clause 6.3 of Eurocode 3, part 1.1) must be checked in all members submitted to compressive stresses, which are:

– members under axial compression N;

– members under bending moment M;

– or under a combination of both (M+N).
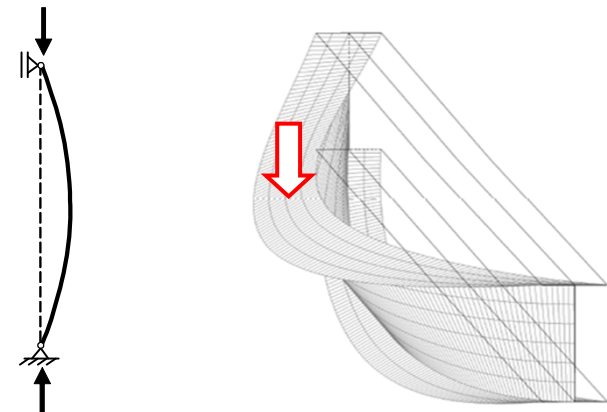
DESIGN OF COLUMNS

Column cross sections and applications

▪ Rolled open or closed sections, welded sections or built-up sections – The objective is to maximize the second moment of area in the relevant buckling plan in order to **maximize the buckling resistance**.
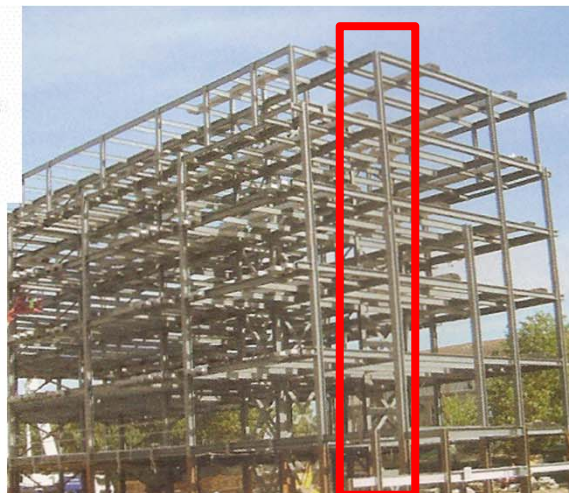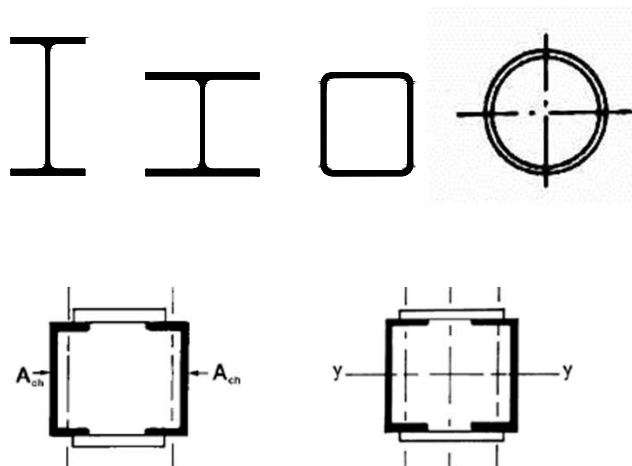
DESIGN OF COLUMNS

Compression resistance (clause 6.2.4 of EC3-1-1)

$$\boxed{\frac{N_{Ed}}{N_{c,Rd}} \le 1.0}$$

N_{Ed} is the design value of the axial compression;

$N_{c,Rd}$ is the design resistance to axial compression, given by the **minimum of**:

i) **Plastic resistance**

$$N_{c,Rd} = A\,f_y / \gamma_{M0} \qquad \text{(class 1, 2 or 3)}$$

$$N_{c,Rd} = A_{eff}\,f_y / \gamma_{M0} \qquad \text{(class 4)}$$
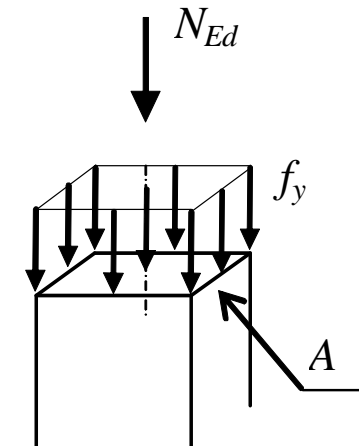
$$\boxed{A_{eff} \text{ - effective area}}$$

ii) **Buckling resistance – $N_{b,Rd}$,** in general the flexural buckling resistance, which is analysed hereafter.

DESIGN OF COLUMNS

Column Buckling

- **Flexural buckling** is in general the buckling mode, which govern the design of a member in pure compression. For this mode in a pinned column, the **elastic critical load N_{cr}**, defined as the maximum load supported by the column, free from any type of imperfections, is given by the well known Euler's formula:

Buckling in a bending mode
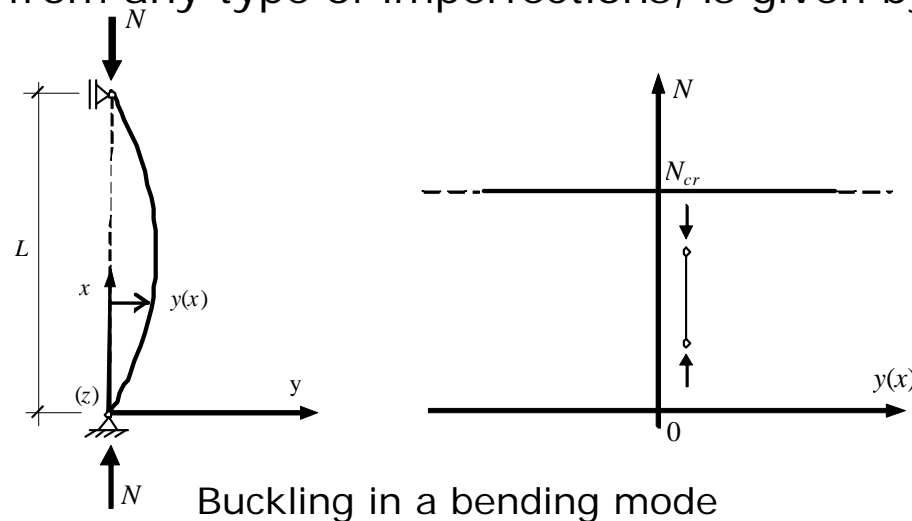
$$E\,I\,\frac{d^2y}{dx^2} + N\,y = 0$$

$$N_{cr} = \frac{\pi^2\,E\,I}{L^2}$$

$E\,I$ – Bending stiffness
L – Buckling length
(L_E for other support conditions)

- In specific cases (e.g. members with cruciform cross sections) buckling may occur in other modes: **torsional buckling or flexural-torsional buckling**.

DESIGN OF COLUMNS

Column Buckling

$$N_{cr} = \frac{\pi^2 E I}{L_E^2}$$

$$\sigma_{cr} = \frac{\pi^2 E I}{A L_E^2} = \frac{\pi^2 E}{\lambda^2}$$

Critical stress

$$\lambda = \frac{L_E}{i}$$

Slenderness

$$i = \sqrt{\frac{I}{A}}$$

Radius of gyration

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda_1^2} = f_y \Rightarrow \lambda_1 = \pi \sqrt{\frac{E}{f_y}}$$

$$\bar{\lambda} = \frac{\lambda}{\lambda_1} = \sqrt{\frac{A f_y}{N_{cr}}}$$

Non-dimensional slenderness

$\sigma = \frac{N}{A}$

Euler's curve

$\sigma = f_y$

$\sigma = \frac{\pi^2 E}{\lambda^2}$

$\lambda_1 = \pi \sqrt{\frac{E}{f_y}}$ $\lambda = \frac{L_E}{i}$

$\frac{\sigma}{f_y}$

1.0

Euler´s curve

1.0

$\bar{\lambda}$
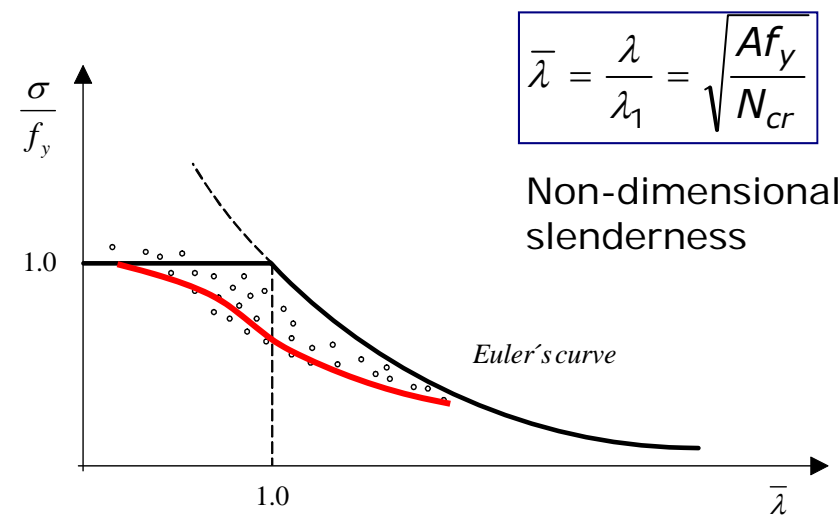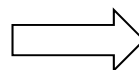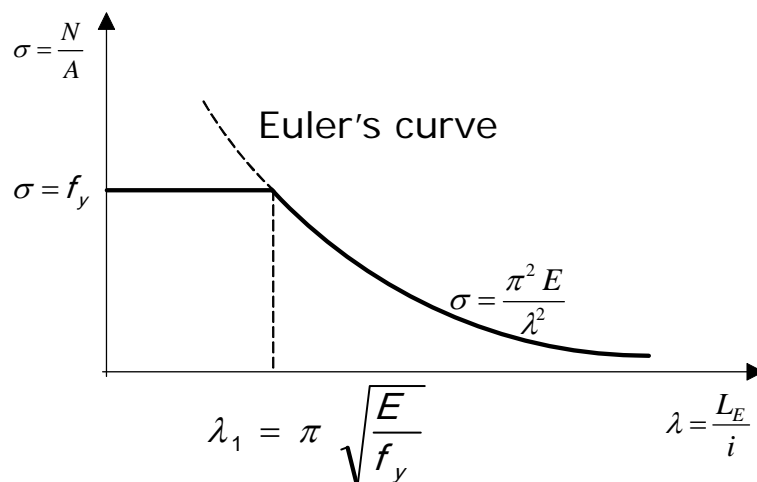
Imperfections or real columns (geometrical imperfections and material imperfections).

