Anchorage and lap splicing
Detailing of slabs, columns, beams, footings

José M. Arrieta
Universidad Politécnica Madrid, E.T.S. Ing. Caminos
PROES Consultores S.A.

1. GENERAL PROVISIONS FOR DETAILING

1.1. Anchorage length

1.2. Lap length

2. DETAILING OF STRUCTURAL MEMBERS

2.1. FOOTING B-2

- 2.1.1. Calculation of the footing
- 2.1.2. Arrangement of the reinforcement

2.2. BEAMS

- 2.2.1. Beam A2 – B2 – C2 for the case 1
- 2.2.2. Beam B1 – B2 – B3 for the case 3

2.3. SLABS

- 2.3.1. Slab AB12 for case 1

2.4. COLUMNS

- 2.4.1. Column B2 for the case 2

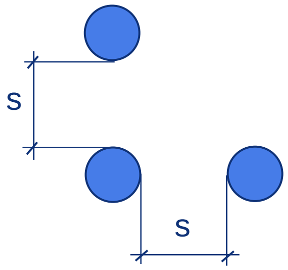
Section 8 EN 1992-1-1

For ribbed reinforcement, mesh and prestressing tendons

Subjected to static loading

Spacing of bars [8.2]

Good concreting $\rightarrow$ adequate bond



$$s_{min} = \max\left[K_1 \cdot \phi;\ d_g + K_2;\ 20\ \text{mm}\right]$$

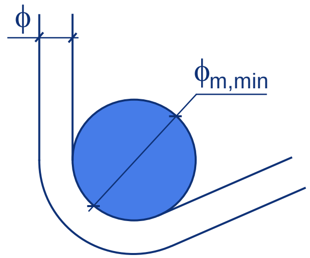
Recom. values: K_1=1,0; K_2=5 mm

$$\text{if} \begin{cases} d_g = 20\ \text{mm} \\ \phi \leq 25\ \text{mm} \end{cases} \Rightarrow s_{min} = 25\ \text{mm}$$

Minimum diameter of the mandrel [8.3]

Avoid:

- Bending cracks in the bar
- Failure of concrete inside the bent



$$\phi_{m,min} = \begin{cases} 4\phi & \text{if } \phi \leq 16 \text{ mm} \\ 7\phi & \text{if } \phi > 16 \text{ mm} \end{cases}$$

Conditions to avoid concrete failure [8.3 (3)]:

- Either not more than 5 ϕ past end bend

  Or bar not positioned at the edge and cross bar $\geq \phi$ inside the bend
- $\phi_m \geq \phi_{m,min}$

Tables for the building

(mm) ϕ	(mm) s_{min}	(mm) $\phi_{mand,min}$
8	25	32
10	25	40
12	25	48
14	25	56
16	25	64
20	25	140
25	25	175

Anchorage of longitudinal reinforcement [8.4]

Transmission forces reinforcement $\rightarrow$ concrete

Transverse tension stresses

Avoid:

- Longitudinal cracks
- Spalling

Methods:

- Straight
- Bend
- Hook
- Loop
- Welded transverse bar

Ultimate bond stress f_{bd}

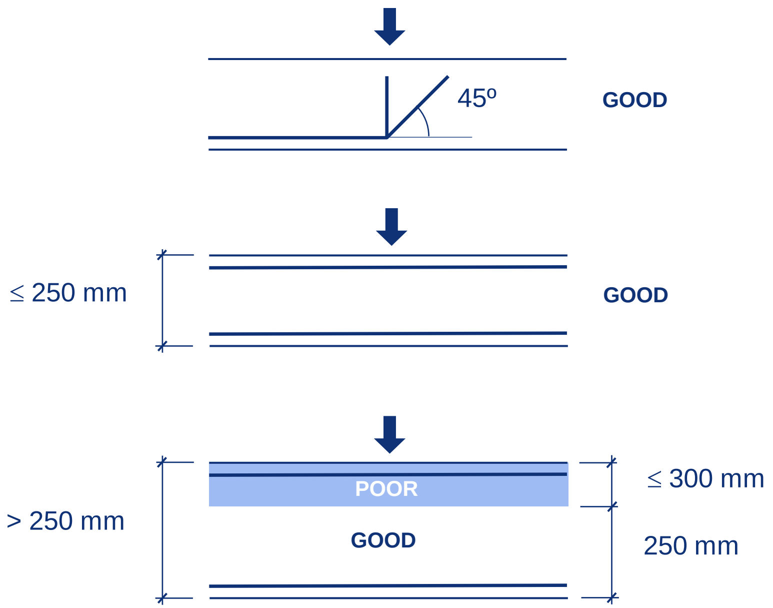
- Mechanism: tangential stresses at interface → bond stresses

$$f_{bd} = 2{,}25 \cdot \eta_1 \cdot \eta_2 \cdot f_{ctd}$$

- η_1: Quality of bond condition after concreting (1.0/0,7)
- η_2: Bar diameter (ϕ > 32 mm)

Basic anchorage length $l_{b,rqd}$

- To anchor $A_s \cdot \sigma_{sd}$ → $l_{b,rqd} = \frac{\phi}{4} \cdot \frac{\sigma_{sd}}{f_{bd}}$



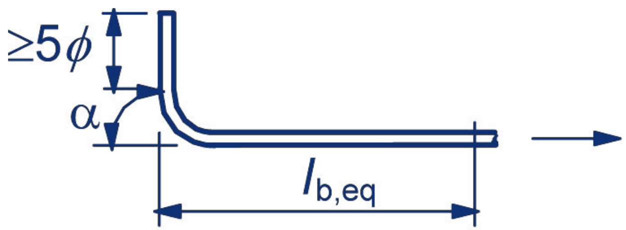
Design anchorage length [Table 8.2]

$$l_{bd} = \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot \alpha_4 \cdot \alpha_5 \cdot l_{b,rqd} \geq l_{b,min}$$

- α_1: Shape of bars
- α_2: Concrete cover
- α_3: Confinement by transv. reinf. not welded
- α_4: Confinement by welded transv. reinf.
- α_5: Confinement by transv. pressure
- $l_{b,min} = \max\left[\alpha \cdot l_{b,rqd}; 10\phi; 100\ \text{mm}\right] \ni \alpha = \begin{cases} 0,30 & \text{if tension} \\ 0,60 & \text{if compression} \end{cases}$

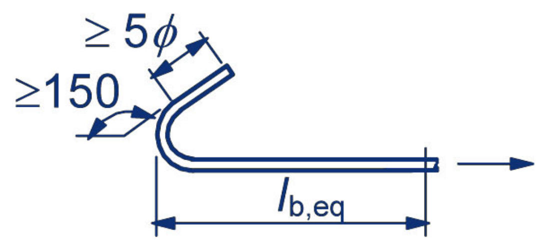
Simplified formulation

- $l_{b,eq} = \alpha_1 \cdot l_{b,rqd}$ Standard bends, hooks, loops
- $l_{b,eq} = \alpha_4 \cdot l_{b,rqd}$ Welded transverse bar

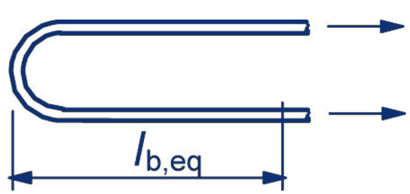


$90^{o} \leq \alpha < 150^{o}$

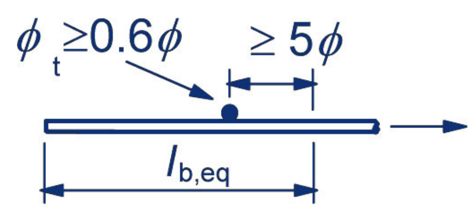
STANDARD BEND



STANDARD HOOK



STANDARD LOOP



WELDED TRANSVERSE BAR

Tables for the building $s_{min} \geq 2 \cdot c_{nom}$

FOOTINGS (C25/30 c_{nom} = 40 mm)

(mm)	$l_{b,d}$ (straight anchorage mm)				$l_{b,eq}$ (std. bend, hook or loop mm)			
	Tension		Compression		Tension		Compression	
ϕ	Good	Poor	Good	Poor	Good	Poor	Good	Poor
8	226	323	323	461	226	323	226	323
10	283	404	404	577	283	404	283	404
12	339	484	484	692	339	484	339	484
14	408	582	565	807	565	807	565	807
16	500	715	646	922	646	922	646	922
20	686	980	807	1153	807	1153	807	1153
25	918	1312	1009	1441	1009	1441	1009	1441

Tables for the building

BEAMS AND SLABS (C25/30 c_{nom} = 30mm)

