Eurocodes
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Eurocode 4
Composite Columns

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Part 1: Introduction
Concrete encased sections

advantages:
- high bearing resistance
- high fire resistance
- economical solution with regard to material costs

disadvantages:
- high costs for formwork
- difficult solutions for connections with beams
- difficulties in case of later strengthening of the column
- in special case edge protection is necessary
Partially concrete encased sections

advantages:
- high bearing resistance, especially in case of welded steel sections
- no formwork
- simple solution for joints and load introduction
- easy solution for later strengthening and additional later joints
- no edge protection

disadvantages:
- lower fire resistance in comparison with concrete encased sections.
Concrete filled hollow sections

advantages:
- high resistance and slender columns
- advantages in case of biaxial bending
- no edge protection

disadvantages:
- high material costs for profiles
- difficult casting
- additional reinforcement is needed for fire resistance
Concrete filled hollow sections with additional inner profiles

advantages:
- extreme high bearing resistance in combination with slender columns
- constant cross section for all stories is possible in high rise buildings
- high fire resistance and no additional reinforcement
- no edge protection

disadvantages:
- high material costs
- difficult casting
Composite columns with hollow sections and additional inner core-profiles
Design of composite columns according to EN 1994-1-1

Verifications for composite columns

- Resistance of the member for structural stability
  - General method
  - Simplified method
- Resistance to local Buckling
- Introduction of loads
- Longitudinal shear outside the areas of load introduction
Methods of verification in accordance with EN 1994-1-1

**Methods of verification**

**general method:**
- any type of cross-section and any combination of materials

**simplified method:**
- double-symmetric cross-section
- uniform cross-section over the member length
- limited steel contribution factor $\delta$
- related Slenderness smaller than 2,0
- limited reinforcement ratio
- limitation of b/t-values

Methods of verification in accordance with EN 1994-1-1
Resistance to local buckling

Concrete encased cross-sections

Verification is not necessary where

\[ c_z \geq \begin{cases} 40 \text{ mm} \\ b \div 6 \end{cases} \]

Concrete filled hollow section

\[ \max \left( \frac{d}{t} \right) = 90 \varepsilon^2 \]

Partially encased I sections

\[ \varepsilon = \frac{f_{y_{k,o}}}{f_{y_k}} \]

\[ f_{y_{k,o}} = 235 \text{ N/mm}^2 \]

\[ \max \left( \frac{d}{t} \right) = 52 \varepsilon \]

\[ \max \left( \frac{d}{t} \right) = 44 \varepsilon \]
Part 2: General design method
Design for structural stability shall take account of

- second-order effects including residual stresses,
- geometrical imperfections,
- local instability,
- cracking of concrete,
- creep and shrinkage of concrete
- yielding of structural steel and of reinforcement.

The design shall ensure that instability does not occur for the most unfavourable combination of actions at the ultimate limit state and that the resistance of individual cross-sections subjected to bending, longitudinal force and shear is not exceeded. Second-order effects shall be considered in any direction in which failure might occur, if they affect the structural stability significantly. Internal forces shall be determined by elasto-plastic analysis. Plane sections may be assumed to remain plane. Full composite action up to failure may be assumed between the steel and concrete components of the member. The tensile strength of concrete shall be neglected. The influence of tension stiffening of concrete between cracks on the flexural stiffness may be taken into account.
General method of design

Cracked concrete
Plastic zones in structural steel
Stresses in structural steel section
Stresses in concrete and reinforcement

Concrete
Reinforcement
Structural steel

\[
\sigma_c = 0.4 f_c
\]

\[
\epsilon_c = \frac{f_c}{E_c}
\]

\[
\epsilon_{c1} = \frac{f_{ct}}{E_{cm}}
\]

\[
\epsilon_{c1u} = \frac{f_{cm}}{E_{cm}}
\]

\[
\sigma_s = \frac{f_{sm}}{E_s}
\]

\[
\epsilon_s = \frac{f_{sm}}{E_s}
\]

\[
\sigma_a = \frac{f_u}{E_a}
\]

\[
\epsilon_a = \frac{f_u}{E_a}
\]
Typical load-deformation behaviour of composite columns in tests

Concrete encased section and bending about the strong axis:
Failure due to exceeding the ultimate strain in concrete, buckling of longitudinal reinforcement and spalling of concrete.

Concrete encased section and bending about the weak axis:
Failure due to exceeding the ultimate strain in concrete.