DESIGN OF COLUMNS

Buckling Resistance

(clause 6.3.1 of EC3-1-1)

$$N_{b.Rd} = \chi\, A\, f_y / \gamma_{M1} \qquad \text{(Class 1, 2 or 3)}$$

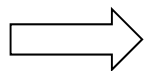$$N_{b.Rd} = \chi\, A_{eff}\, f_y / \gamma_{M1} \qquad \text{(Class 4)}$$

χ is the **reduction factor** for the relevant buckling mode

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \qquad \text{but } \chi \leq 1.0$$

$$\Phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$$

Table 6.1: Imperfection factors for buckling curves

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

Theoretical behaviour
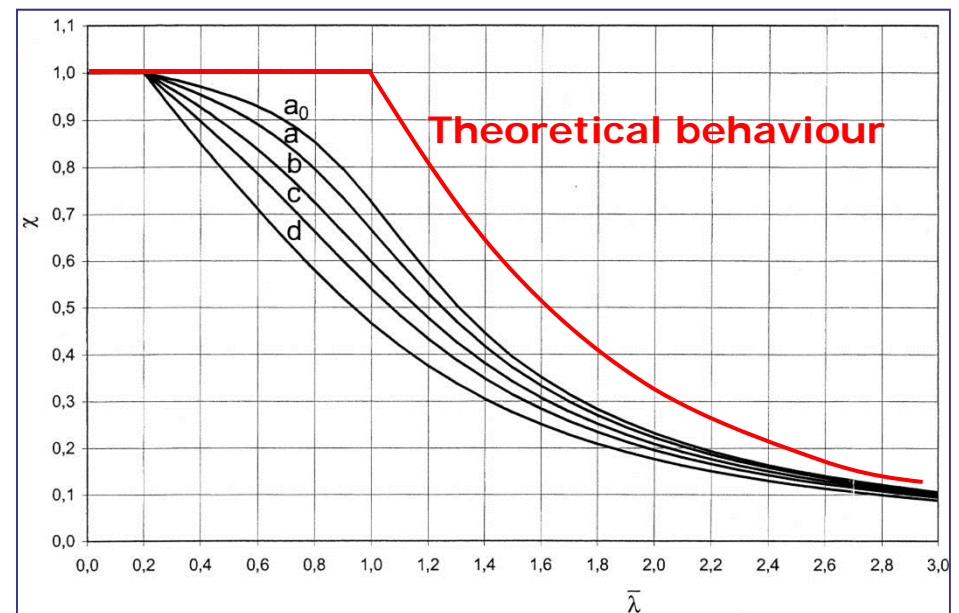
Neglect BUCKLING if:

$$\bar{\lambda} \leq 0.2 \qquad \text{or} \qquad N_{Ed}/N_{cr} \leq 0.04$$

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DESIGN OF COLUMNS

Buckling Resistance

(clause 6.3.1 of EC3-1-1)

Flexural buckling

$$\bar{\lambda} = \sqrt{A f_y / N_{cr}} = \frac{L_{cr}}{i} \frac{1}{\lambda_1} \qquad \text{(Class 1, 2 or 3)}$$

$$\bar{\lambda} = \sqrt{A_{eff} f_y / N_{cr}} = \frac{L_{cr}}{i} \frac{\sqrt{A_{eff}/A}}{\lambda_1} \qquad \text{(Class 4)}$$

$$\lambda_1 = \pi \sqrt{E/f_y} = 93.9\, \varepsilon \qquad \varepsilon = \sqrt{235/f_y}$$
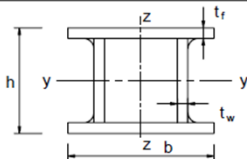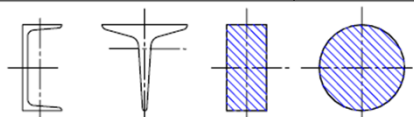
Torsional or flexural-torsional buckling

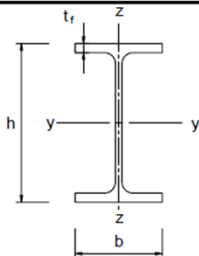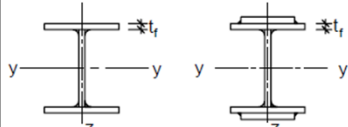$$\bar{\lambda}_T = \sqrt{A f_y / N_{cr}} \qquad \text{(Class 1, 2 or 3)}$$

$$\bar{\lambda}_T = \sqrt{A_{eff} f_y / N_{cr}} \qquad \text{(Class 4)}$$

α - buckling in flexural buckling mode about z axis

Table 6.2: Selection of buckling curve for a cross-section

Cross section		Limits	Buckling about axis	Buckling curve S 235 S 275 S 355 S 420	S 460
Rolled sections	h/b > 1,2	$t_f \le 40$ mm	y − y / z − z	a / b	a₀ / a₀
		40 mm < t_f ≤ 100	y − y / z − z	b / c	a / a
	h/b ≤ 1,2	$t_f \le 100$ mm	y − y / z − z	b / c	a / a
		$t_f > 100$ mm	y − y / z − z	d / d	c / c
Welded I-sections		$t_f \le 40$ mm	y − y / z − z	b / c	b / c
		$t_f > 40$ mm	y − y / z − z	c / d	c / d
Hollow sections		hot finished	any	a	a₀
		cold formed	any	c	c
Welded box sections		generally (except as below)	any	b	b
		thick welds: a > 0,5t_f b/t_f < 30 h/t_w <30	any	c	c
U-, T- and solid sections			any	c	c
L-sections			any	b	b

DESIGN OF COLUMNS

EXAMPLE 1

Safety verification of a column member of the building represented in the figure.
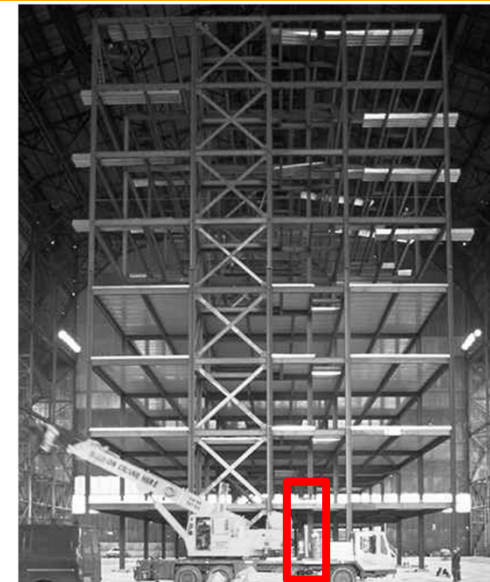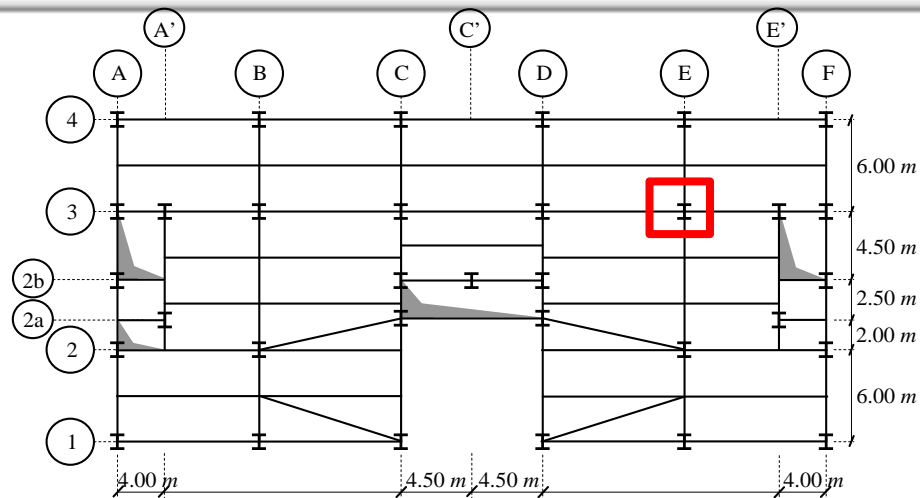
Building – master example

i) The inner column E-3 represented in the figure, at base level, is selected. This member has a length of 4.335 m and is composed by a section HEB 340 in steel S 355.

In this column the bending moments (and the shear force) may be neglected; the **design axial force** (compression) obtained from the previous analysis is given by N_{Ed} = **3326.0 kN**.

DESIGN OF COLUMNS

EXAMPLE 1

ii) Cross section classification – section HEB 340 in pure compression.

Geometric characteristics: A = 170.9 cm², b = 300 mm, h = 340 mm, t_f = 21.5 mm, t_w = 12 mm, r = 27 mm, I_y = 36660 cm⁴, i_y = 14.65 cm, I_z = 9690 cm⁴, i_z = 7.53 cm.
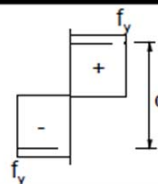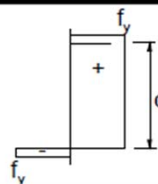
Mechanical properties of the steel: f_y = 355 MPa and E = 210 GPa.
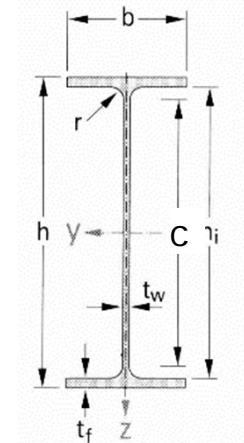
Web in compression (Table 5.2 of EC3-1-1)
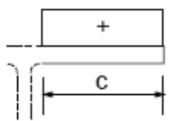
$$\frac{c}{t} = \frac{(340 - 2 \times 21.5 - 2 \times 27)}{12} = 20.25 < 33\varepsilon$$

$$= 33 \times 0.81 = 26.73 \quad \text{(class 1)}$$

Class	Part subject to bending	Part subject to compression
Stress distribution in parts (compression positive)		
1	$c/t \le 72\varepsilon$	$c/t \le 33\varepsilon$

Flange in compression (Table 5.2 of EC3-1-1)

$$\frac{c}{t} = \frac{300/2 - 12/2 - 27}{21.5} = 5.44 < 9\varepsilon = 9 \times 0.81 = 7.29 \quad \text{(class 1)}$$

HEB 340 cross section, steel S 355, in pure compression is **class 1**.

Class	Part subject to compression
Stress distribution in parts (compression positive)	
1	$c/t \le 9\varepsilon$

DESIGN OF COLUMNS

EXAMPLE 1

iii) Cross section verification - class 1 in pure compression.

$$N_{Ed} = 3326.0 \, kN < N_{c,Rd} = \frac{A f_y}{\gamma_{M0}} = \frac{170.9 \times 10^{-4} \times 355 \times 10^3}{1.0} = 6067.0 \, kN.$$

iv) Buckling resistance.

Buckling lengths – Assuming that the design forces were obtained by a second order structural analysis, the buckling lengths are considered (conservatively) equal to the real lengths (mid-distance between floors), given by:

Buckling in the plan x-z (around y) - $L_{Ey} = 4.335 \, m$

Buckling in the plan x-y (around z) - $L_{Ez} = 4.335 \, m$

Determination of the slenderness coefficients

$$\lambda_1 = \pi \sqrt{\frac{210 \times 10^6}{355 \times 10^3}} = 76.41$$

$$\lambda_y = \frac{L_{Ey}}{i_y} = \frac{4.335}{14.65 \times 10^{-2}} = 29.59 \qquad \bar{\lambda}_y = \frac{\lambda_y}{\lambda_1} = 0.39$$

$$\lambda_z = \frac{L_{Ez}}{i_z} = \frac{4.335}{7.53 \times 10^{-2}} = 57.57 \qquad \bar{\lambda}_z = \frac{\lambda_z}{\lambda_1} = 0.75$$

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DESIGN OF COLUMNS

EXAMPLE 1

Calculation of the reduction factor χ_{min}

$$\frac{h}{b} = \frac{340}{300} = 1.13 < 1.2 \qquad \text{and} \qquad t_f = 21.5\,mm < 100\,mm$$

$$\Rightarrow \quad \begin{array}{l} \textit{flexural buckling around } y - \textit{curve } b \, (\alpha = 0.34) \\ \textit{flexural buckling around } z - \textit{curve } c \, (\alpha = 0.49). \end{array}$$
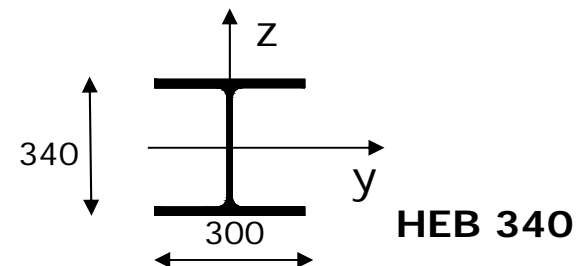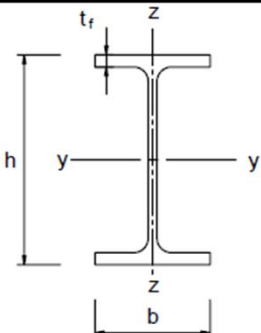
As $\overline{\lambda}_z = 0.75 > \overline{\lambda}_y = 0.39$

and

$$\alpha_{\textit{curve } c} > \alpha_{\textit{curve } b}$$

$$\Rightarrow \quad \chi_{min} => \chi_z$$

Table 6.2: Selection of buckling curve for a cross-section

	Cross section		Limits	Buckling about axis	Buckling curve S 235 S 275 S 355 S 420	S 460
Rolled sections		h/b > 1,2	$t_f \leq 40$ mm	y − y z − z	a b	a₀ a₀
			40 mm < $t_f \leq 100$	y − y z − z	b c	a a
		h/b ≤ 1,2	$t_f \leq 100$ mm	y − y z − z	b c	a a
			$t_f > 100$ mm	y − y z − z	d d	c c