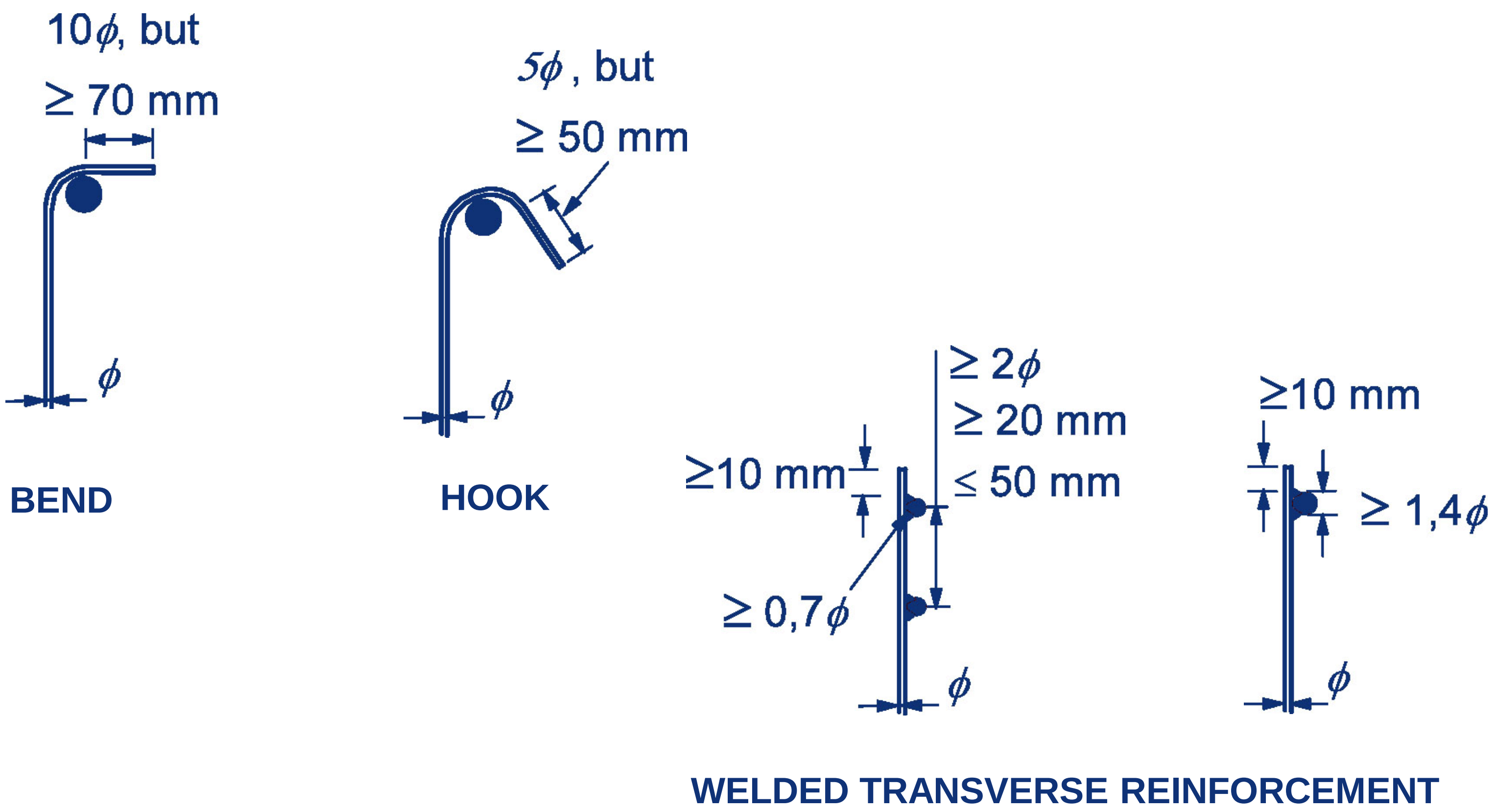
	$l_{b,d}$ (straight anchorage mm)				$l_{b,eq}$ (std. bend, hook or loop mm)			
(mm)	Tension		Compression		Tension		Compression	
ϕ	Good	Poor	Good	Poor	Good	Poor	Good	Poor
8	226	323	323	461	226	323	226	323
10	283	404	404	577	404	577	404	577
12	375	536	484	692	484	692	484	692
14	468	669	565	807	565	807	565	807
16	561	801	646	922	646	922	646	922
20	747	1067	807	1153	807	1153	807	1153
25	979	1398	1009	1441	1009	1441	1009	1441

Tables for the building

COLUMNS (C30/37 c_{nom} = 30mm)

	$l_{b,d}$ (straight anchorage mm)				$l_{b,eq}$ (std. bend, hook or loop mm)			
(mm)	Tension		Compression		Tension		Compression	
ϕ	Good	Poor	Good	Poor	Good	Poor	Good	Poor
8	200	286	286	408	200	286	200	286
10	250	357	357	511	357	511	357	511
12	332	475	429	613	429	613	429	613
14	415	592	500	715	500	715	500	715
16	497	710	572	817	572	817	572	817
20	661	945	715	1021	715	1021	715	1021
25	867	1238	893	1276	893	1276	893	1276

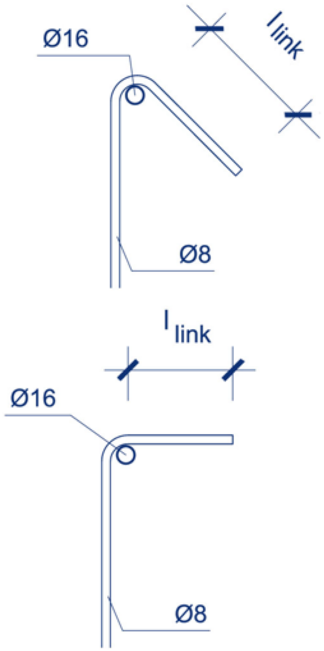
Anchorage of links and shear reinf. [8.5]



Tables for the building

LENGTH AFTER THE CURVE

(mm)	l_{link} (mm)	
ϕ	Bend	Hook
6	70	50
8	80	50
10	100	50
12	120	60



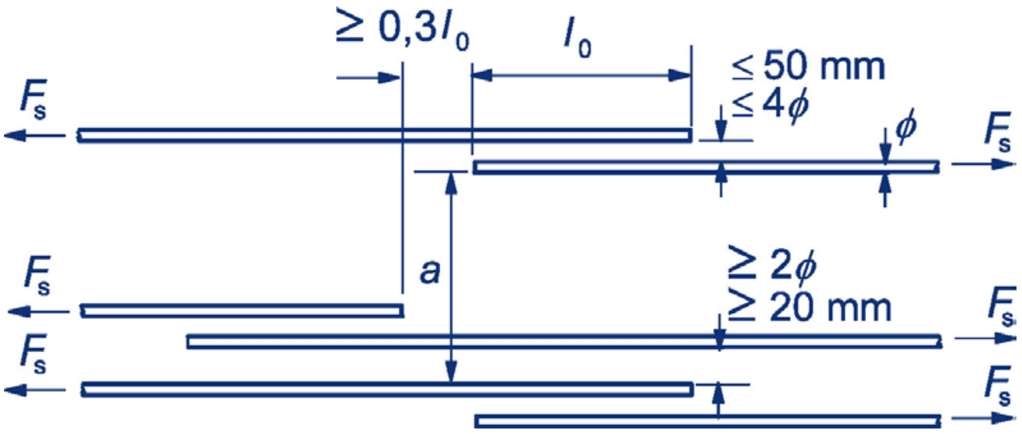
Laps [8.7]

Transmission forces reinforcement $\rightarrow$ reinforcement

Transverse tension stresses

Avoid:

- Large cracks
- Spalling



Methods:

- Lapping of bars
- Welding
- Mechanical couplers

Arrangement

- Should be staggered
- Not located in areas of high moments
- Symmetrically at any section

Design lap length [Table 8.2]