Concrete filled hollow section:
Cross-section with high ductility and rotation capacity. Fracture of the steel profile in the tension zone at high deformations and local buckling in the compression zone of the structural steel section.
General Method – Safety concept based on DIN 18800-5 (2004) and German national Annex for EN 1994-1-1

Verification $\lambda_u \geq \gamma_R$

$\lambda_u$: amplification factor for ultimate system capacity

$\gamma_R = \frac{R_{pl,m}}{R_{pl,d}}$

$N_{Ed}$ $M_{Ed}$ $E_d$ $W_{o}=L/1000$

$R_{pl,m}$ $R_{pl,d}$

$\epsilon_{c1}$ $\epsilon_{c1u}$ $E_{cm}$ $f_{cm}$ $0.4f_c$

$\epsilon_{c}$ $\sigma_c$

$\epsilon_{s}$ $\sigma_s$

$\epsilon_{v}$ $\sigma_a$

$\lambda_u E_d$

$N$ $E$

$\sigma_{cm}$ $f_{tm}$ $f_{sm}$ $E_s$ $\lambda_u$

$\sigma_{s}$ $f_u$ $f_y$

$E_v$ $\sigma_a$

structural steel

reinforcement

concrete

geometrical Imperfection

Residual stresses
Composite columns for the central station in Berlin

- Buckling curves:
  - Curve a
  - Curve b
  - Curve c
  - Curve d

- Materials:
  - S235
  - S355

- Dimensions:
  - Thickness: t=25mm, t=50mm
  - Size: 800x550, 700x1200

- Residual stresses

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Part IV-3:

Plastic resistance of cross-sections and interaction curve
Resistance of cross-sections

Design value of the plastic resistance to compressive forces:

Characteristic value of the plastic resistance to compressive forces:

Design strength:

Increase of concrete strength due to better curing conditions in case of concrete filled hollow sections:
Confinement effects in case of concrete filled tubes

For concrete stresses $\sigma_{c}>0.8\ f_{ck}$ the Poisson’s ratio of concrete is higher than the Poisson’s ratio of structural steel. The confinement of the circular tube causes radial compressive stresses $\sigma_{c,r}$. This leads to an increased strength and higher ultimate strains of the concrete. In addition the radial stresses cause friction in the interface between the steel tube and the concrete and therefore to an increase of the longitudinal shear resistance.
Confinement effect acc. to Eurocode 4-1-1

Design value of the plastic resistance to compressive forces taking into account the confinement effect:

\[
N_{pl,Rd} = \eta_a f_{yd} A_a + A_c f_{cd} \left( 1 + \eta_c \frac{t}{d} \frac{f_{yk}}{f_{ck}} \right)
\]

Basic values \( \eta \) for stocky columns centrically loaded:

- \( \eta_{ao} = 0.25 \)
- \( \eta_{co} = 4.9 \)

Influence of slenderness for \( \bar{\lambda} \leq 0.5 \):

- \( \eta_{a,\lambda} = \eta_{ao} + 0.5 \bar{\lambda}_K \leq 1.0 \)
- \( \eta_{c,\lambda} = \eta_{co} - 18.5 \bar{\lambda}_K \left( 1 - 0.92 \bar{\lambda}_K \right) \geq 0 \)

Influence of load eccentricity:

- \( \eta_a = \eta_{a,\lambda} + 10 \left( 1 - \eta_{ao} \right) \frac{e}{d} \)
- \( \eta_c = \eta_{c,\lambda} \left( 1 - 10 \frac{e}{d} \right) \)

For \( e/d \geq 0.1 \): \( \eta_a = 1.0 \) and \( \eta_c = 0 \)
Plastic resistance to combined bending and compression

The resistance of a cross-section to combined compression and bending and the corresponding interaction curve may be calculated assuming rectangular stress blocks.

The tensile strength of the concrete should be neglected.

The influence of transverse shear forces on the resistance to bending and normal force should be considered when determining the interaction curve, if the shear force $V_{a,Ed}$ on the steel section exceeds 50% of the design shear resistance $V_{pl,a,Rd}$ of the steel section. The influence of the transverse shear on the resistance in combined bending and compression should be taken into account by a reduced design steel strength $(1 - \rho) f_{yd}$ in the shear area $A_v$.

\[
V_{a,Ed} \leq 0.5 \ V_{pla,Rd} \Rightarrow \rho = 0
\]
\[
V_{a,Ed} > 0.5 \ V_{pla,Rd} \Rightarrow \rho = \left( \frac{2 V_{a,Ed}}{V_{pla,Rd}} - 1 \right)^2
\]
Influence of vertical shear

The shear force $V_{a,Ed}$ should not exceed the resistance to shear of the steel section. The resistance to shear $V_{c,Ed}$ of the reinforced concrete part should be verified in accordance with EN 1992-1-1, 6.2.