HEB 340

DESIGN OF COLUMNS

EXAMPLE 1

$$\Phi_z = 0.5\left[1 + \alpha\left(\overline{\lambda}_z - 0.2\right) + \overline{\lambda}_z{}^2\right]$$

$$\Phi_z = 0.5 \times \left[1 + 0.49 \times \left(0.75 - 0.2\right) + 0.75^2\right] = 0.92$$

$$\chi_z = \frac{1}{0.92 + \sqrt{0.92^2 - 0.75^2}} = 0.69$$

$$\chi_{min} = \chi_z = 0.69$$

$\chi_z = 0.69$

$\overline{\lambda}_z = 0.75$
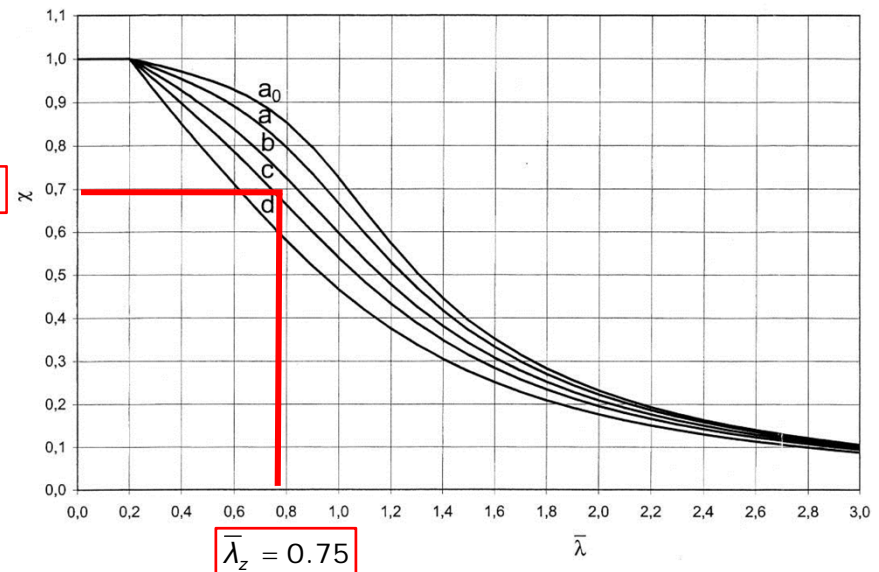
v) Safety verification

$$N_{b,Rd} = \chi_z\, Af_y / \gamma_{M1} = 0.69 \times 170.9 \times 10^{-4} \times 355 \times 10^3 / 1.0 = 4186.2\,\text{kN}$$

As, $N_{Ed} = 3326.0\,\text{kN} < N_{b,Rd} = 4186.2\,\text{kN}$

safety is verified with the cross section HEB 340 in S 355 steel.

DESIGN OF BEAMS

Beam cross sections and applications

- A beam may be defined as a member subjected essentially to bending and shear force.
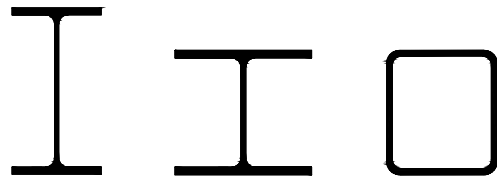
Hot-rolled sections (IPE, HEA or HEB, RHS,....)

Castellated beams

Welded sections

Welded sections in non-uniform beams

DESIGN OF BEAMS

Cross section resistance

Uniaxial bending (clause 6.2.5 of EC3-1-1)

$$\frac{M_{Ed}}{M_{c.Rd}} \leq 1.0$$

- Class 1 or 2 $M_{c.Rd} = W_{pl}\, f_y / \gamma_{M0}$

- Class 3 $M_{c.Rd} = W_{el.min}\, f_y / \gamma_{M0}$

- Class 4 $M_{c.Rd} = W_{eff.min}\, f_y / \gamma_{M0}$

Long Carbon Europe

ArcelorMittal

**Profilés et Aciers Marchands
Sections and Merchant Bars
Profil- und Stabstahl**

Programme de Vente / Sales Programme / Verkaufsprogramm

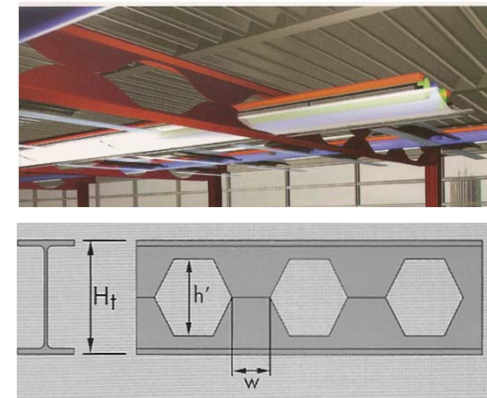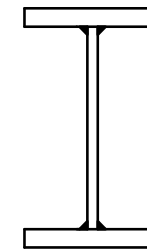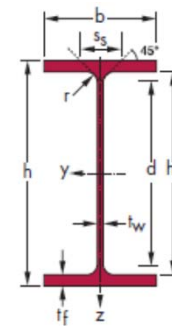| Désignation Designation Bezeichnung		axe fort y-y strong axis y-y starke Achse y-y					axe faible z-z weak axis z-z schwache Achse z-z						
	G	I_y	$W_{el,y}$	$W_{pl,y}$	i_y	A_{vz}	I_z	$W_{el,z}$	$W_{pl,z}$	i_z	s_s	I_t	I_w
	kg/m	mm⁴	mm³	mm³	mm	mm²	mm⁴	mm³	mm³	mm	mm	mm⁴	mm⁶
		x10⁴	x10³	x10³	x10	x10²	x10⁴	x10³	x10³	x10		x10⁴	x10⁹
IPE AA 240	24,9	3154	267	298	9,97	15,3	231	38,6	60,0	2,70	38,4	7,33	30,1
IPE A 240	26,2	3290	278	312	9,94	16,3	240	40,0	62,4	2,68	39,4	8,35	31,3
IPE 240	30,7	3892	324	367	9,97	19,1	284	47,3	73,9	2,69	43,4	12,9	37,4
IPE O 240	34,3	4369	361	410	10,0	21,4	329	53,9	84,4	2,74	46,2	17,2	43,7
IPE A 270	30,7	4917	368	413	11,2	18,8	358	53,0	82,3	3,02	40,5	10,3	59,5
IPE 270	36,1	5790	429	484	11,2	22,1	420	62,2	97,0	3,02	44,6	15,9	70,6
IPE O 270	42,3	6947	507	575	11,4	25,2	514	75,5	118	3,09	49,5	24,9	87,6

Bi-axial bending (clause 6.2.9 of EC3.1.1)

$$\left[\frac{M_{y,Ed}}{M_{pl,y.Rd}}\right]^{\alpha} + \left[\frac{M_{z,Ed}}{M_{pl,z.Rd}}\right]^{\beta} \leq 1.0$$

I or H $\alpha = 2;\ \beta = 5n$ but $\beta \geq 1$

CHS $\alpha = \beta = 2$

RHS $\alpha = \beta = \dfrac{1.66}{1 - 1.13\,n^2}$ but $\alpha = \beta \leq 6$

$$n = N_{Ed} / N_{pl,Rd}$$

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DESIGN OF BEAMS

Cross section resistance

Shear (clause 6.2.6 of EC3-1-1)

$$\frac{V_{Ed}}{V_{c,Rd}} \le 1.0$$

PLASTIC RESISTANCE $V_{pl.Rd}$

$$V_{pl.Rd} = A_v \left(f_y / \sqrt{3} \right) / \gamma_{M0}$$

A_v – Shear area
(obtained from clause 6.2.6 (3) of EC3-1-1 or from tables of profiles).

V_{Ed}

A_v

$\left(f_y / \sqrt{3} \right)$
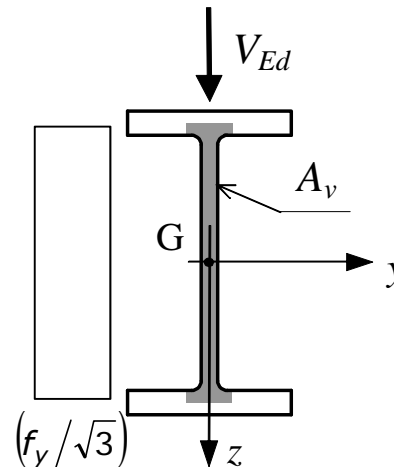
ELASTIC RESISTANCE

$$\frac{\tau_{Ed}}{f_y / \left(\sqrt{3}\, \gamma_{M0} \right)} \le 1.0 \qquad \tau_{Ed} = \frac{V_{Ed}\, S}{I\, t}$$

V_z

M_y G   e. n. a.

τ

$\left(f_y / \sqrt{3} \right)$

Shear stresses - τ
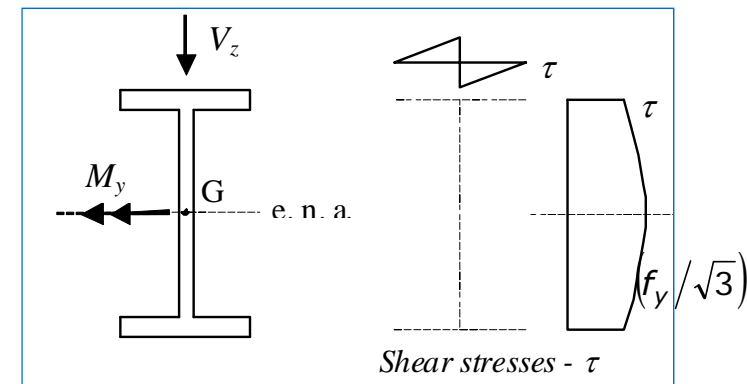
- <u>Shear buckling</u> for webs without stiffeners should be verified in accordance with EC3-1-5, if:

$$\frac{h_w}{t_w} > 72\, \frac{\varepsilon}{\eta} \qquad \varepsilon = \sqrt{235 / f_y}$$

h_w and t_w are the height and thickness of the web and η is in accordance with EC3-1-5.

DESIGN OF BEAMS

Cross section resistance

Bending and Shear Interaction

(clause 6.2.8 of EC3-1-1)

$V_{Ed} \leq 50\% V_{pl,Rd}$ **NO REDUCTION**

$V_{Ed} > 50\% V_{pl,Rd}$ **REDUCED MOMENT**

$$f_{yr} = (1 - \rho)f_y \qquad \rho = (2V_{Ed}/V_{pl.Rd} - 1)^2$$

For **I and H cross sections of equal flanges**, with bending about the major axis y, the bending moment resistance $M_{y,V,Rd}$ is given by (clause 6.2.8 of EC3-1-1):

$$M_{y,V.Rd} = \left(W_{pl,y} - \frac{\rho A_w^{\;2}}{4 t_w} \right) \frac{f_y}{\gamma_{M0}} \leq M_{y,c,Rd} \qquad A_W = h_w t_w$$
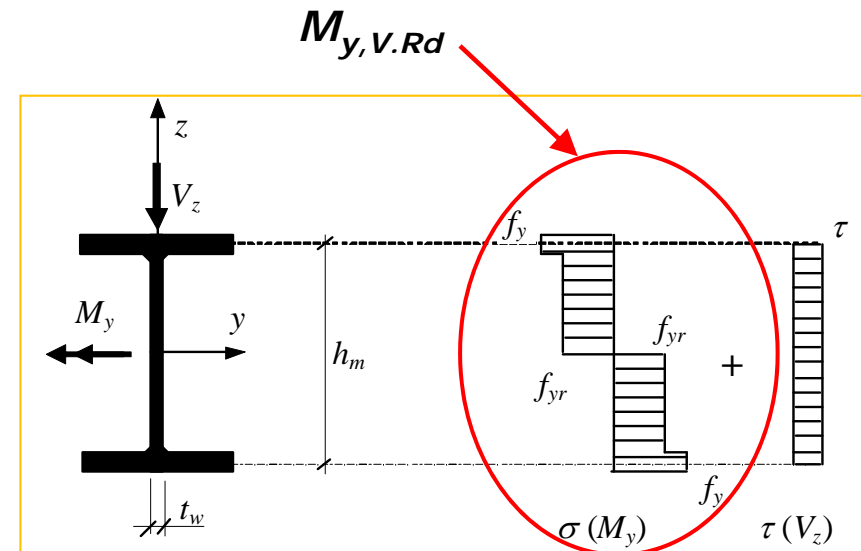
DESIGN OF BEAMS

Lateral-Torsional Buckling

▪ Instability phenomenon characterized by the occurrence of large transversal displacements and rotation about the member axis, under bending moment about the major axis (y axis).