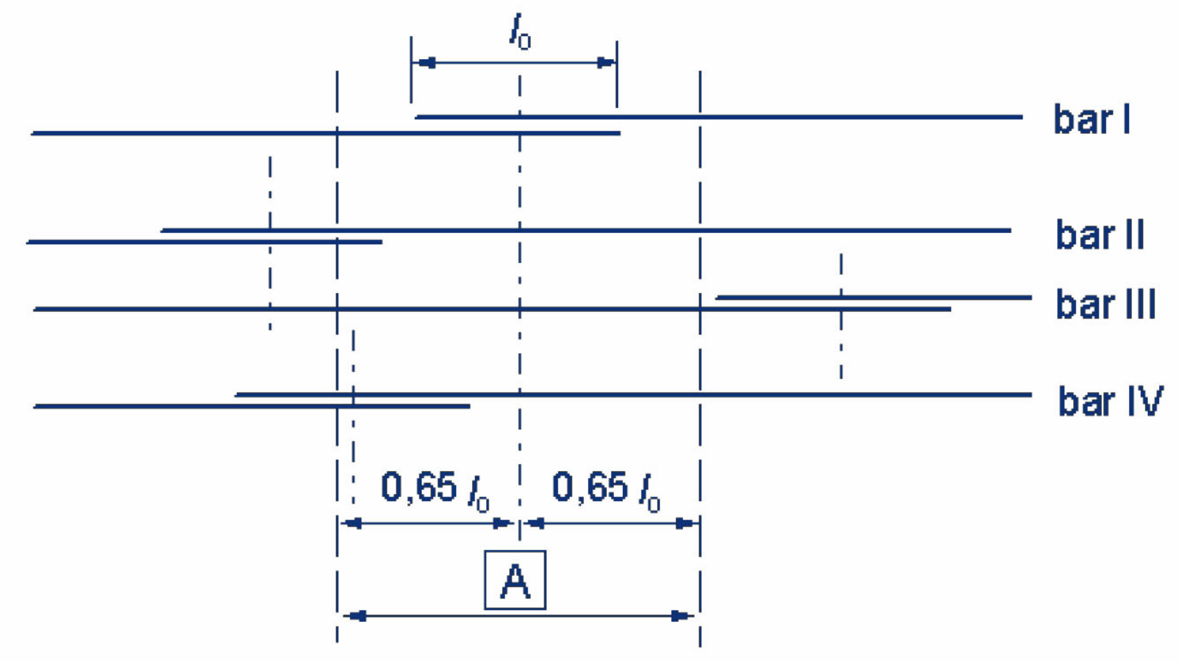
$$l_0 = \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot \alpha_5 \cdot \alpha_6 \cdot l_{b,rqd} \geq l_{0,min}$$

- α_1: Shape of bars
- α_2: Concrete cover
- α_3: Confinement by transv. reinf. not welded
- α_5: Confinement by transv. pressure
- α_6: Percentage of lapped bars within a zone

$$\alpha_6 = \sqrt{\frac{\rho_1}{25}}$$

- $l_{0,min} = \max\left[0{,}30 \cdot \alpha_6 \cdot l_{b,rqd}; 15\phi; 200\text{ mm}\right]$

Design lap length



A: SECTION CONSIDERED

bar II: OUTSIDE

bar III: OUTSIDE

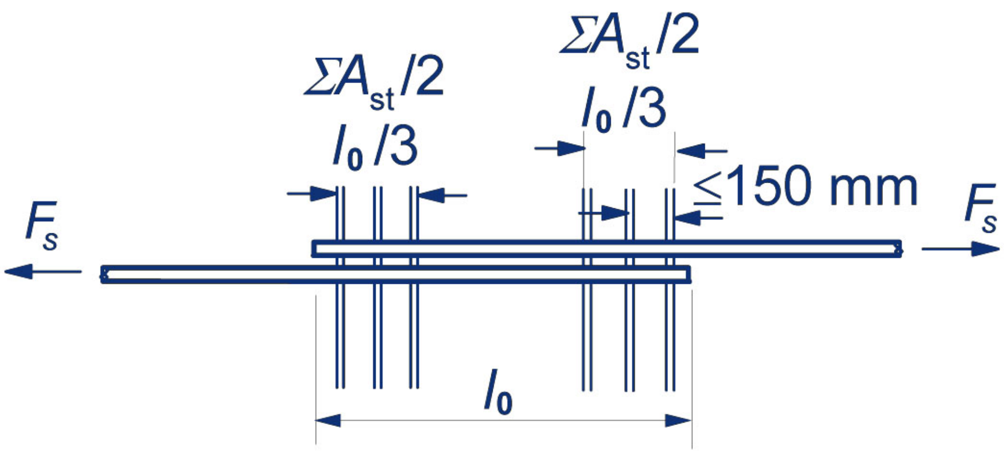
bar IV: INSIDE

ρ_1 = 50 %

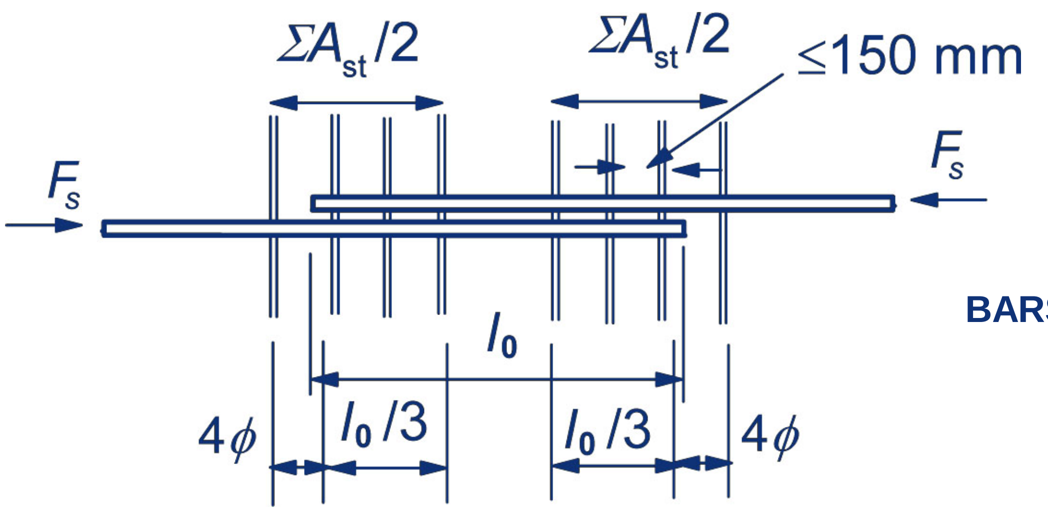
α_6 = 1,41

Transverse reinforcement

- No transv. reinf. if:
  - Either $\phi \leq 20$ mm
  - Or $\rho_1 < 25\%$



BARS IN TENSION



BARS IN COMPRESSION

Tables for the building

BEAMS AND SLABS (C25/30 c_{nom} = 30mm) - TENSION

	Lap length l_o (mm)							
(mm)	Good bond conditions				Poor bond conditions			
ϕ	ρ_1<25	ρ_1=33	ρ_1=50	ρ_1>50	ρ_1<25	ρ_1=33	ρ_1=50	ρ_1>50
8	226	260	316	339	323	371	452	484
10	283	325	396	424	404	464	565	605
12	375	432	525	563	536	617	751	804
14	468	538	655	702	669	769	936	1003
16	561	645	785	841	801	922	1122	1202
20	747	859	1045	1120	1067	1227	1493	1600
25	979	1126	1370	1468	1398	1608	1957	2097

Tables for the building

BEAMS AND SLABS (C25/30 c_{nom} = 30mm) - COMPRESSION

(mm)	Lap length l_o (mm)							
	Good bond conditions				Poor bond conditions			
ϕ	ρ_1<25	ρ_1=33	ρ_1=50	ρ_1>50	ρ_1<25	ρ_1=33	ρ_1=50	ρ_1>50
8	323	371	452	484	461	530	646	692
10	404	464	565	605	577	663	807	865
12	484	557	678	726	692	796	969	1038
14	565	650	791	848	807	928	1130	1211
16	646	743	904	969	922	1061	1291	1384
20	807	928	1130	1211	1153	1326	1614	1730
25	1009	1160	1413	1513	1441	1658	2018	2162

Tables for the building

COLUMNS (C30/37 c_{nom} = 30mm) – TENSION

	Lap length l_o (mm)							
(mm)	Good bond conditions				Poor bond conditions			
ϕ	ρ_1<25	ρ_1=33	ρ_1=50	ρ_1>50	ρ_1<25	ρ_1=33	ρ_1=50	ρ_1>50
8	200	230	280	300	286	329	400	429
10	250	288	350	375	357	411	500	536
12	332	382	465	499	475	546	665	712
14	415	477	580	622	592	681	829	888
16	497	571	695	745	710	816	994	1065
20	661	760	926	992	945	1086	1322	1417
25	867	997	1213	1300	1238	1424	1733	1857

Tables for the building

COLUMNS (C30/37 c_{nom} = 30mm) – COMPRESSION

(mm)	Lap length l_o (mm)							
	Good bond conditions				Poor bond conditions			
ϕ	ρ_1<25	ρ_1=33	ρ_1=50	ρ_1>50	ρ_1<25	ρ_1=33	ρ_1=50	ρ_1>50
8	286	329	400	429	408	470	572	613
10	357	411	500	536	511	587	715	766
12	429	493	600	643	613	705	858	919
14	500	575	701	751	715	822	1001	1072
16	572	658	801	858	817	939	1144	1225
20	715	822	1001	1072	1021	1174	1430	1532
25	893	1028	1251	1340	1276	1468	1787	1915

DETAILING OF STRUCTURAL MEMBERS [9]

Satisfy the requirements of:

- Safety
- Serviceability
- Durability

Consistency with design models

Minimum areas of reinforcement to:

- Prevent brittle failure
- Prevent wide cracks
- Resist forces from restrained actions

Reflections

- No unique solutions
- Need to simplify; usually many bars to dispose
- Arrangement: complexity vs simplicity
- Economics and sustainability
- Aspects of cost
    - Quantity of reinforcement
    - Labour force
    - Size of the structure

STRUCTURAL MEMBERS

FOOTING B-2

- Reinforcement calculation
- Detailing

BEAMS

- BEAM 2 – Case 1
- BEAM B – Case 3

SLABS

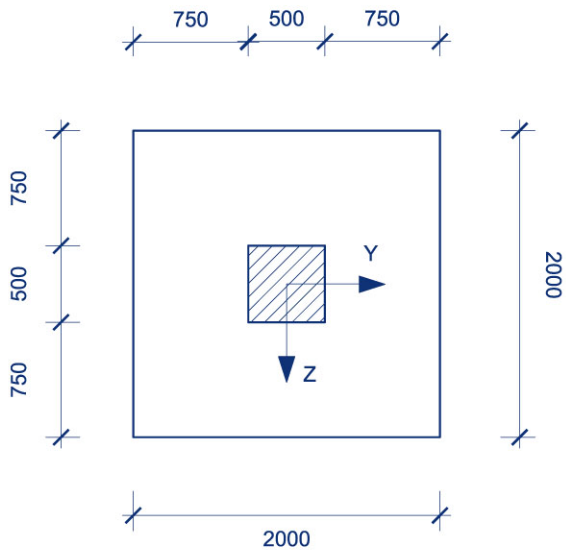
- SLAB AB12 – Case 1

COLUMNS

- COLUMN B-2 – Case 2

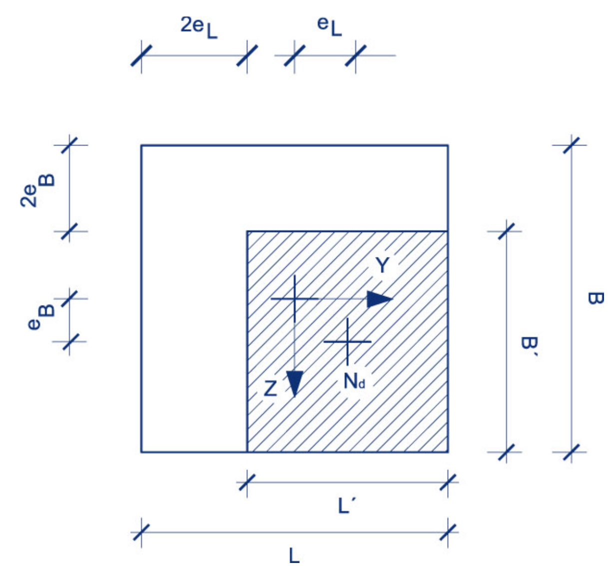
FOOTING B-2

h = 800 mm
Concrete: C25/30
Steel: B500



Model for soil bearing resistance

$$\sigma_{Ed} = \frac{N_{Ed}}{B' \cdot L'} \qquad \begin{cases} L' = L - 2 \cdot e_L \text{ and } e_L = -\dfrac{M_{Ed,z}}{|N_{Ed}|} \\ B' = L - 2 \cdot e_B \text{ and } e_B = \dfrac{M_{Ed,y}}{|N_{Ed}|} \end{cases}$$



FOOTING B-2

Internal forces ground level ULS $\leftarrow$ structural analysis

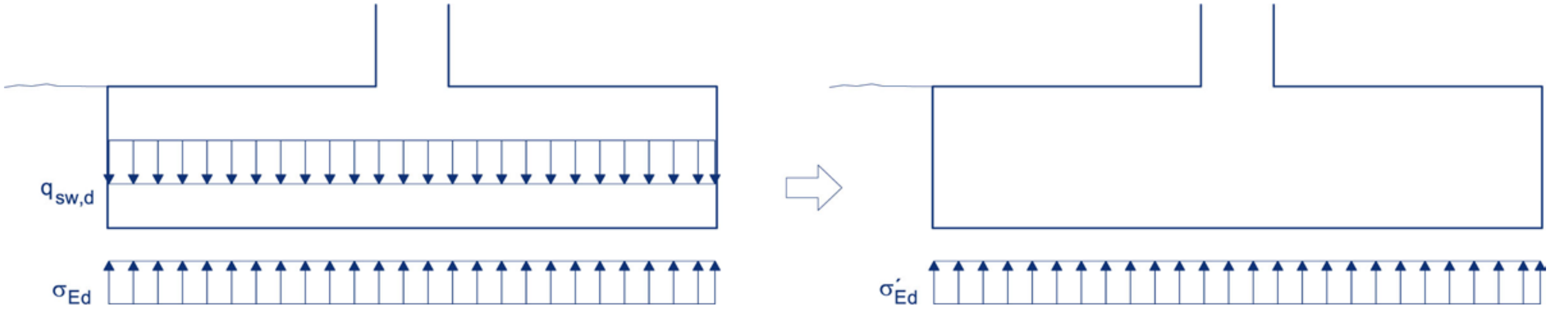
Combination	(kN) N_{Ed}	(m·kN) $M_{Ed,y}$	(m·kN) $M_{Ed,z}$	(mm) e_L	(mm) e_B
1	-4554,80	3,82	-3,78	0,8	0,8
2	-4837,96	-3,11	0,60	-0,1	-0,6
3	-4990,35	-2,71	0,66	-0,1	-0,5
4	-4985,91	-3,26	-2,31	0,5	-0,7
5	-4491,62	-2,73	-1,38	0,3	-0,6
6	-5435,54	-2,98	-3,46	0,6	-0,5
7	-5359,70	1,44	0,71	-0,1	0,3
8	-5359,70	1,44	0,71	-0,1	0,3
9	-4502,78	4,08	-3,50	0,8	0,9
10	-5780,18	-2,36	-4,49	0,8	-0,4

$$B' \approx L' \approx 2{,}00 \text{ m}$$

$$\sigma_{Ed} = \frac{N_{Ed}}{B' \cdot L'} = 1445 \text{ kN/m}^2$$

FOOTING B-2

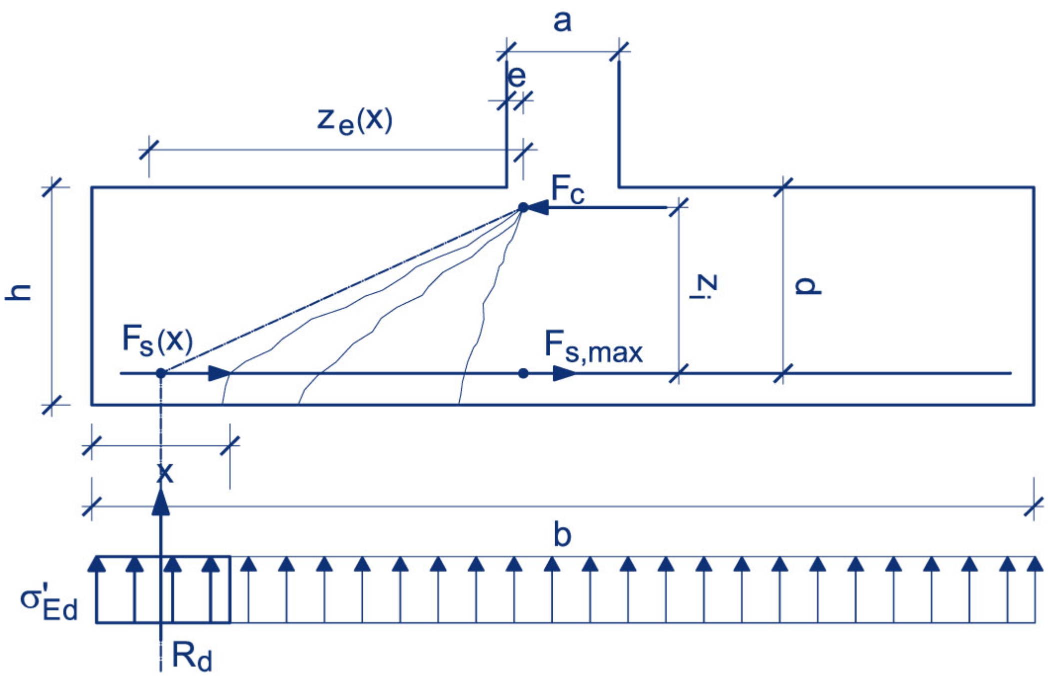
Soil pressure + Self weight = Effective pressure



$$\sigma'_{Ed} = \sigma_{Ed} - q_{sw,d} = 1418\,kN/m^2$$

FOOTING B-2

Model for anchorage of bars [9.2.2.2]