Unless a more accurate analysis is used, $V_{Ed}$ may be distributed into $V_{a,Ed}$ acting on the structural steel and $V_{c,Ed}$ acting on the reinforced concrete section by:

$$V_{a,Ed} = V_{Ed} \frac{M_a}{M_{Rd}} \approx \frac{M_{pla,Rd}}{M_{pl,Rd}}$$

$$V_{c,Ed} = V_{Ed} - V_{a,Ed}$$

$M_{pl,a,Rd}$ is the plastic resistance moment of the steel section.

$M_{pl,Rd}$ is the plastic resistance moment of the composite section.

Verification for vertical shear:

$V_{a,Ed} \leq V_{pla,Rd}$  
$V_{c,Ed} \leq V_{c,Rd}$
Determination of the resistance to normal forces and bending (example)

Position of the plastic neutral axis: \[ \sum N_i = N_{Ed} \]

\[ N_c + N_{aw,c} - N_{aw,t} = N_{Ed} \]

\[ (b - t_w)z_{pl} 0,85f_{cd} + t_w z_{pl} (1-\rho)f_{yd} - t_w (h_w - z_{pl}) (1-\rho)f_{yd} = N_{Ed} \]

Plastic resistance to bending \( M_{pl,N,Rd} \) in case of the simultaneously acting compression force \( N_{Ed} \) and the vertical shear \( V_{Ed} \):

\[ M_{pl,N,Rd} = N_c z_c + N_{aw,c} z_{aw,c} + N_{aw,t} z_{aw,t} + N_{af} (h_w + t_f) + 2N_s z_s \]

\[ N_{aw,c} = z_{pl} t_w (1-\rho)f_{yd} \]

\[ N_{aw,t} = (h_w - z_{pl}) t_w (1-\rho)f_{yd} \]

\[ N_{af} = b t_f f_{yd} \]

\[ N_c = (b - t_w) z_{pl} 0,85 f_{cd} \]

\[ N_s = 2A_s f_{sd} \]
Simplified determination of the interaction curve

As a simplification, the interaction curve may be replaced by a polygonal diagram given by the points A to D.
Resistance at points A and D

Point A

- \( N_{\text{pla,Rd}} \)
- \( N_{\text{plc,Rd}} \)
- \( N_{\text{pls,Rd}} \)

\[
N_{\text{pl,Rd}} = N_{\text{pla,Rd}} + N_{\text{plc,Rd}} + N_{\text{pls,Rd}}
\]

\( M_{A,Rd} = 0 \)

Point D

- \( M_{\text{pla,Rd}} \)
- \( M_{\text{plc,Rd}} \)
- \( M_{\text{pls,Rd}} \)

\[
N_{\text{D,Rd}} = 0.5 \times N_{\text{plc,Rd}}
\]

\( M_{D,Rd} = M_{\text{max,Rd}} \)

\[
M_{\text{max,Rd}} = M_{\text{pla,Rd}} + M_{\text{pls,Rd}} + 1/2 \times M_{\text{plc,Rd}}
\]

- \( W_{\text{pl,a}} \) plastic section modulus of the structural steel section
- \( W_{\text{pl,s}} \) plastic section modulus of the cross-section of reinforcement
- \( W_{\text{pl,c}} \) plastic section modulus of the concrete section

\[
M_{\text{pla,Rd}} = W_{\text{pl,a}} \times f_{yd} = \left[ \frac{(h - 2t_f)^2 \times t_w}{4} + b t_f (h - t_f) \right] f_{yd}
\]

\[
M_{\text{plc,Rd}} = W_{\text{pl,c}} \times f_{sd} = \left[ \sum A_{z_{si}} z_{si} \right] f_{ys}
\]

\[
M_{\text{pls,Rd}} = W_{\text{pl,s}} \times f_{sd} = \left[ \sum A_{z_{si}} z_{si} \right] f_{ys}
\]

\[
M_{\text{plc,Rd}} = W_{\text{pl,c}} \times 0.85 f_{cd} = \left[ \frac{b c h_c^2}{4} - W_{\text{pl,a}} - W_{\text{pl,s}} \right] 0.85 f_{cd}
\]
At point B is no resistance to compression forces. Therefore the resistance to compression forces at point D results from the additional cross-section zones in compression. With $N_{D,Rd}$ the depth $h_n$ and the position of the plastic neutral axis at point B can be determined. With the plastic bending moment $M_{n,Rd}$ resulting from the stress blocks within the depth $h_n$ the plastic resistance moment $M_{pl,Rd}$ at point B can be calculated by:

$$M_{pl,Rd} = M_{D,Rd} - M_{pln,Rd}$$
The bending resistance at point C is the same as the bending resistance at point B.