▪ This instability phenomenon involves lateral bending (about z axis) and torsion of cross section.
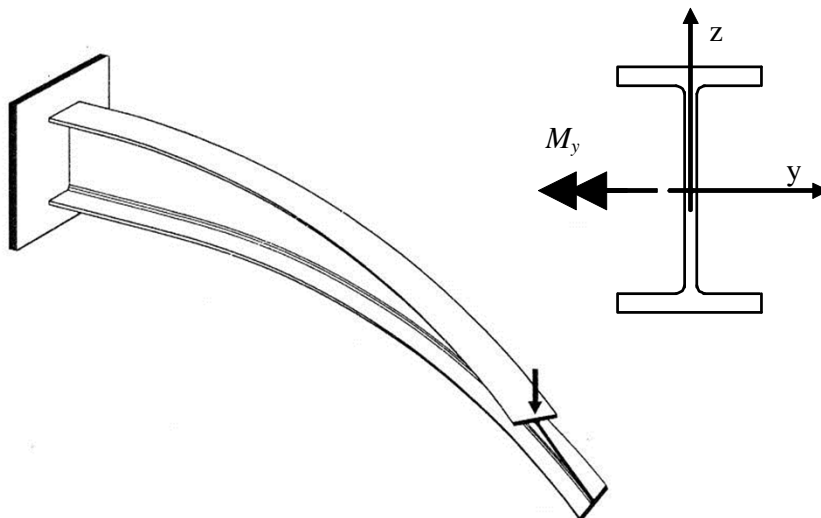
DESIGN OF BEAMS

Lateral-Torsional Buckling

▪ In the study of lateral-torsional buckling of beams, the **Elastic Critical Moment M_{cr}** plays a fundamental role; this quantity is defined as the maximum value of bending moment supported by a beam, free from any type of imperfections.

▪ For a simple supported beam with a double symmetric section, with supports prevent lateral displacements and rotation around member axis (twist rotations), but allowing warping and rotations around cross section axis (y and z), submitted to a uniform bending moment M_y ("standard case"), the **elastic critical moment** is given by:

a) Elevation
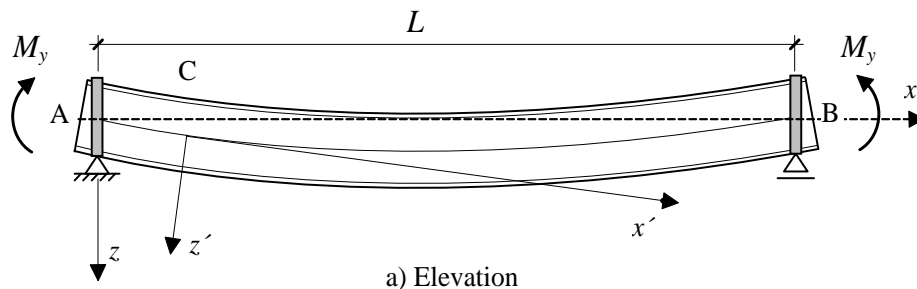
$$M_{cr}^E = \frac{\pi}{L}\sqrt{G\,I_T\,E\,I_z\left(1 + \frac{\pi^2\,E\,I_W}{L^2 G\,I_T}\right)}$$

Which depend mainly of:
Loading and support conditions;
Length between lateral braced sections (L);
Lateral bending stiffness ($E\,I_z$);
Torsional stiffness ($G\,I_T$);
Warping stiffness ($E\,I_w$).

DESIGN OF BEAMS

Lateral-Torsional Buckling

Elastic critical moment
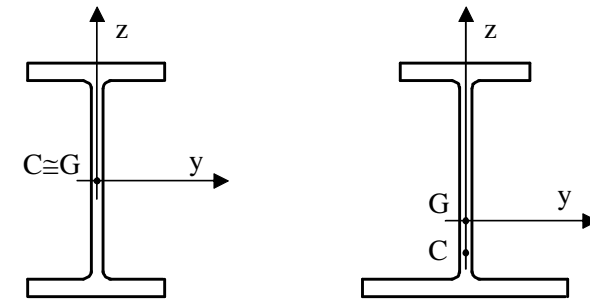
$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2} \left\{ \left[\left(\frac{k_z}{k_w} \right)^2 \frac{I_W}{I_z} + \frac{(k_z L)^2 G I_T}{\pi^2 E I_z} + \left(C_2 z_g - C_3 z_j \right)^2 \right]^{0.5} - \left(C_2 z_g - C_3 z_j \right) \right\}$$

$$z_g = \left(z_a - z_s \right)$$

$$z_j = z_s - \left(0.5 \int_A \left(y^2 + z^2 \right) z \, dA \right) \Big/ I_y$$

$M_{cr,1} > M_{cr}$ M_{cr} $M_{cr,2} < M_{cr}$
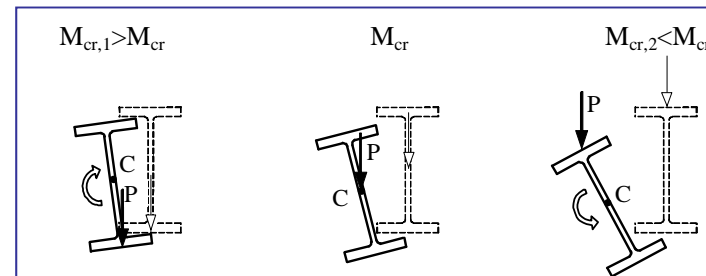
- applicable to member with symmetric and mono-symmetric cross sections,
- include the effects of the loading applied below or above the shear centre;
- several degrees of restriction to lateral bending (k_z) and warping (k_w);
- several shapes of bending moment diagram (C_1, C_2 and C_3 in the next tables).

DESIGN OF BEAMS

Lateral-Torsional Buckling

Loading and support conditions	Diagram of moments	k_z	C_1	C_2	C_3
p		1.0	1.12	0.45	0.525
		0.5	0.97	0.36	0.478
P		1.0	1.35	0.59	0.411
		0.5	1.05	0.48	0.338
P P		1.0	1.04	0.42	0.562
$d\;d\;d\;d$		0.5	0.95	0.31	0.539

Elastic critical moment

- **Publication n° 119 do ECCS (Boissonnade et al. 2006).**

- **LTBeam software**
http://www.cticm.com

LTBeam

Loading and support conditions	Diagram of moments	k_z	C_1	C_3 $\psi_f \leq 0$	C_3 $\psi_f > 0$
$\Psi = +1$		1.0	1.00	1.000	
		0.5	1.05	1.019	
$\Psi = +3/4$		1.0	1.14	1.000	
		0.5	1.19	1.017	
$\Psi = +1/2$		1.0	1.31	1.000	
		0.5	1.37	1.000	
$\Psi = +1/4$		1.0	1.52	1.000	
		0.5	1.60	1.000	
$\Psi = 0$		1.0	1.77	1.000	
		0.5	1.86	1.000	
$\Psi = -1/4$		1.0	2.06	1.000	0.850
		0.5	2.15	1.000	0.650
$\Psi = -1/2$		1.0	2.35	1.000	$1.3 - 1.2\psi_f$
		0.5	2.42	0.950	$0.77 - \psi_f$
$\Psi = -3/4$		1.0	2.60	1.000	$0.55 - \psi_f$
		0.5	2.45	0.850	$0.35 - \psi_f$
$\Psi = -1$		1.0	2.60	$-\psi_f$	$-\psi_f$
		0.5	2.45	$-0.125 - 0.7\psi_f$	$-0.125 - 0.7\psi_f$

M ΨM

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DESIGN OF BEAMS

Lateral-Torsional Buckling

Lateral-torsional buckling resistance (clause 6.3.2 of EC3-1-1)

$$\frac{M_{Ed}}{M_{b.Rd}} \leq 1.0$$

$$M_{b.Rd} = \chi_{LT} W_y f_y / \gamma_{M1}$$

$W_y = W_{pl.y}$ Class 1 and 2;

$W_y = W_{el.y}$ Class 3;

$W_y = W_{eff.y}$ Class 4.

χ_{LT} is the **reduction factor** for lateral-torsional buckling, which can be calculated by one of two methods, depending of member cross section.

DESIGN OF BEAMS

Lateral-Torsional Buckling

i) General method

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \left(\Phi_{LT}^2 - \bar{\lambda}_{LT}^2\right)^{0.5}}$$ $\chi_{LT} \leq 1.0$

$$\Phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right]$$

Table 6.3: Recommended values for imperfection factors for lateral torsional buckling curves

Buckling curve	a	b	c	d
Imperfection factor α_{LT}	0,21	0,34	0,49	0,76

$$\bar{\lambda}_{LT} = \left[W_y\, f_y / M_{cr}\right]^{0.5}$$

Table 6.4 - Buckling curves for lateral-torsional buckling (General method)

Section	Limits	Buckling curve
I or H sections rolled	$h/b \leq 2$	a
	$h/b > 2$	b
I or H sections welded	$h/b \leq 2$	c
	$h/b > 2$	d
Other sections	---	d

M_{cr} - Elastic critical moment

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DESIGN OF BEAMS

Lateral-Torsional Buckling

ii) Alternative method (rolled sections or equivalent welded sections)

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \left(\Phi_{LT}^2 - \beta\,\overline{\lambda}_{LT}^2\right)^{0.5}}$$

$$\chi_{LT} \leq 1.0$$
$$\chi_{LT} \leq 1/\overline{\lambda}_{LT}^2$$

$$\overline{\lambda}_{LT,0} \leq 0.4$$
$$\beta \geq 0.75$$

$$\Phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\overline{\lambda}_{LT} - \overline{\lambda}_{LT,0}\right) + \beta\,\overline{\lambda}_{LT}^2\right]$$

(may be specified in National Annexes of Eurocode 3)

$$\overline{\lambda}_{LT} = \left[W_y\, f_y / M_{cr}\right]^{0.5}$$

M_{cr} - Elastic critical moment

Table 6.5 - Buckling curves for lateral-torsional buckling (Alternative method)

Section		Limits	Buckling curve (EC3-1-1)
I or H sections	rolled	$h/b \leq 2$	b
		$h/b > 2$	c
I or H sections	welded	$h/b \leq 2$	c
		$h/b > 2$	d

DESIGN OF BEAMS

Lateral-Torsional Buckling

$$\chi_{LT,mod} = \frac{\chi_{LT}}{f}$$

$\chi_{LT,mod} \leq 1.0$
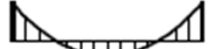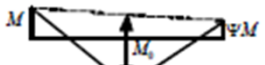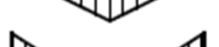
$$f = 1 - 0.5(1 - k_c)\left[1 - 2.0\left(\overline{\lambda}_{LT} - 0.8\right)^2\right]$$

$f \leq 1.0$

Neglect LTB if:

$\overline{\lambda}_{LT} \leq \overline{\lambda}_{LT,0}$

$M_{Ed}/M_{cr} \leq \overline{\lambda}_{LT,0}^2$

Diagram of bending moments	k_c
$\Psi = +1$	1.0
$-1 \leq \Psi \leq 1$	$\dfrac{1}{1.33 - 0.33\Psi}$
	0.94
	0.90
	0.91
	0.86
	0.77
	0.82
Ψ - ratio between end moments, with -1≤Ψ≤1.	