$$F_s(x) = R_d(x) \cdot \frac{z_e(x)}{z_i}$$

$$F_{s,max} = F_s\left(\frac{b}{2} - 0{,}35 \cdot a\right)$$

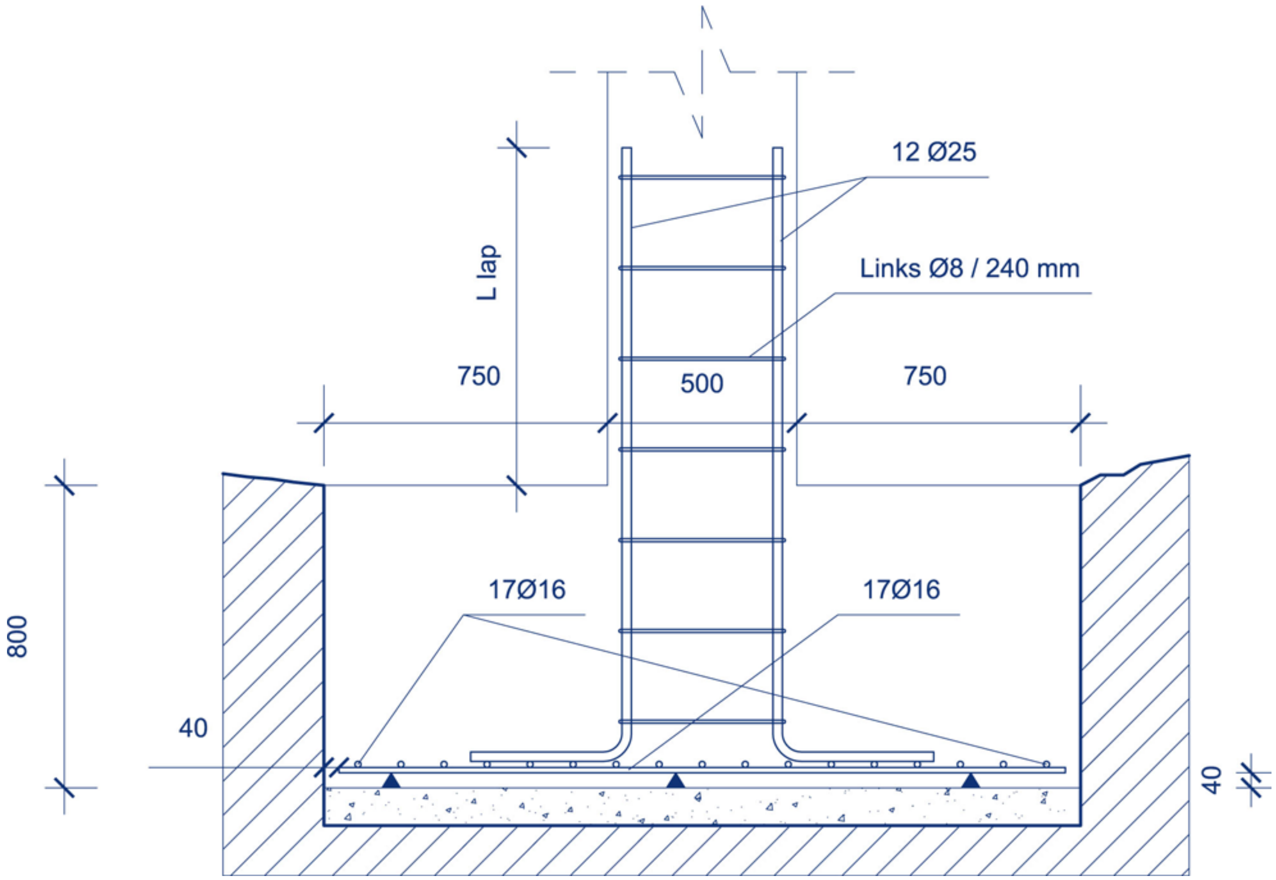
$$F_{s,max} = F_s(0{,}825) = 1457{,}9 \text{ kN}$$

$$A_s = \frac{F_{s,max}}{f_{yd}} = 3353 \text{ mm}^2$$

$$(7\phi16)$$

FOOTING B-2

Arrangement of reinforcement



FOOTING B-2

Verification straight anchorage first inclined crack: $l_b + c_{nom} \leq x_{min}$

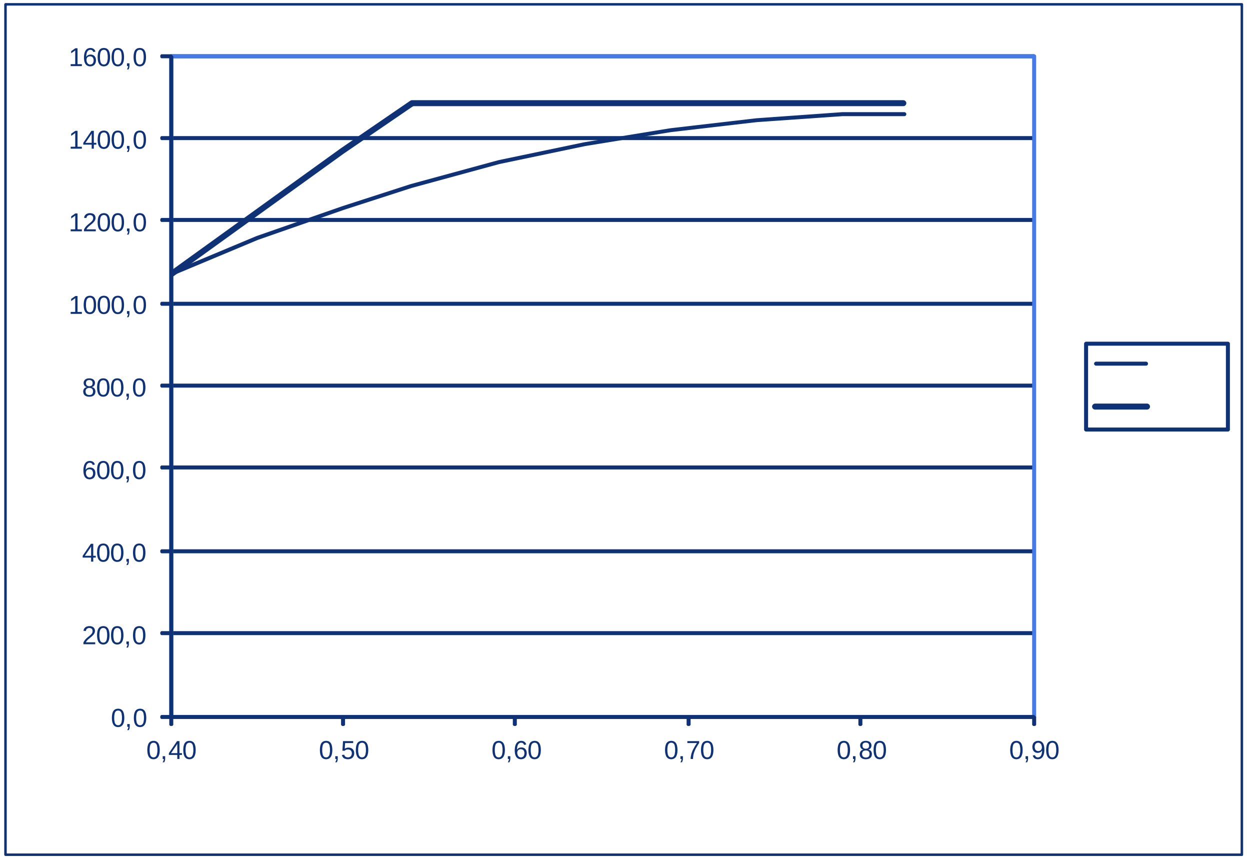
$x_{min} = h/2 = 400$ mm

$$F_s\left(x_{min}\right) = F_s\left(0,40\right) = 1071,0\ \text{kN}$$

$$l_b = \frac{F_s\left(x_{min}\right)}{A_s f_{yd}} \cdot l_{bd} = \frac{1071,0}{1485,7} \cdot 500 = 360\ \text{mm}$$