\[ M_{C,Rd} = M_{pl,Rd} \]

The normal force results from the stress blocks in the zone 2\( h_n \).

\[ N_{C,Rd} = 2N_{D,Rd} = N_{cpl,Rd} = N_{pm,Rd} \]
Part 4: Simplified design method
Simplified Method

Methods of verification acc. to the simplified method

- Axial compression
  - Design based on the European buckling curves
- Resistance of member in combined compression and bending
  - Design based on second order analysis with equivalent geometrical bow imperfections
  - Design based on second order analysis with equivalent geometrical bow imperfections
Scope of the simplified method

- double symmetrical cross-section
- uniform cross-sections over the member length with rolled, cold-formed or welded steel sections
- steel contribution ratio
- relative slenderness
- longitudinal reinforcement ratio
- the ratio of the depth to the width of the composite cross-section should be within the limits 0,2 and 5,0
The horizontal deflection and the second order bending moments increase under permanent loads due to creep of concrete. This leads to a reduction of the ultimate load.

The effects of creep of concrete are taken into account in design by a reduced flexural stiffness of the composite cross-section.
The effects of creep of concrete are taken into account by an effective modulus of elasticity of concrete

\[ E_{c,\text{eff}} = \frac{E_{cm}}{1 + \frac{N_{G,Ed}}{N_{Ed}} \varphi(t,t_o)} \]

notional size of the cross-section for the determination of the creep coefficient \( \varphi(t,t_o) \)

\[ h_o = \frac{2A_c}{U} \]

effective perimeter \( U \) of the cross-section

\[ U = 2(b + h) \quad U \approx 2h + 0.5b \]

In case of concrete filled hollow section the drying of the concrete is significantly reduced by the steel section. A good estimation of the creep coefficient can be achieved, if 25% of that creep coefficient is used, which results from a cross-section, where the notional size \( h_o \) is determined neglecting the steel hollow section.

\[ \varphi_{t,\text{eff}} = 0.25 \varphi(t,t_o) \]
Verification for axial compression with the European buckling curves

<table>
<thead>
<tr>
<th>cross-section</th>
<th>buckling curve</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="I-shaped cross-section" /></td>
<td><img src="image2" alt="Cross-section b" /></td>
</tr>
<tr>
<td><img src="image3" alt="H-shaped cross-section" /></td>
<td><img src="image4" alt="Cross-section c" /></td>
</tr>
<tr>
<td><img src="image5" alt="Round cross-section" /></td>
<td><img src="image6" alt="Cross-section a" /></td>
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<td><img src="image2" alt="Cross-section b" /></td>
</tr>
</tbody>
</table>

**Verification:**

\[
\frac{N_{\text{Ed}}}{N_{\text{Rd}}} \leq 1.0
\]

**Design value of resistance**

\[
N_{\text{pl,Rd}} = \chi \cdot N_{\text{pl,Rd}} = \lambda \cdot N_{\text{cr}}
\]

\[
N_{\text{pl,Rd}} = A_a f_{yd} + A_s f_{sd} + \nu A_c f_{cd}
\]
Relative slenderness

- relative slenderness:
  \[ \lambda = \sqrt{\frac{N_{pl,Rk}}{N_{cr}}} \leq 2,0 \]

- characteristic value of the plastic resistance to compressive forces
  \[ N_{pl,Rk} = A_a f_{yk} + A_c \nu f_{ck} + A_s f_{sk} \]

- elastic critical normal force
  \[ N_{cr} = \frac{\pi^2 (EJ)_{eff}}{(\beta L)^2} \]

- buckling length factor

- effective flexural stiffness
  \[ (EJ)_{eff} = (E_a J_a + K_e E_{c,eff} J_c + E_s J_s) \]

- \( K_e = 0,6 \)
Verification for combined compression and bending

\[ w_0 \text{ equivalent geometrical bow imperfection} \]

**Verification**

\[ \max M_{Ed} \leq M_{Rd} = \alpha_M \mu M_{pl,Rd} \]

\[ \alpha_M = 0,9 \text{ for S235 and S355} \]

\[ \alpha_M = 0,8 \text{ for S420 and S460} \]

bending moments taking into account second order effects:

\[ \max M_{Ed} = N_{Ed} w_0 \frac{1}{1 - \frac{N_{Ed}}{N_{cr}}} \]