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DESIGN OF BEAMS

EXAMPLE 2

Safety check of a beam of the building illustrated in the figure (along line E). The beam is composed by a IPE 600 with 9 m length at the central span; the lateral spans with 6 m length (the governing spans) are composed by a section IPE 400 in steel S 355. For the lateral buckling check, two cases are considered:

a) a beam with 6 m length, laterally braced only at the end support sections;

b) a beam with 6 m length, laterally braced at the end support sections and at mid-span section.

The geometrical and mechanical properties of the section IPE 400 in S 355 steel are:

A = 84.46 cm², b = 180 mm, h = 400 mm,
t_f = 13.5 mm, t_w = 8.6 mm, I_y = 23130 cm⁴,
i_y = 16.55 cm, I_z = 1318 cm⁴, i_z = 3.95 cm,
I_T = 51.08 cm⁴ ; Iw = 490x10³ cm⁶;
f_y = 355 MPa and E = 210 GPa.
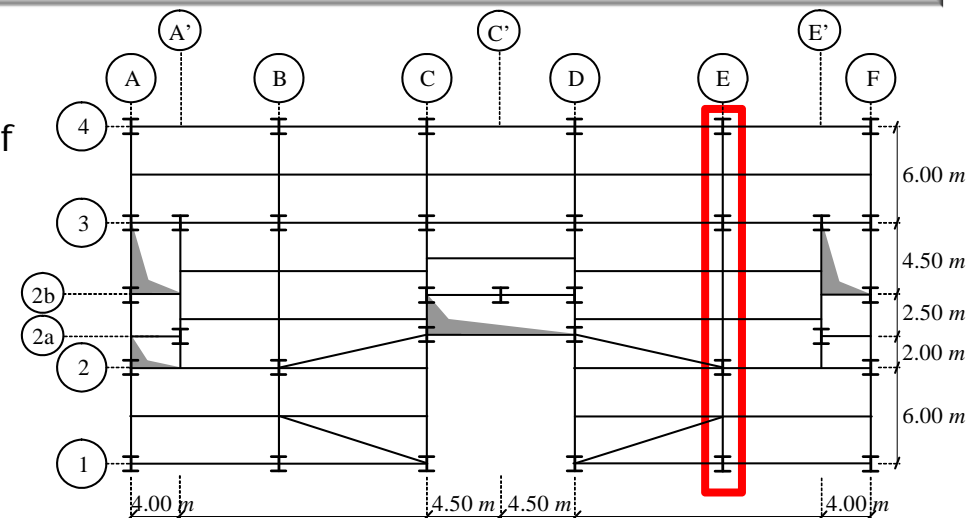
Building plan – master example

DESIGN OF BEAMS

EXAMPLE 2

a) Beam laterally braced at supports

i) The internal forces (neglecting the axial force) are represented in the figure. The design values are M_{Ed} = **114.3 kNm** and V_{Ed} = **75.9 kN**.
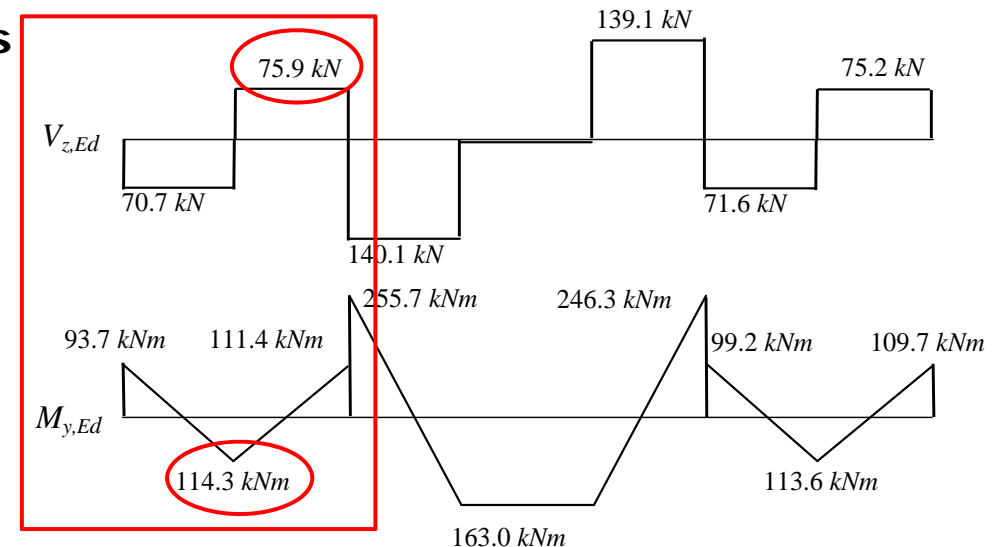
ii) Cross section classification
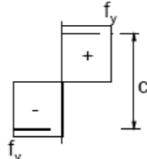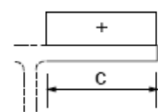
Web (an internal part) in bending:

$$\frac{c}{t} = \frac{331}{8.6} = 38.49 < 72\,\varepsilon = 72 \times 0.81 = 58.32$$

Flange (outstand part) in compression:

$$\frac{c}{t} = \frac{(180 - 2 \times 21 - 8.6)/2}{13.5} = 4.79 < 9\,\varepsilon = 9 \times 0.81 = 7.29$$

The cross section is class 1

$V_{z,Ed}$

139.1 kN
75.9 kN
75.2 kN
70.7 kN
140.1 kN
71.6 kN

255.7 kNm   246.3 kNm
93.7 kNm   111.4 kNm   99.2 kNm   109.7 kNm

$M_{y,Ed}$

114.3 kNm   113.6 kNm
163.0 kNm

Class	Part subject to bending		Class	Part subject to compression
Stress distribution in parts (compression positive)		c	Stress distribution in parts (compression positive)	c
1	$c/t \le 72\varepsilon$		1	$c/t \le 9\varepsilon$

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DESIGN OF BEAMS

EXAMPLE 2

iii) Cross section verification

Bending resistance:

$$M_{Ed} = 114.3 \text{ kNm} < W_{pl,y}\, f_y / \gamma_{M0} = 1307 \times 10^{-6} \times 355 \times 10^3 / 1.0 = 464.0\,kNm$$

Shear resistance:

$$V_{Ed} = 75.9 \text{ kN} < V_{pl,Rd} = \frac{A_v\, f_y}{\gamma_{M0}\,\sqrt{3}} = \frac{42.69 \times 10^{-4} \times 355 \times 10^3}{1.0 \times \sqrt{3}} = 875.0 \text{ kN}$$

$$\frac{h_w}{t_w} = \frac{373.0}{8.6} = 43.4 < 72\,\frac{\varepsilon}{\eta} = 72 \times \frac{0.81}{1.0} = 58.3$$

So, it is not necessary to verify the shear buckling resistance.

Bending + Shear:

$$V_{Ed} = 75.9 \text{ kN} < 0.50 \times V_{pl,Rd} = 0.50 \times 875.0 = 437.5 \text{ kN}$$

So, it is not necessary to reduce the bending resistance to account for the shear force.

Cross section resistance is verified.

DESIGN OF BEAMS

EXAMPLE 2

iv) Lateral buckling resistance

Assuming the support conditions of the "standard case" and the loading applied at the upper flange level, the **elastic critical moment** can be obtained from the following equation, with $L = 6.00$ m, $k_z = k_w = 1.0$, $C_1 \approx 1.80$ and $C_2 \approx 1.60$ (Boissonnade et al., 2006) and $z_g = 200$ mm.

$z_g = 200$ mm

$C \cong G$

400

180

IPE 400

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2} \left\{ \left[\left(\frac{k_z}{k_w} \right)^2 \frac{I_W}{I_z} + \frac{(k_z L)^2 G I_T}{\pi^2 E I_z} + \left(C_2 z_g - C_3 z_j \right)^2 \right]^{0.5} - \left(C_2 z_g - C_3 z_j \right) \right\}$$

$M_{cr} = 164.7 \ kNm$

A   C

3 m   3 m

6 m

Critical Moment

| Critical value of maximum moment | Mcr | = | 175,64 | kN.m |
| Abscissa/L | xf | = | 0,500 | |

Eigenmode

— v   — θ   — v'   — θ'

(Using LTBeam
−> M_{cr} = 175.64 kNm)

93.7 kNm   111.4 kNm

$M_{y,Ed}$

114.3 kNm
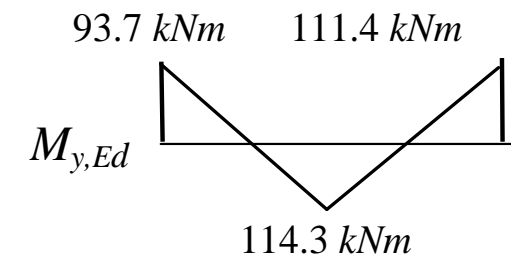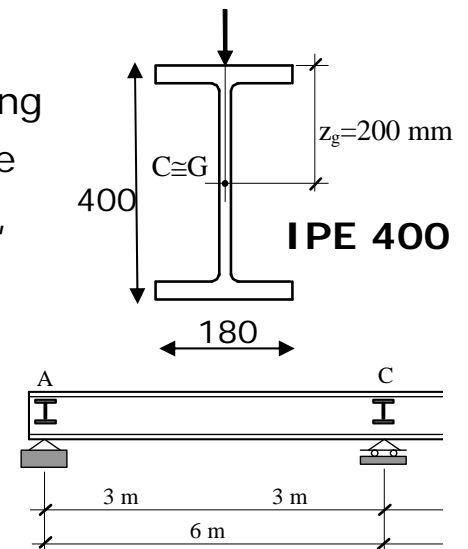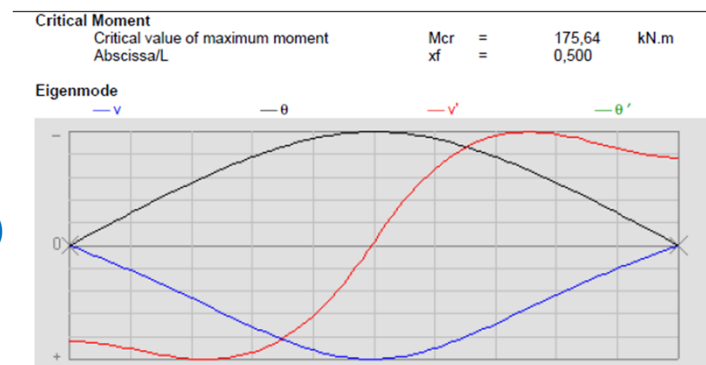
$\Psi = 93.7/111.4 = 0.84$

DESIGN OF BEAMS

EXAMPLE 2

$$M_{cr} = 164.7\,kNm; \quad W_y = W_{pl,y} = 1307\,cm^3 \quad \Rightarrow \quad \overline{\lambda}_{LT} = 1.68$$

$$\overline{\lambda}_{LT} = \left[W_y\,f_y\,/M_{cr}\right]^{0.5}$$

General method:

Rolled cross section IPE 400 with

h/b=400/180=2.2>2   - Curve *b*

$$\Rightarrow \quad \alpha_{LT} = 0.34$$

$$\Phi_{LT} = 2.16$$

$$\Rightarrow \quad \chi_{LT} = 0.28$$

$$M_{b,Rd} = 0.28 \times 1307 \times 10^{-6} \times \frac{355 \times 10^3}{1.0} = 129.9 \text{ kNm} > 114.3 \text{ kNm}$$

So, the safety is verified (utilization ratio = 114.3/129.9=0.88).