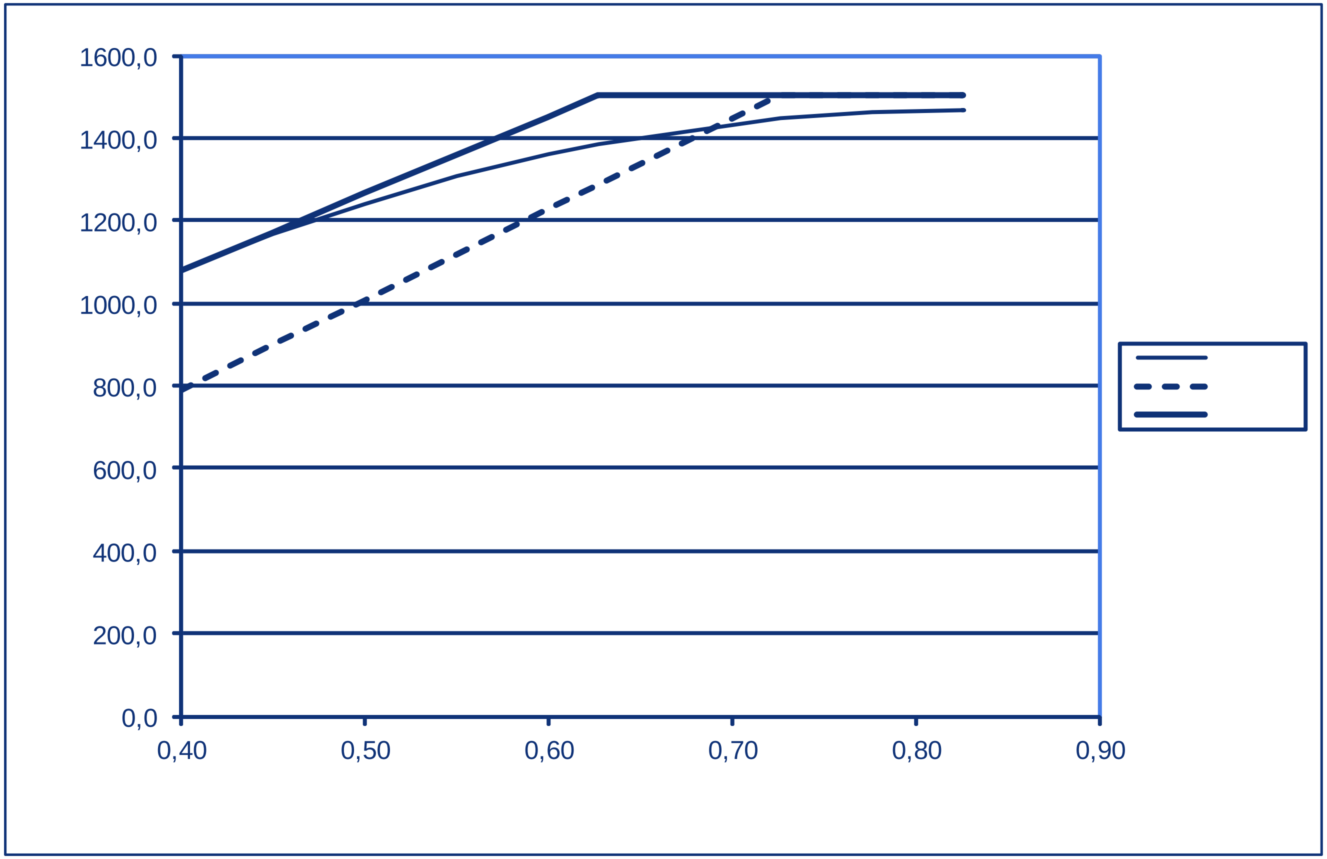
$$360 + 40 = 400 \quad \text{Ok}$$

FOOTING B-2

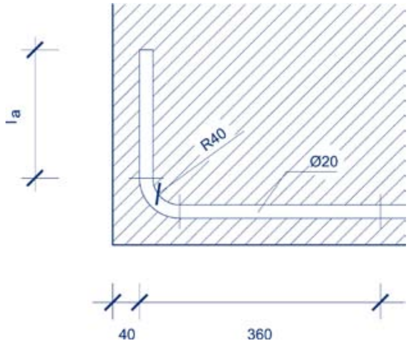

1600,0
1400,0
1200,0
1000,0
800,0
600,0
400,0
200,0
0,0
0,40
0,50
0,60
0,70
0,80
0,90

FOOTING B-2

If we dispose 11ϕ20



$F_{s,Rd1}$: straight anch.



$F_{s,Rd2}$: bend anch.

l_a = 195 mm

BEAMS [9.2]

Materials

- Concrete: f_{ck} = 25 N/mm^2 γ_c = 1,50
- Steel: f_{yk} = 500 N/mm^2 γ_s = 1,15

Longitudinal reinforcement

$$A_{s,min} = 0,26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b_t \cdot d \not< 0,0013 \cdot b_t \cdot d \quad \Rightarrow \quad A_{s,min} = 0,00133 \cdot b_t \cdot d$$

$$A_{s,max} = 0,04 \cdot A_c$$

$$a_l = z \cdot \frac{(\cot\theta - \cot\alpha)}{2}$$

BEAMS [9.2]

Transverse reinforcement

$$\rho_{w,min} = \frac{0{,}08 \cdot \sqrt{f_{ck}}}{f_{yk}} \quad \Rightarrow \quad \rho_{w,min} = 0{,}0008$$

$$\rho_w = \frac{A_{sw}}{s \cdot b_w \cdot \sin\alpha}$$

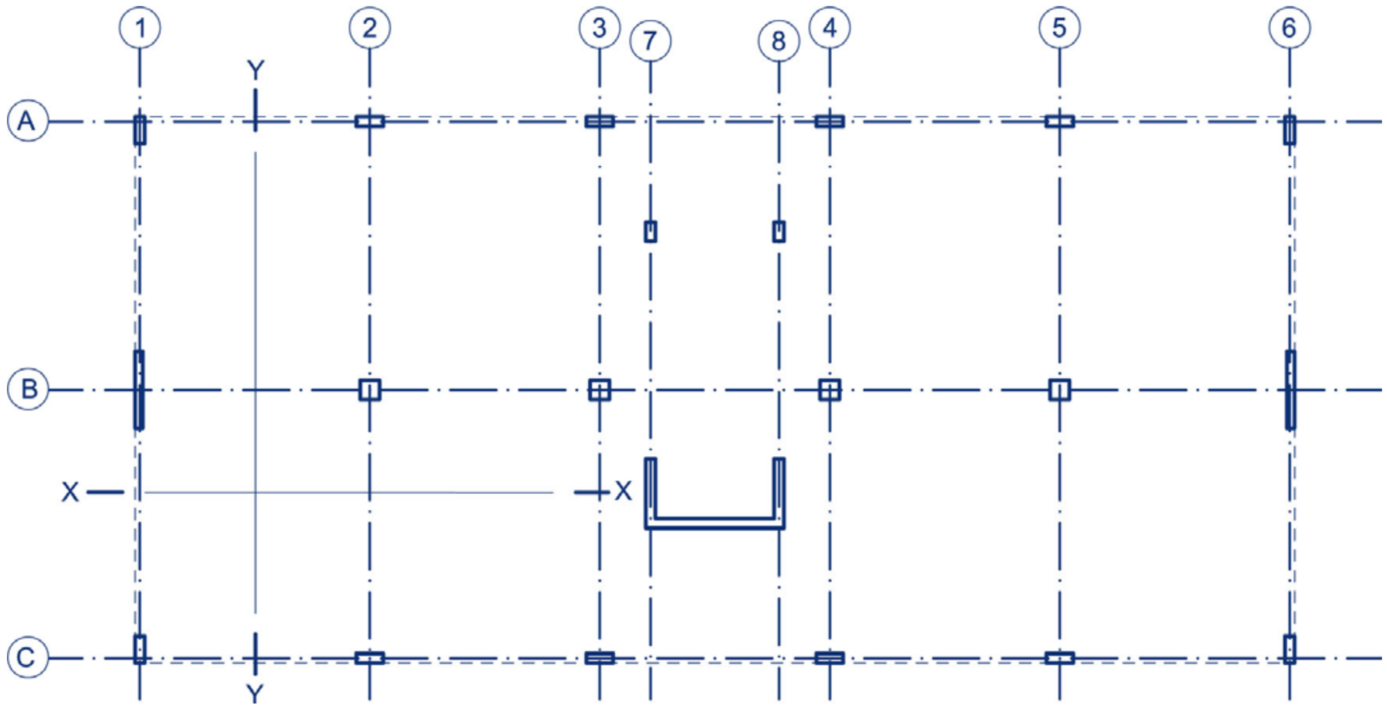
$$\left(\frac{A_{sw}}{s}\right)_{min} = 0{,}0008 \cdot b_w$$