Effective flexural stiffness

\[ N_{cr} = \frac{\pi^2 (E J)_{eff,II}}{\beta^2 L^2} \]

\[(E I)_{eff,II} = K_o (E_a J_a + K_e E_{c,eff} J_c + E_s J_s)\]

with \( K_{e,II} = 0,5 \), \( K_o = 0,9 \)

The factor \( \alpha_M \) takes into account the difference between the full plastic and the elasto-plastic resistance of the cross-section resulting from strain limitations for concrete.
Equivalent initial bow imperfections

Buckling curve

$\rho_s \leq 3\%$

$3\% < \rho_s \leq 6\%$

Member imperfection

$w_o = L/300$

$w_o = L/200$

$w_o = L/150$
Imperfections for global analysis of frames

Global initial sway imperfection acc. to EN 1993-1-1:

\[ \phi = \phi_0 \cdot \alpha_m \cdot \alpha_h \]

- \( \phi_0 \) basic value with \( \phi_0 = 1/200 \)
- \( \alpha_h \) reduction factor for the height \( h \) in [m]
- \( \alpha_m \) reduction factor for the number of columns in a row

\[ \alpha_h = \frac{2}{\sqrt{h}} \text{ but } \frac{2}{3} \leq \alpha_h \leq 1,0 \]

\[ \alpha_m = \sqrt{0.5 \left[ 1 + \frac{1}{m} \right]} \]

- \( m \) is the number of columns in a row including only those columns which carry a vertical load \( N_{Ed} \) not less than 50% of the average value of the column in a vertical plane considered.
Frames sensitive against second order effects

Within a global analysis, member imperfections in composite compression members may be neglected where first-order analysis may be used. Where second-order analysis should be used, member imperfections may be neglected within the global analysis if:

\[ \lambda \leq 0.5 \left( \frac{N_{pl,Rk}}{N_{Ed,i}} \right)^{1/2} \]

\[ \lambda = \sqrt{\frac{N_{pl,Rk}}{N_{cr}}} \]

\[ N_{cr} = \frac{\pi^2 \left( EJ \right)_{eff}}{L_i^2} \]

\[ (EJ)_{eff} = (E_a J_a + 0.6 E_{c,eff} J_c + E_s J_s) \]
Second order analysis

Bending moments including second order effects:

\[ M(\xi) = M_R \left( \frac{r \sin \varepsilon (1 - \xi) + \sin \varepsilon \xi}{\sin \varepsilon} \right) + \frac{M_0 \left( \cos \varepsilon (0.5 - \xi) \right)}{\cos \left( \varepsilon / 2 \right)} - 1 \]

\[ V_z(\xi) = \frac{M_R}{L} \left( \frac{r \cos \varepsilon (1 - \xi) + \cos \varepsilon \xi}{\sin \varepsilon} \right) + \frac{M_0 \left( \sin \varepsilon (0.5 - \xi) \right)}{\cos \left( \varepsilon / 2 \right)} - 1 \]

\[ M_0 = (q L^2 + 8Nw_0) \frac{1}{\varepsilon^2} \]

\[ \varepsilon = L \frac{f_{Ed}}{(EJ)_{eff, ll}} \]

Maximum bending moment at the point \( \xi_M \):

\[ \frac{dM}{d\xi} = 0 \]

\[ M_{max} = \left[ 0.5M(1+r) + M_0 \right] \frac{\sqrt{1 + c^2}}{\cos(0.5 \varepsilon)} - M_0 \]

\[ c = \frac{M (r - 1)}{M(1+r) + 2M_0} \]

\[ \xi_M = 0.5 + \frac{\arctan c}{\varepsilon} \]
Simplified calculation of second order effects

Exact solution:

\[ M_{\text{max}} = 0.5M_R (1 + r) \frac{\sqrt{1 + c^2}}{\cos(0.5 \varepsilon)} \]

\[ c = \frac{r - 1}{1 + r} \frac{1}{\tan(0.5 \varepsilon)} \]

\[ \xi_M = 0.5 + \frac{\arctan c}{\varepsilon} \]

\[ \varepsilon = L \frac{N_{\text{Ed}}}{(E J)_{\text{eff,ll}}} \]

simplified solution:

\[ k = \frac{M_{\text{max}}}{M_R} = \frac{\beta}{1 - \frac{N_{\text{Ed}}}{N_{\text{cr}}}} \]

\[ \beta = 0.66 + 0.44 r \]

\[ \beta \geq 0.44 \]
Background of the member imperfections

The initial bow imperfections were recalculated from the resistance to compression calculated with the European buckling curves.