Table 6.4 - Buckling curves for lateral-torsional buckling (General method)

Section		Limits	Buckling curve
I or H sections rolled		$h/b \le 2$	a
		$h/b > 2$	b
I or H sections welded		$h/b \le 2$	c
		$h/b > 2$	d
Other sections		---	d

Buckling curve	a	b	c	d
Imperfection factor α_{LT}	0,21	0,34	0,49	0,76

$$\Phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\overline{\lambda}_{LT} - 0.2\right) + \overline{\lambda}_{LT}^{\,2}\right]$$

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \left(\Phi_{LT}^{\,2} - \overline{\lambda}_{LT}^{\,2}\right)^{0.5}}$$

DESIGN OF BEAMS

EXAMPLE 2

b) Beam laterally braced at supports and <u>mid-span</u>
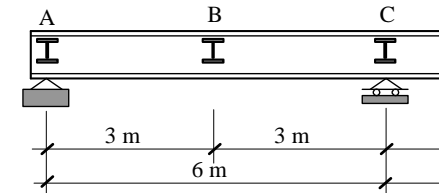
i) Cross section verifications are not changed.

ii) Lateral buckling check:

As the beam is laterally braced at mid span cross section, the **critical moment** can be evaluated with L = 3.00 m and a conservative hypothesis of $k_z = k_w$ = 1.0. For the given bending moment shape between lateral braced cross sections, C_1 = 2.6 (Boissonnade et al., 2006) .

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2} \left\{ \left[\left(\frac{k_z}{k_w} \right)^2 \frac{I_W}{I_z} + \frac{(k_z L)^2 G I_T}{\pi^2 E I_z} + (C_2 z_g - C_3 z_j)^2 \right]^{0.5} - (C_2 z_g - C_3 z_j) \right\}$$

$$M_{cr} = 1778.8 \, kNm$$
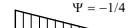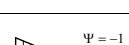
(Using LTBeam – M_{cr} = 1967.7 kNm)

$M_{y,Ed}$

93.7 kNm

Ψ = 93.7/114.3 = 0.82

114.3 kNm

$\Psi = -1/4$	1.0	2.06	1.000	0.850
	0.5	2.15	1.000	0.650
$\Psi = -1/2$	1.0	2.35	1.000	$1.3 - 1.2\psi_f$
	0.5	2.42	0.950	$0.77 - \psi_f$
$\Psi = -3/4$	1.0	2.60	1.000	$0.55 - \psi_f$
	0.5	2.45	0.850	$0.35 - \psi_f$
$\Psi = -1$	1.0	2.60	$-\psi_f$	$-\psi_f$
	0.5	2.45	$-0.125 - 0.7\psi_f$	$-0.125 - 0.7\psi_f$

DESIGN OF BEAMS

EXAMPLE 2

$$M_{cr} = 1778.8 \, kNm; \quad W_y = W_{pl,y} = 1307 \, cm^3 \quad \Rightarrow \quad \overline{\lambda}_{LT} = 0.51 \qquad \boxed{\overline{\lambda}_{LT} = \left[W_y f_y / M_{cr}\right]^{0.5}}$$

Table 6.4 - Buckling curves for lateral-torsional buckling (General method)

Section		Limits	Buckling curve
I or H sections rolled		$h/b \leq 2$	a
		$h/b > 2$	b
I or H sections welded		$h/b \leq 2$	c
		$h/b > 2$	d
Other sections		---	d

General method:

Rolled cross section IPE 400 with

$h/b = 400/180 = 2.2 > 2$ - Curve b

$\Rightarrow \quad \alpha_{LT} = 0.34$

Buckling curve	a	b	c	d
Imperfection factor α_{LT}	0,21	0,34	0,49	0,76

$\Phi_{LT} = 0.68$

$\Rightarrow \quad \chi_{LT} = 0.89$

$$\boxed{\Phi_{LT} = 0.5 \left[1 + \alpha_{LT} \left(\overline{\lambda}_{LT} - 0.2\right) + \overline{\lambda}_{LT}^2\right]}$$

$$M_{b,Rd} = 0.89 \times 1307 \times 10^{-6} \times \frac{355 \times 10^3}{1.0} = 412.9 \text{ kNm} > 114.3 \text{ kNm}$$

$$\boxed{\chi_{LT} = \frac{1}{\Phi_{LT} + \left(\Phi_{LT}^2 - \overline{\lambda}_{LT}^2\right)^{0.5}}}$$

So, the safety is verified (utilization ratio = 114.3/412.9=0.28).

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DESIGN OF BEAM-COLUMNS

Cross section resistance (clause 6.2.9 of EC3-1-1)

▪ **Class 1 or 2 – Uniaxial bending**

$$\boxed{M_{Ed} \leq M_{N,Rd}}$$

Double-symmetric I or H sections

$$M_{N,y,Rd} = M_{pl,y,Rd} \frac{1-n}{1-0.5 \cdot a} \quad \text{but} \quad M_{N,y,Rd} \leq M_{pl,y,Rd}$$

$$M_{N,z,Rd} = M_{pl,z,Rd} \qquad \text{if} \quad n \leq a$$

$$M_{N,z,Rd} = M_{pl,z,Rd}\left[1 - \left(\frac{n-a}{1-a}\right)^2\right] \quad \text{if} \quad n > a$$

$$n = N_{Ed}/N_{pl.Rd} \qquad a = (A - 2\,b\,t_f)/A \leq 0.50$$

No reduction if

$$N_{Ed} \leq 0.25\,N_{pl,Rd}$$

$$\left.\begin{array}{l} N_{Ed} \leq 0.5\,h_w\,t_w\,f_y/\gamma_{M0} \end{array}\right\} \text{(y axis)}$$

$$N_{Ed} \leq h_w\,t_w\,f_y/\gamma_{M0} \quad \text{(z axis)}$$

Bending about minor axis - z

Bending about major axis - y

HEA
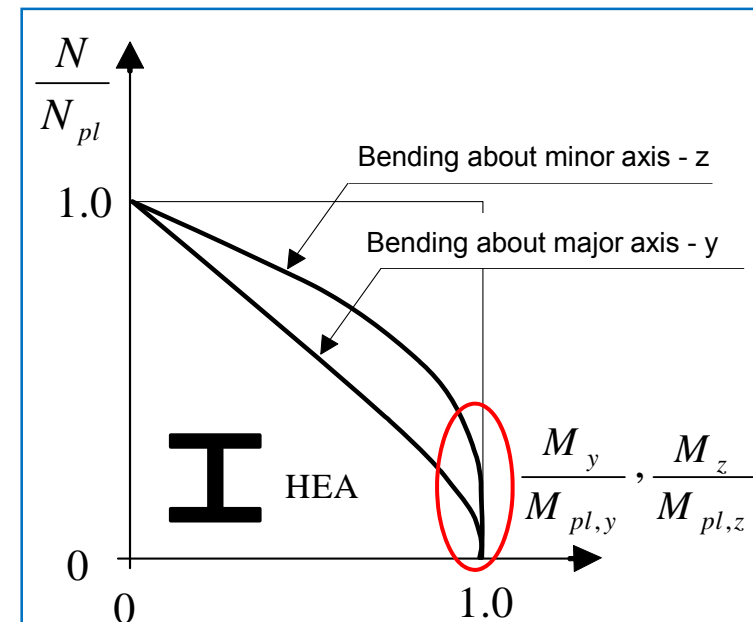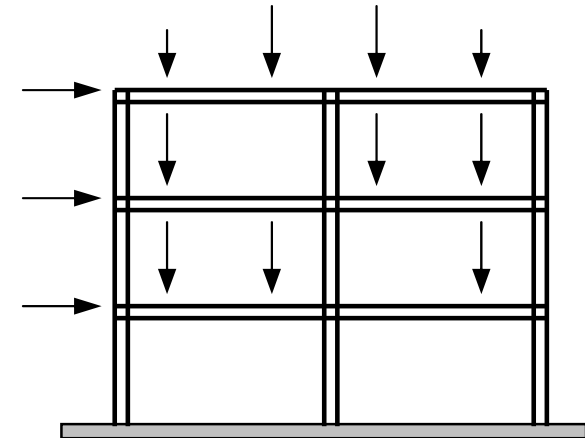
DESIGN OF BEAM-COLUMNS

Cross section resistance (clause 6.2.9 of EC3-1-1)

- **Class 1 or 2 – Bi-axial bending**

$$\left[\frac{M_{y,Ed}}{M_{N,y.Rd}}\right]^{\alpha} + \left[\frac{M_{z,Ed}}{M_{N,z.Rd}}\right]^{\beta} \leq 1.0$$

$$n = N_{Ed}/N_{pl,Rd}$$

I or H $\quad \alpha = 2; \; \beta = 5\,n \quad$ but $\quad \beta \geq 1$

Circular hollow sections $\quad \alpha = \beta = 2$

Rectangular hollow sections $\quad \alpha = \beta = \dfrac{1.66}{1-1.13\,n^2} \leq 6$
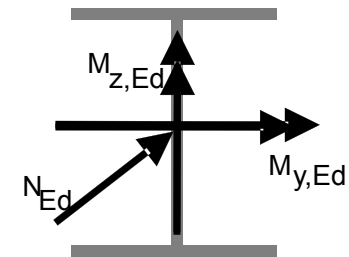
- **Class 3 or 4** $\quad \boxed{\sigma_{x,Ed} \leq \dfrac{f_y}{\gamma_{M0}}}$

$$\sigma_{x,Ed} = \frac{N_{Ed}}{A} + \frac{M_{y,Ed}}{I_y}z + \frac{M_{z,Ed}}{I_z}y$$

$M_{z,Ed}$

N_{Ed}

$M_{y,Ed}$

Bending, shear and axial force (clause 6.2.10 of EC3-1-1) – Similar to bending and shear interaction.

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DESIGN OF BEAM-COLUMNS

Member stability

Members with high slenderness subjected to bending and compression, may fail by flexural buckling or lateral-torsional buckling.

Flexural buckling and **lateral-torsional buckling** (doubly-symmetric cross-section):

$$\frac{N_{Ed}}{\chi_y \, N_{Rk}/\gamma_{M1}} + k_{yy} \frac{M_{y.Ed} + \Delta M_{y,Ed}}{\chi_{LT} \, M_{y,Rk}/\gamma_{M1}} + k_{yz} \frac{M_{z.Ed} + \Delta M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \leq 1.0$$ (Eq. 6.61 of EC3-1-1)

$e_{N,y} \, N_{Ed}$ (class 4)

$$\frac{N_{Ed}}{\chi_z \, N_{Rk}/\gamma_{M1}} + k_{zy} \frac{M_{y.Ed} + \Delta M_{y,Ed}}{\chi_{LT} \, M_{y,Rk}/\gamma_{M1}} + k_{zz} \frac{M_{z.Ed} + \Delta M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \leq 1.0$$ (Eq. 6.62 of EC3-1-1)

k_{yy}, k_{yz}, k_{zy} and k_{zz} - interaction factors, which are dependent of instability phenomena and plasticity – **Annex A** of EC3-1-1 (Method 1) or **Annex B** (Method 2).

DESIGN OF BEAM-COLUMNS

Member stability

i) Members with closed hollow sections or open sections restrained to torsion are **not susceptible** to torsional deformation.

ii) Members with open sections (I or H sections) are **susceptible** to torsional deformation.

Members not susceptible to torsional deformation – checking of flexural buckling against *y-axis* and *z-axis*, considering eqs. (6.61) and (6.62) with χ_{LT} = 1.0 and interaction factors k_{yy}, k_{yz}, k_{zy} and k_{zz} in members not susceptible to torsional deformation.

Members susceptible to torsional deformation – checking of lateral-torsional buckling, considering eqs (6.61) and (6.62) with χ_{LT} according to 6.3.2 of EC3-1-1 and interaction factors k_{yy}, k_{yz}, k_{zy} and k_{zz} in members susceptible to torsional deformation.

DESIGN OF BEAM-COLUMNS

Member stability

Method 2
(Annex B of EC3-1-1)

Interaction factors for members **not susceptible** to torsional deformations (Table B.1 of EC3-1-1).