$$s_{l,max} = 0{,}75 \cdot d \cdot (1 + \cot\alpha) = 0{,}75 \cdot d$$

$$s_{t,max} = 0{,}75 \cdot d \not> 600 \text{ mm}$$

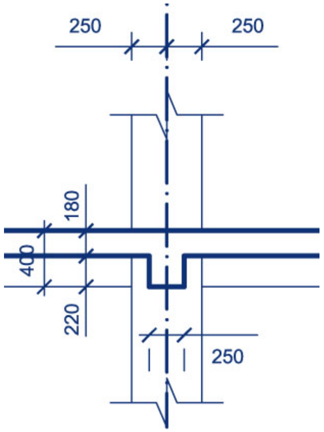
BEAM 2 – Case 1

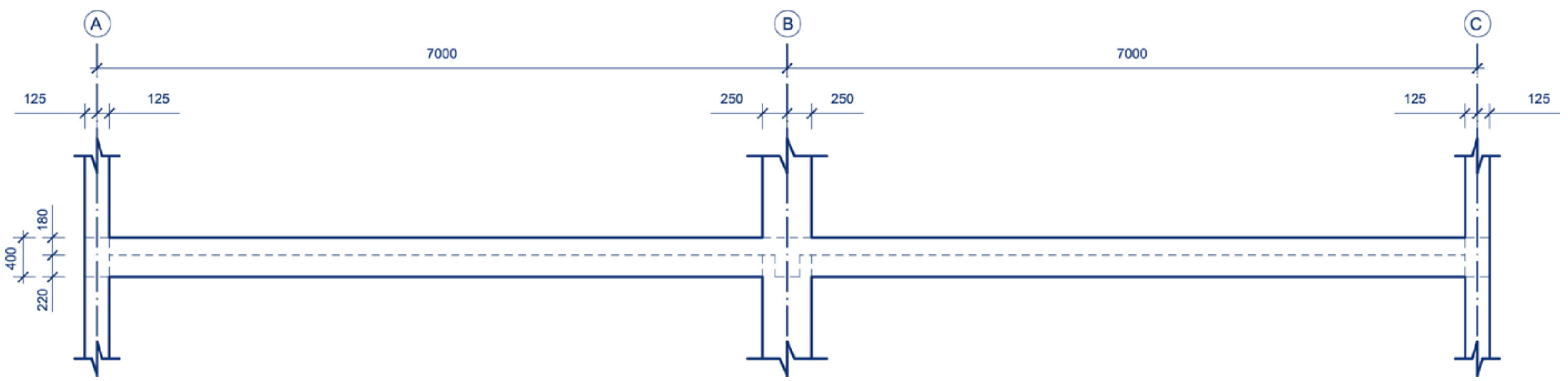
Two way slab on beams



Geometry

Two way slab on beams





Longitudinal reinforcement

If we suppose ϕ = 16 mm ϕ_w = 8 mm

$$d = h - c_{nom} - \phi_w - \frac{\phi}{2} = 354 \text{ mm}$$

$$b_t = \begin{cases} 250 \text{ mm} & \text{for positive moments} \\ 1100 \text{ mm} & \text{for negative moments} \end{cases}$$

$$b_w = 250 \text{ mm}$$

$$A_{s,min} = 0{,}00133 \cdot b_t \cdot d = \begin{cases} 118 \text{ mm}^2 & \text{for positive moments} \\ 518 \text{ mm}^2 & \text{for negative moments} \end{cases}$$

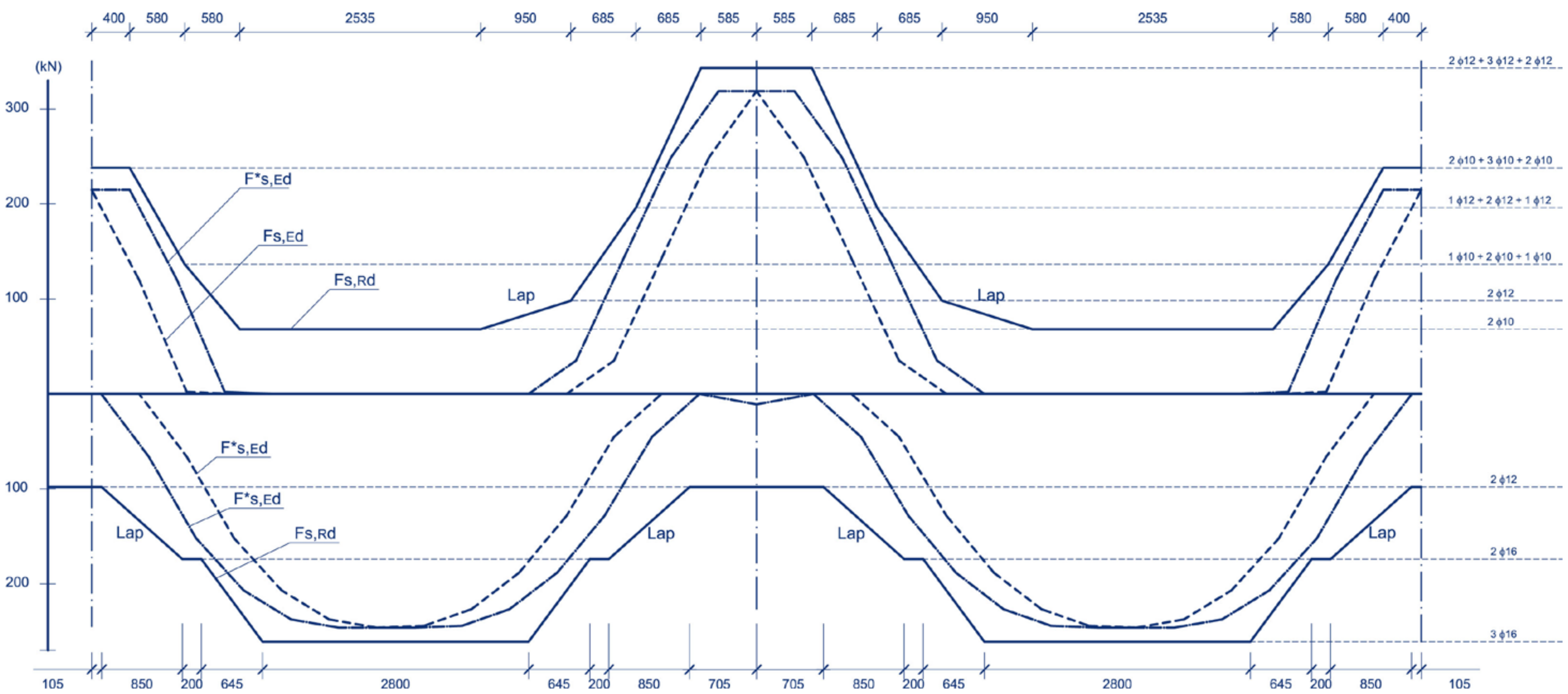
$$A_{s,max} = 0{,}04 \cdot A_c = 6520 \text{ mm}^2$$

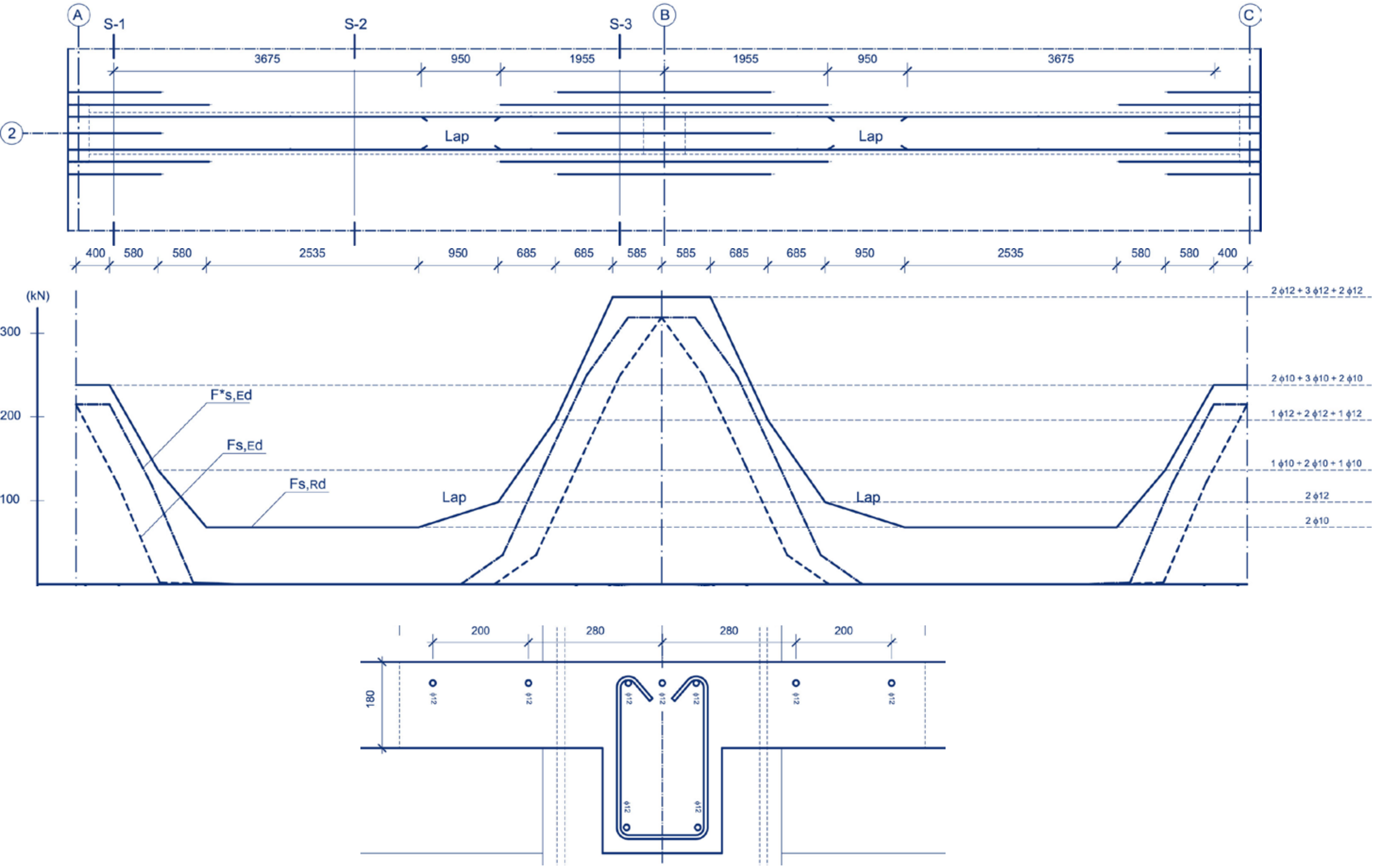
$$a_l = 0{,}45 \cdot d \cdot \cot\theta = 400 \text{ mm} \quad \text{for } \cot\theta = 2{,}5$$