Bending moment based on second order analysis:

\[
M = \frac{8 \omega_0 (EJ)_{\text{eff},\text{II}}}{L^2} \left[ \frac{1}{\cos(\varepsilon/2)} - 1 \right]
\]

Resistance to axial compression based on the European buckling curves:

\[
N_{\text{Rd}} = \chi N_{pI,\text{Rd}}
\]

Bending resistance:

\[
M_{\text{Rd}} = \alpha M \mu M_{pI,\text{Rd}}
\]

Determination of the equivalent bow imperfection:

\[
\omega_0 = \frac{\alpha M \mu d M_{pI,\text{Rd}} L^2}{8 (EJ)_{\text{eff},\text{II}}} \left[ \frac{1}{1 - \cos(\varepsilon/2)} - 1 \right]
\]
The initial bow imperfection is a function of the related slenderness and the resistance of cross-sections. In Eurocode 4 constant values for $w_0$ are used.

The use of constant values for $w_0$ leads to maximum differences of 5% in comparison with the calculation based on the European buckling curves.
Comparison of the simplified method with non-linear calculations for combined compression and bending.

Graph showing resistance as a function of the related slenderness.

Plastic cross-section resistance

\[ \bar{\lambda} = \sqrt[\lambda]{\frac{N_{pl,Rk}}{N_{cr}}} \]

Comparison of the simplified method with non-linear calculations for combined compression and bending.
Resistance to combined compression and biaxial bending

The resistance is given by a three-dimensional interaction relation. For simplification a linear interaction between the points A and B is used.

\[ M_{y,Rd} \left( N_{Ed} \right) = \mu_{dy} M_{pl,y,Rd} \]
\[ M_{z,Rd} \left( N_{Ed} \right) = \mu_{dz} M_{pl,y,Rd} \]
\[ M_{y,Ed} = \mu_{y,Ed} M_{y,Rd} \]
\[ M_{z,Ed} = \mu_{z,Ed} M_{y,Rd} \]

Approximation for the interaction curve:

\[ \frac{\mu_{y,Ed}}{\mu_{dy}} + \frac{\mu_{z,Ed}}{\mu_{dz}} \leq 1,0 \]

\[ \frac{M_{y,Ed}}{\mu_{dy} M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} M_{pl,z,Rd}} \leq 1,0 \]
Verification in case of compression an biaxial bending

For both axis a separate verification is necessary.

\[ \frac{M_{y,Ed}}{\mu_{dy} M_{pl,y,Rd}} \leq \alpha_M \]
\[ \frac{M_{z,Ed}}{\mu_{dz} M_{pl,y,Rd}} \leq \alpha_M \]

Verification for the interaction of biaxial bending.

\[ \frac{M_{y,Ed}}{\mu_{dy} M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} M_{pl,y,Rd}} \leq 1,0 \]

Imperfections should be considered only in the plane in which failure is expected to occur. If it is not evident which plane is the most critical, checks should be made for both planes.

\[ \alpha_M = 0,9 \text{ for S235 and S355} \]
\[ \alpha_M = 0,8 \text{ for S420 and S460} \]
Part 5:

Special aspects of columns with inner core profiles
Composite columns – General Method

- Commerzbank Frankfurt
- Millennium Tower Vienna
- New railway station in Berlin (Lehrter Bahnhof)

Highlight Center Munich
Composite columns with concrete filled tubes and steel cores – special effects

Resistance based on stress blocks (plastic resistance)

Cross-sections with massive inner cores have a very high plastic shape factor and the cores can have very high residual stresses. Therefore these columns cannot be design with the simplified method according to EN 1944-1-1.

Non linear resistance with strain limitation for concrete

\[ \alpha_M = \frac{M_{Rd}}{\mu M_{pl,Rd}} \]
Residual stresses and distribution of the yield strength

\[ \sigma_{ED} [N/mm^2] \]

\[ U = \pi d_k \]
\[ A = \pi d_k^2 / 4 \]

\[ f_y(r) = f_y(r) = f_{yk} - \text{characteristic value of the yield strength} \]
\[ \frac{f_y(r)}{f_{yk}} = 0.95 + 0.1 \left( \frac{r}{r_k} \right)^2 \]

\[ \sigma_{E}(r) = \sigma_{ED} \left[ 1 - \frac{2r^2}{r_k^2} \right] \]

\[ r, r_k \]

\[ d_k \]

\[ d \]

\[ \pi d_k \]

\[ d_k^2 / 4 \]
General method – Finite Element Model

load introduction

stresses in the tube

initial bow imperfection

cross-section

stresses in concrete

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Part 6: Load introduction and longitudinal shear
Load introduction over the steel section