Interaction factors	Type of section	Elastic sectional properties (Class 3 or 4 sections)	Plastic sectional properties (Class 1 or 2 sections)
k_{yy}	I or H sections and rectangular hollow sections	$C_{my}\left(1+0.6\,\overline{\lambda}_y\dfrac{N_{Ed}}{\chi_y\,N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{my}\left(1+0.6\dfrac{N_{Ed}}{\chi_y\,N_{Rk}/\gamma_{M1}}\right)$	$C_{my}\left(1+\left(\overline{\lambda}_y-0.2\right)\dfrac{N_{Ed}}{\chi_y\,N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{my}\left(1+0.8\dfrac{N_{Ed}}{\chi_y\,N_{Rk}/\gamma_{M1}}\right)$
k_{yz}	I or H sections and rectangular hollow sections	k_{zz}	$0.6\,k_{zz}$
k_{zy}	I or H sections and rectangular hollow sections	$0.8\,k_{yy}$	$0.6\,k_{yy}$
k_{zz}	I or H sections	$C_{mz}\left(1+0.6\,\overline{\lambda}_z\dfrac{N_{Ed}}{\chi_z\,N_{Rk}/\gamma_{M1}}\right)$	$C_{mz}\left(1+\left(2\,\overline{\lambda}_z-0.6\right)\dfrac{N_{Ed}}{\chi_z\,N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{mz}\left(1+1.4\dfrac{N_{Ed}}{\chi_z\,N_{Rk}/\gamma_{M1}}\right)$
	rectangular hollow sections	$\leq C_{mz}\left(1+0.6\dfrac{N_{Ed}}{\chi_z\,N_{Rk}/\gamma_{M1}}\right)$	$C_{mz}\left(1+\left(\overline{\lambda}_z-0.2\right)\dfrac{N_{Ed}}{\chi_z\,N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{mz}\left(1+0.8\dfrac{N_{Ed}}{\chi_z\,N_{Rk}/\gamma_{M1}}\right)$

In I or H sections and rectangular hollow sections under axial compression and uniaxial bending ($M_{y,Ed}$), k_{zy} may be taken as zero.

DESIGN OF BEAM-COLUMNS

Member stability

Method 2 (Annex B of EC3-1-1)

Interaction factors for members **susceptible** to torsional deformations (Table B.2 of EC3-1-1).

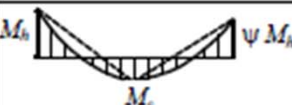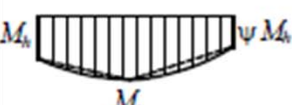| Interaction factors	Elastic sectional properties (Class 3 or 4 sections)	Plastic sectional properties (Class 1 or 2 sections)
k_{yy}	k_{yy} of Table 3.16	k_{yy} of Table 3.16
k_{yz}	k_{yz} of Table 3.16	k_{yz} of Table 3.16
k_{zy}	$\left[1 - \dfrac{0.05\bar{\lambda}_z}{(C_{mLT} - 0.25)} \dfrac{N_{Ed}}{\chi_z\, N_{Rk}/\gamma_{M1}}\right]$ $\geq \left[1 - \dfrac{0.05}{(C_{mLT} - 0.25)} \dfrac{N_{Ed}}{\chi_z\, N_{Rk}/\gamma_{M1}}\right]$	$\left[1 - \dfrac{0.1\bar{\lambda}_z}{(C_{mLT} - 0.25)} \dfrac{N_{Ed}}{\chi_z\, N_{Rk}/\gamma_{M1}}\right]$ $\geq \left[1 - \dfrac{0.1}{(C_{mLT} - 0.25)} \dfrac{N_{Ed}}{\chi_z\, N_{Rk}/\gamma_{M1}}\right]$ for $\bar{\lambda}_z < 0.4 : k_{zy} = 0.6 + \bar{\lambda}_z$ $\leq 1 - \dfrac{0.1\bar{\lambda}_z}{(C_{mLT} - 0.25)} \dfrac{N_{Ed}}{\chi_z\, N_{Rk}/\gamma_{M1}}$
k_{zz}	k_{zz} of Table 3.16	k_{zz} of Table 3.16

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DESIGN OF BEAM-COLUMNS

Member stability

Method 2 (Annex B of EC3-1-1)

Equivalent factors of uniform moment C_{mi} (Table B.3 of EC3-1-1)

- Equivalent factors of uniform moment C_{mi}

Diagram of moments	Range		C_{my}, C_{mz} and C_{mLT}	
			Uniform loading	Concentrated load
M ΨM	$-1 \leq \Psi \leq 1$		$0.6 + 0.4\Psi \geq 0.4$	
M_h ΨM_h M_s $\alpha_s = M_s/M_h$	$0 \leq \alpha_s \leq 1$	$-1 \leq \Psi \leq 1$	$0.2 + 0.8\alpha_s \geq 0.4$	$0.2 + 0.8\alpha_s \geq 0.4$
	$-1 \leq \alpha_s < 0$	$0 \leq \Psi \leq 1$	$0.1 - 0.8\alpha_s \geq 0.4$	$-0.8\alpha_s \geq 0.4$
		$-1 \leq \Psi < 0$	$0.1(1-\Psi) - 0.8\alpha_s \geq 0.4$	$0.2(-\Psi) - 0.8\alpha_s \geq 0.4$
M_h ΨM_h M_s $\alpha_h = M_h/M_s$	$0 \leq \alpha_h \leq 1$	$-1 \leq \Psi \leq 1$	$0.95 + 0.05\alpha_h$	$0.90 + 0.10\alpha_h$
	$-1 \leq \alpha_h < 0$	$0 \leq \Psi \leq 1$	$0.95 + 0.05\alpha_h$	$0.90 + 0.10\alpha_h$
		$-1 \leq \Psi < 0$	$0.95 + 0.05\alpha_h(1+2\Psi)$	$0.90 + 0.10\alpha_h(1+2\Psi)$

In the calculation of α_s or α_h parameters, a hogging moment should be taken as negative and a sagging moment should be taken as positive.

For members with sway buckling mode, the equivalent uniform moment factor should be taken as $C_{my} = 0.9$ or $C_{mz} = 0.9$, respectively.

Factors C_{my}, C_{mz} and C_{mLT} should be obtained from the diagram of bending moments between the relevant braced sections, according to the following:

| Moment factor	bending axis	points braced in direction
C_{my}	y-y	z-z
C_{mz}	z-z	y-y
C_{mLT}	y-y	y-y

DESIGN OF BEAM-COLUMNS

EXAMPLE 3

Safety check of a beam-column of the first storey of the building illustrated in the figure. The member, composed by a HEB 320 cross section in steel S 355, has a length of 4.335.

The relevant geometric characteristics of HEB 320 cross section are: $A = 161.3$ cm^2; $W_{pl,y} = 2149$ cm^3, $I_y = 30820$ cm^4, $i_y = 13.82$ cm; $I_z = 9239$ cm^4, $i_z = 7.57$ cm; $I_T = 225.1$ cm^4 and $I_W = 2069 \times 10^3$ cm^6.

The mechanical characteristics of the material are:
$f_y = 355$ MPa, $E = 210$ GPa and $G = 81$ GPa.

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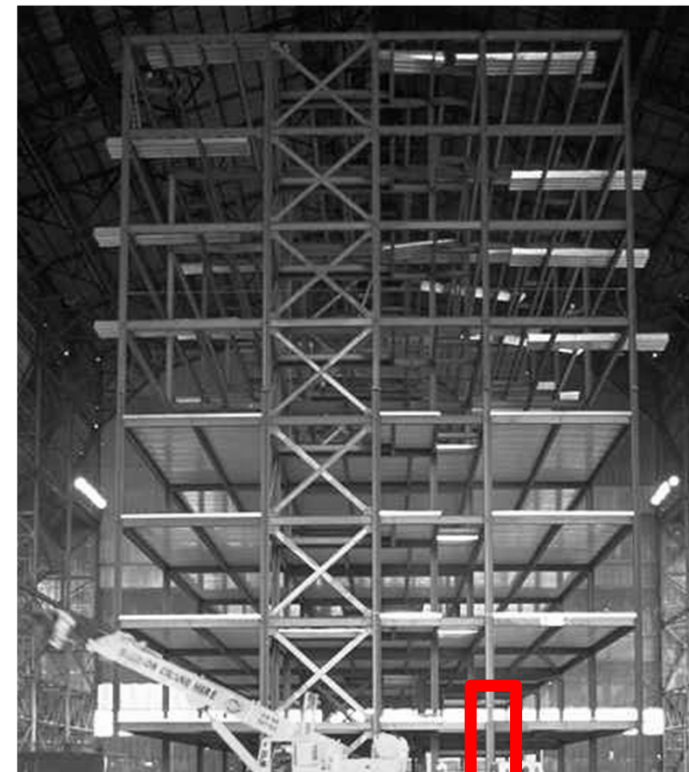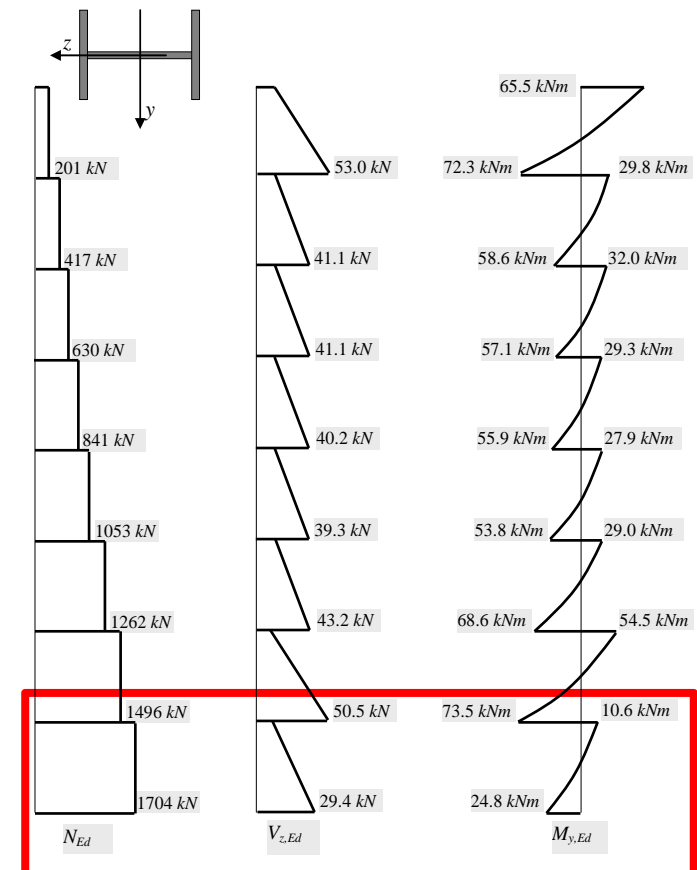
DESIGN OF BEAM-COLUMNS

EXAMPLE 3

The design internal forces obtained through the structure analysis (second order) for the various load combinations are illustrated in the figure. Two simplification are assumed for the subsequent design verifications: i) the shear force is sufficient small so can be neglected; ii) the shape of the bending moment diagram is linear.

Design values are: N_{Ed} = **1704 kN**; $M_{y,Ed}$ = **24.8 kNm** at the base cross section.

i) Cross section classification

As the compression force is high, the cross section is classified under compression only (conservative approach). As the section HEB 320 is a stocky section, even under this load condition, is **class 1.**

DESIGN OF BEAM-COLUMNS

EXAMPLE 3

ii) Cross section resistance

The design internal forces: $M_{y,Ed}$ = **24.6 kNm** and N_{Ed} = **1704 kN** (compression).

$$N_{pl,Rd} = A f_y / \gamma_{M0} = 161.3 \times 10^{-4} \times 355 \times 10^3 / 1.0 = 5726.2 \, \text{kN}$$

As, $N_{Ed} = 1704 \, kN < N_{pl,Rd} = 5726.2 \, \text{kN}$, the axial force resistance is verified.