Load introduction by headed studs within the load introduction length $L_E$

- $N_{Ed}$
- $N_{c,Ed}$
- $N_{s,Ed}$
- $N_{a,Ed}$

$d$ minimum transverse dimension of the cross-section

$L$ member length of the column

Sectional forces of the cross-section:

$N_{a,Ed} = N_{Ed} \frac{N_{pl,a}}{N_{pl,Rd}}$

$N_{s,Ed} = N_{Ed} \frac{N_{pl,s}}{N_{pl,Rd}}$

$N_{c,Ed} = N_{Ed} \frac{N_{pl,c}}{N_{pl,Rd}}$

Required number of studs $n$ resulting from the sectional forces $N_{Ed,c} + N_{Ed,s}$:

$V_{L,Ed} = N_{c,Ed} + N_{s,Ed} = N_{Ed} \left[ 1 - \frac{N_{pl,a}}{N_{pl,Rd}} \right]$  

$V_{L,Rd} = nP_{Rd}$

$P_{Rd}$ – design resistance of studs
Load introduction for combined compression and bending

sectional forces due to $N_{Ed}$ and $M_{Ed}$

sectional forces based on plastic theory

$$
M_{Rd} = M_{a,Rd} + M_{c+s,Rd}
$$

$$
N_{Rd} = N_{a,Rd} + N_{c+s,Rd}
$$

$$
E_d = \frac{N_{a,Ed}}{N_{pl,Rd}} = \frac{M_{a,Ed}}{M_{a,Rd}} = \frac{N_{c+s,Ed}}{N_{c+s,Rd}} = \frac{M_{c+s,Ed}}{M_{c+s,Rd}}
$$

$$
R_d = \sqrt{\left(\frac{M_{Ed}}{M_{pl,Rd}}\right)^2 + \left(\frac{N_{Ed}}{N_{pl,Rd}}\right)^2}
$$

$$
E_d = \sqrt{\left(\frac{M_{Rd}}{M_{pl,Rd}}\right)^2 + \left(\frac{N_{Rd}}{N_{pl,Rd}}\right)^2}
$$
**Load introduction – Example**

**sectional forces based on stress blocks:**

\[ N_{c+s,Rd} = N_c + \sum N_{s,i} \]
\[ M_{c+s,Rd} = N_c z_c + \sum N_{s,i} z_{s,i} \]

**shear forces of studs based on elastic theory**

\[ \max P_{Ed} = \sqrt{\frac{N_{c+s,Ed}}{n} + \frac{M_{c+s,Ed}}{\sum r_i^2} x_i} \]
\[ + \frac{M_{c+s,Ed}}{\sum r_i^2} z_i \]

**shear forces of studs based on plastic theory**

\[ \max P_{Ed} = \frac{N_{c+s,Ed}}{n} + \frac{M_{c+s,Ed}}{e_h 0.5n} \]

- \( N_{c+s,Ed} \)
- \( M_{c+s,Ed} \)
- \( P_{Ed} \)
- \( P_{ed,v} \)
- \( P_{ed,h} \)
- \( e_h \)
- \( n \) – number of studs within the load introduction length

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Shear resistance of stud connectors welded to the web of partially encased I-Sections

Where stud connectors are attached to the web of a fully or partially concrete encased steel I-section or a similar section, account may be taken of the frictional forces that develop from the prevention of lateral expansion of the concrete by the adjacent steel flanges. This resistance may be added to the calculated resistance of the shear connectors. The additional resistance may be assumed to be on each flange and each horizontal row of studs, where $\mu$ is the relevant coefficient of friction that may be assumed. For steel sections without painting, $\mu$ may be taken as 0.5. $P_{Rd}$ is the resistance of a single stud.
Shear resistance of stud connectors welded to the web of partially encased I-Sections

\[ V_{L,Rd} = nP_{Rd} + V_{LR,Rd} \quad V_{LR,Rd} = \mu P_{Rd} \]

In absence of better information from tests, the clear distance between the flanges should not exceed the values given above.

\[ P_{Rd} = \min \begin{cases} P_{Rd,1} = 0.29 \alpha d^2 \sqrt{f_{ck}} E_{cm} \frac{1}{\gamma_v} \\ P_{Rd,2} = 0.8 \cdot f_u \left( \frac{\pi d^2}{4} \right) \frac{1}{\gamma_v} \end{cases} \]
Shear resistance of stud connectors welded to the web of partially encased I-sections
Load introduction – longitudinal shear forces in concrete