Since, $N_{Ed} = 1704 \, \text{kN} > 0.25 N_{pl,Rd} = 1431.5 \, \text{kN}$,

in accordance with clause 6.2.9.1(4) of EC3-1-1, it is necessary to reduce the **plastic bending resistance** (to $M_{N,y,Rd}$):
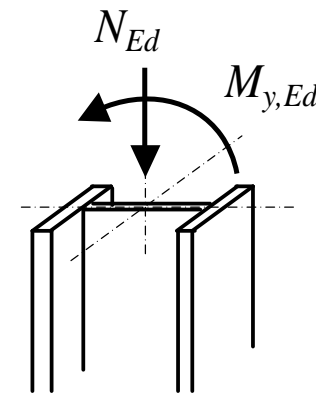
$$M_{pl,y,Rd} = \frac{W_{pl,y} \, f_y}{\gamma_{Mo}} = \frac{2149 \times 10^{-6} \times 355 \times 10^3}{1.0} = 762.9 \, kNm$$

$$M_{N,y,Rd} = M_{pl,y,Rd} \frac{1-n}{1-0.5\,a}$$

$$a = \frac{A - 2\,b\,t_f}{A} = \frac{161.3 - 2 \times 30 \times 2.05}{161.3} = 0.24$$

$$n = \frac{N_{Ed}}{N_{pl,Rd}} = \frac{1704}{5726.2} = 0.30$$

As, $M_{y,Ed} = 24.8 \, kNm < M_{N,y,Rd} = 606.9 \, \text{kNm}$, the bending resistance is verified.

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EXAMPLE 3

iii) Verification of the stability of the member

In this example the Method 2 is applied. As the member is susceptible to torsional deformations (thin-walled open cross section), it is assumed that lateral-torsional buckling constitutes the relevant instability mode. Since $M_{z,Ed} = 0$, the following conditions must be verified:

$$\frac{N_{Ed}}{\chi_y \, N_{Rk}/\gamma_{M1}} + k_{yy} \frac{M_{y,Ed}}{\chi_{LT} \, M_{y,Rk}/\gamma_{M1}} \leq 1.0$$

$$\frac{N_{Ed}}{\chi_z \, N_{Rk}/\gamma_{M1}} + k_{zy} \frac{M_{y,Ed}}{\chi_{LT} \, M_{y,Rk}/\gamma_{M1}} \leq 1.0$$

Step 1: characteristic resistance of the cross section

$N_{Rk} = A f_y = 161.3 \times 10^{-4} \times 355 \times 10^3 = 5726.2 \, \text{kN}$

$M_{y,Rk} = W_{pl,y} \, f_y = 2149 \times 10^{-6} \times 355 \times 10^3 = 762.9 \, \text{kNm}$

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EXAMPLE 3

Step 2: reduction coefficients due to flexural buckling, χ_y and χ_z

$$\frac{h}{b} = \frac{320}{300} = 1.07 < 1.2 \quad \text{and} \quad t_f = 20.5\,\text{mm} < 100\,\text{mm}$$

$$\Rightarrow \quad \begin{array}{l} \textit{flexural buckling around } y - \textit{curve } b\,(\alpha = 0.34) \\ \textit{flexural buckling around } z - \textit{curve } c\,(\alpha = 0.49). \end{array}$$

HEB 320

Table 6.2: Selection of buckling curve for a cross-section

	Cross section		Limits	Buckling about axis	Buckling curve S 235 S 275 S 355 S 420	S 460
Rolled sections		h/b > 1,2	$t_f \le 40$ mm	y – y z – z	a b	a_0 a_0
			40 mm < $t_f \le$ 100	y – y z – z	b c	a a
		h/b ≤ 1,2	$t_f \le 100$ mm	y – y z – z	b c	a a
			$t_f > 100$ mm	y – y z – z	d d	c c

<u>Plane *xz*</u> - $L_{E,y} = 4.335\ m$.

$$\bar{\lambda}_y = \frac{L_{E,y}}{i_y}\frac{1}{\lambda_1} = \frac{4.335}{13.82 \times 10^{-2}} \times \frac{1}{93.9 \times 0.81} = 0.41$$

$$\Phi_y = 0.62 \qquad \Rightarrow \qquad \boxed{\chi_y = 0.92}$$

<u>Plane *xy*</u> - $L_{E,z} = 4.335\ m$

$$\bar{\lambda}_z = \frac{L_{E,z}}{i_z}\frac{1}{\lambda_1} = \frac{4.335}{7.57 \times 10^{-2}} \times \frac{1}{93.9 \times 0.81} = 0.75$$
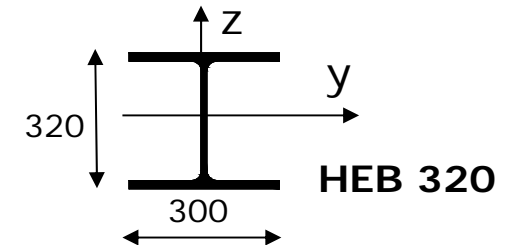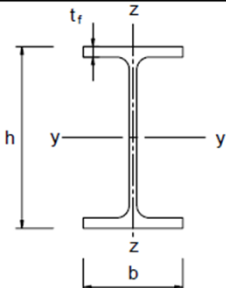
$$\Phi_z = 0.92 \qquad \Rightarrow \qquad \boxed{\chi_z = 0.69}$$

$$\phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$$

$$\chi = \frac{1}{\Phi + \left(\Phi^2 - \bar{\lambda}^2\right)^{0.5}}$$

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DESIGN OF BEAM-COLUMNS

EXAMPLE 3

Step 3: calculation of the χ_{LT} using the alternative method applicable to rolled or equivalent welded sections (clause 6.3.2.3 of EC3-1-1)

The length between braced sections is L = 4.335 m. The critical moment M_{cr} assuming a linear diagram, in this example obtained just by **LTBeam software**, is given by:

$$M_{cr} = 5045.1 \text{ kNm} \quad \Rightarrow \quad \overline{\lambda}_{LT} = \left[2149 \times 10^{-6} \times 355 \times 10^3 / 5045.1\right]^{0.5} = 0.39$$

Rolled I or H sections with $h/b = 320/300 = 1.07 < 2 \Rightarrow$

curve b, and $\alpha_{LT} = 0.34$

Taking $\overline{\lambda}_{LT,0} = 0.4$ and $\beta = 0.75$

$$\Phi_{LT} = 0.5 \times \left[1 + 0.34 \times (0.39 - 0.4) + 0.75 \times 0.39^2\right]$$
$$= 0.56$$

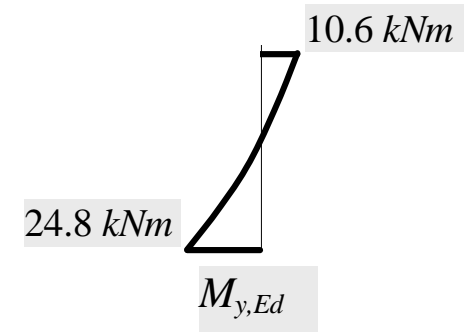$$\chi_{LT} = \frac{1}{0.56 + \left(0.56^2 - 0.75 \times 0.39^2\right)^{0.5}} = 0.99$$

10.6 kNm

24.8 kNm

$M_{y,Ed}$

Table 6.4 - Buckling curves for lateral-torsional buckling (Alternative method)

Section	Limits	Buckling curve (EC3-1-1)
I or H sections rolled	$h/b \leq 2$	b
	$h/b > 2$	c
I or H sections welded	$h/b \leq 2$	c
	$h/b > 2$	d

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EXAMPLE 3

Step 3: calculation of the χ_{LT} using the alternative method applicable to rolled or equivalent welded sections (clause 6.3.2.3 of EC3-1-1)

The correction factor k_c, according to Table 6.6 of EC3-1-1, with $\Psi = 10.6/(-24.8) = -0.43$, is given by:

$$k_c = \frac{1}{1.33 - 0.33\,\Psi} = \frac{1}{1.33 - 0.33 \times (-0.43)} = 0.68$$
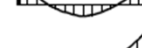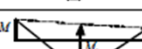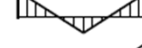
$$f = 1 - 0.5 \times (1 - k_c) \times \left[1 - 2.0 \times \left(\overline{\lambda}_{LT} - 0.8\right)^2\right]$$

$$= 1 - 0.5 \times (1 - 0.68) \times \left[1 - 2.0 \times (0.39 - 0.8)^2\right] = 0.89$$

The modified lateral-torsional buckling reduction factor is given by:

$$\chi_{LT,\text{mod}} = 0.99/0.89 = 1.11 > 1.00$$

So, $\boxed{\chi_{LT,\text{mod}} = 1.00}$ must be adopted.

Diagram of bending moments	k_c
$\Psi = +1$ (uniform)	1.0
$-1 \leq \Psi \leq 1$ (triangular)	$\dfrac{1}{1.33 - 0.33\,\Psi}$
M ... ΨM	0.94
	0.90
	0.91
M ... ΨM	0.86
	0.77
	0.82

Ψ - ratio between end moments, with $-1 \leq \Psi \leq 1$.

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EXAMPLE 3

Step 4: interaction factors k_{yy} and k_{zy}.

The equivalent factors of uniform moment C_{my} and C_{mLT} are obtained based on the bending moment diagram, between braced sections according to the z direction in case of C_{my} and laterally in case of C_{mLT}. Assuming the member braced in z direction and laterally just at the base and top cross sections, the factors C_{my} and C_{mLT} must be calculated based on the bending moment diagram along the total length of the member.

Since the bending moment diagram is assumed linear, defined by:

$M_{y,Ed,base}$ = -24.8 kNm;

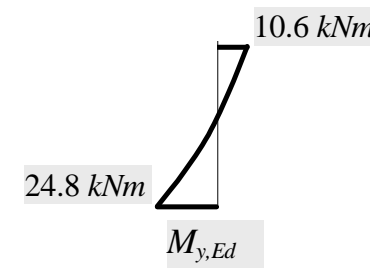$M_{y,Ed,top}$ = 10.4 kNm, from Table B.3 of EC3-1-1, is obtained:

Table B.3: Equivalent uniform moment factors C_m in Tables B.1 and B.2

Moment diagram	range	C_{my} and C_{mz} and C_{mLT}	
		uniform loading	concentrated load
M ⬛ ψM	$-1 \leq \psi \leq 1$		$0.6 + 0.4\psi \geq 0.4$

$\Psi = M_{y,Ed,top}/M_{y,Ed,base} = (10.6)/(-24.8) = -0.43 \Rightarrow$

$\boxed{C_{my} = C_{mLT} = 0.60 + 0.4 \times (-0.43) = 0.43}$ (> 0.40)

10.6 kNm

24.8 kNm

$M_{y,Ed}$

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EXAMPLE 3

Because the member is susceptible to torsional deformations, the interaction factors k_{yy} and k_{zy} are obtained from Table B.2 of EC3-1-1, through the following calculations:

$$k_{yy} = C_{my}\left[1 + \left(\bar{\lambda}_y - 0.2\right)\frac{N_{Ed}}{\chi_y\, N_{Rk}/\gamma_{M1}}\right] = 0.43 \times \left[1 + \left(0.41 - 0.2\right)\times\frac{1704}{0.92\times5726.2/1.0}\right] = 0.46;$$

As $\quad k_{yy} \le C_{my}\left(1 + 0.8\frac{N_{Ed}}{\chi_y\, N_{Rk}/\gamma_{M1}}\right) = 0.54 \quad$, then $\quad \boxed{k_{yy} = 0.46}$

$$k_{zy} = \left[1 - \frac{0.1\bar{\lambda}_z}{\left(C_{mLT} - 0.25\right)}\frac{N_{Ed}}{\chi_z\, N_{Rk}/\gamma_{M1}}\right] =$$

$$= \left[1 - \frac{0.1\times0.75}{\left(0.43 - 0.25\right)}\frac{1704}{0.69\times5726.2/1.0}\right] = 0.82$$

As $\quad k_{zy} \ge \left[1 - \frac{0.1}{\left(C_{mLT} - 0.25\right)}\frac{N_{Ed}}{\chi_z\, N_{Rk}/\gamma_{M1}}\right] = 0.76$

then $\quad \boxed{k_{zy} = 0.82}$

Step 5: Finally, the verification of equations 6.61 and 6.62 of EC3-1-1 yields:

$$\frac{1704}{0.92\times5726.2/1.0} + 0.46\times\frac{24.8}{1.00\times762.9/1.0} = 0.34 < 1.0$$

$$\frac{1704}{0.69\times5726.2/1.0} + 0.82\times\frac{24.8}{1.00\times762.1/1.0} = 0.46 < 1.0$$

Section O.K.

❑ **Free software for design of steel members in accordance with EC3-1-1.**

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Thank you for your attention

rads@dec.uc.pt