Longitudinal shear force in section I-I:

$$V_{L,Ed} = N_{Ed} \left[ 1 - \frac{N_{pl,a}}{N_{pl,Rd}} \right] \frac{A_{c1}}{A_c} \frac{0,85 f_{cd} + A_{s1} f_{sd}}{0,85 f_{cd} + A_s f_{sd}}$$

Longitudinal shear resistance of concrete struts:

$$V_{L,Rd,max} = 4 \frac{c_y \nu 0,85 f_{cd}}{\cot \theta + \tan \theta} \frac{L_E}{\theta = 45^\circ}$$

$$\nu = 0,6 (1 - (f_{ck} / 250)) \text{ with } f_{ck} \text{ in N/mm}^2$$

Longitudinal shear resistance of the stirrups:

$$V_{L,Rd,s} = 4 \frac{A_s}{s_w} f_{yd} \cot \theta L_E$$

$A_s$ - cross-section area of the stirrups

$s_w$ - spacing of stirrups

not directly connected concrete area $A_{s1}$
Load introduction – longitudinal shear forces in concrete – test results

$F_u = 1608 \text{ kN}$
Load introduction – Examples (Airport Hannover)

Load introduction with gusset plates
Load introduction with partially loaded end plates
Load introduction with distance plates for columns with inner steel cores
Composite columns with hollow sections –
Load introduction

Gusset plate
Stiffeners and end plates
Distance plates

\( \sigma_c \)

Stiffener
Distance plate
Mechanical model

\[ P_{cR,m} = f_c A_1 \sqrt{\frac{A_c}{A_1}} \left[ 1 + \eta_{cL} \frac{t}{d} \frac{f_y}{f_c} \right] \]

Effect of partially loaded area

Effect of confinement by the tube

\[ \eta_{cL} = 3.5 \]

\[ \eta_{cL} = 4.9 \]
Typical load-deformation curves

P [MN]

series SXIII

series SV

δ [mm]
Test evaluation according to EN 1990

41 tests
\[ V_r = 0.14 \]

\[ P_{c,Rm} = 0.78 P_{c,Rm} \]
\[ P_{c,Rd} = 0.66 P_{c,Rm} \]

\[ P_{c,Rm} = f_c A_1 \left[ 1 + \eta_{CL} \frac{t}{d} \frac{f_y}{f_c} \right] \sqrt{\frac{A_c}{A_1}} \]
Load distribution by end plates

$F_u = 6047 \text{ kN}$
$F_{u,\text{stat}} = 4750 \text{ kN}$
$\delta_u = 7.5 \text{ mm}$

$b_c = t_s + 5t_p$
Design rules according to EN 1994-1-1

\[
\sigma_{c,Rd} = f_{cd} \left[ 1 + \eta_{cL} \frac{t}{d} \frac{f_{yk}}{f_{ck}} \right] \sqrt{\frac{A_c}{A_1}} \leq \frac{A_c f_{cd}}{A_1} \leq f_{yd}
\]

- \(f_{ck}\) concrete cylinder strength
- \(t\) wall thickness of the tube
- \(d\) diameter of the tube
- \(f_{yk}\) yield strength of structural steel
- \(A_1\) loaded area
- \(A_c\) cross section area of the concrete
- \(\eta_{c,L}\) confinement factor
  \(\eta_{c,L} = 4,9\) (tube)
  \(\eta_{c,L} = 3,5\) (square hollow sections)

Load distribution 1:2.5

\[b_c = t_s + 5 t_p\]
Outside the area of load introduction, longitudinal shear at the interface between concrete and steel should be verified where it is caused by transverse loads and / or end moments. Shear connectors should be provided, based on the distribution of the design value of longitudinal shear, where this exceeds the design shear strength $\tau_{Rd}$.

In absence of a more accurate method, elastic analysis, considering long term effects and cracking of concrete may be used to determine the longitudinal shear at the interface.
Design shear strength $\tau_{Rd}$

- **Concrete encased sections**
  - $\beta_c = 1 + 0.02 \frac{c_z}{c_{z,\text{min}}} \left[ 1 - \frac{c_{z,\text{min}}}{c_z} \right] \leq 2.5$
  - $c_{z,\text{min}} = 40\text{mm}$ (minimum value)

- **Concrete filled tubes**
  - $\tau_{Rd} = 0.55 \text{N/mm}^2$

- **Concrete filled rectangular hollow sections**
  - $\tau_{Rd} = 0.40 \text{N/mm}^2$

- **Flanges of partially encased I-sections**
  - $\tau_{Rd} = 0.20 \text{N/mm}^2$

- **Webs of partially encased I-sections**
  - $\tau_{Rd} = 0.0 \text{N/mm}^2$
Thank you very much for your kind attention