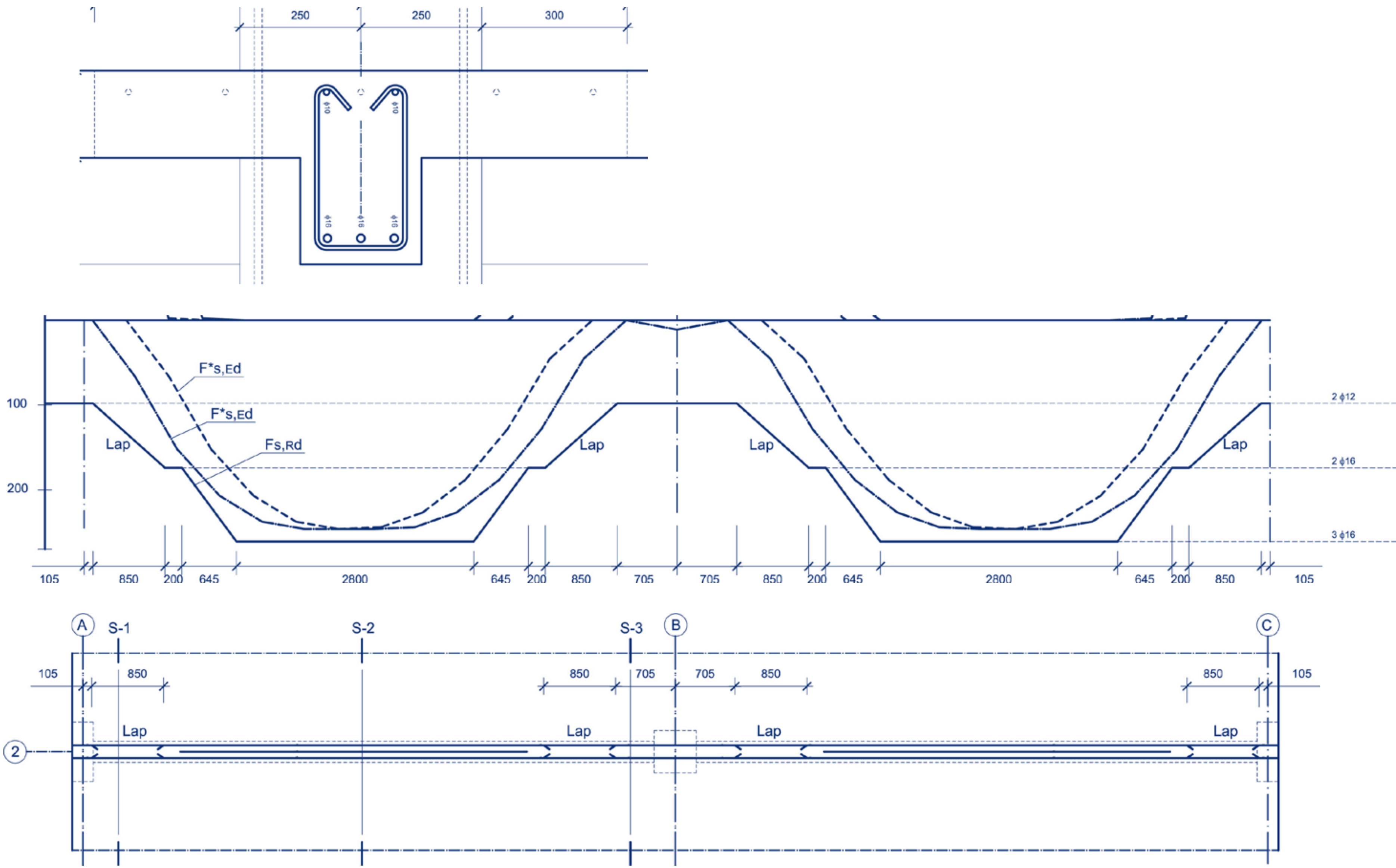
BEAM 2 – Case 1

STRUCTURAL ANALISYS | SHIFTING a_l | DISPOSE REINFORCEMENT

Envelop internal forces → Envelop $F_{s,Ed}$ → Envelop $F^{*}_{s,Ed}$ → $F_{s,Rd}$




A
B
C
S-1
S-2
S-3
3675
950
1955
1955
950
3675
2
Lap
Lap
400
580
580
2535
950
685
685
585
585
685
685
950
2535
580
580
400
(kN)
300
200
100
F*s,Ed
Fs,Ed
Fs,Rd
Lap
Lap
2 ϕ12 + 3 ϕ12 + 2 ϕ12
2 ϕ10 + 3 ϕ10 + 2 ϕ10
1 ϕ12 + 2 ϕ12 + 1 ϕ12
1 ϕ10 + 2 ϕ10 + 1 ϕ10
2 ϕ12
2 ϕ10
200
280
280
200
180

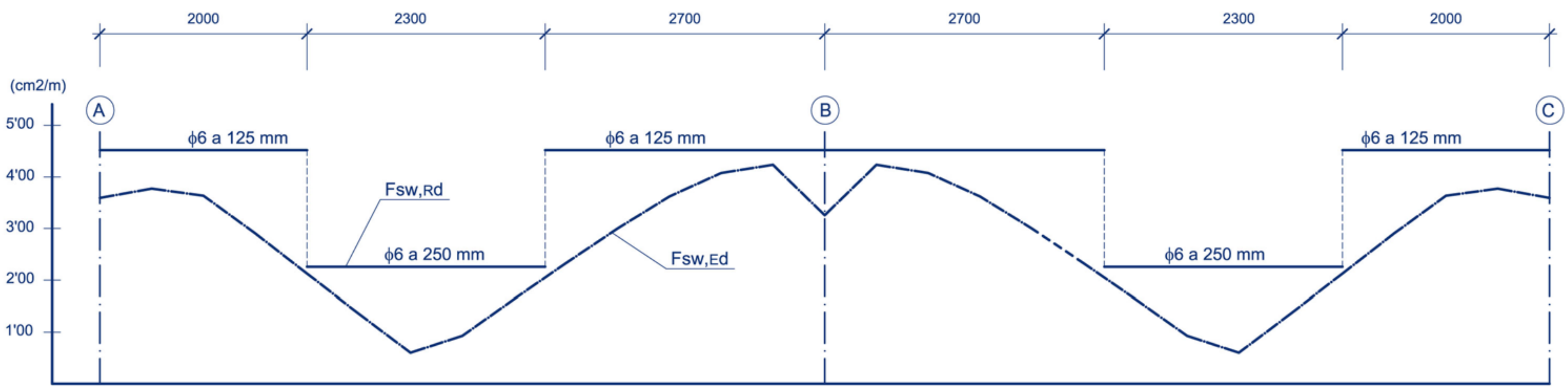

250
250
300
F*s,Ed
F*s,Ed
Fs,Rd
100
200
Lap
Lap
Lap
Lap
2 ϕ12
2 ϕ16
3 ϕ16
105
850
200
645
2800
645
200
850
705
705
850
200
645
2800
645
200
850
105
A
S-1
S-2
S-3
B
C
105
850
850
705
705
850
850
105
Lap
Lap
Lap
Lap
2

Shear reinforcement $F_{sw,Rd} \geq F_{sw,Ed}$

$$\left(\frac{A_{sw}}{s}\right)_{min} = 0{,}0008 \cdot b_w = 0{,}20 \ \frac{mm^2}{mm}$$

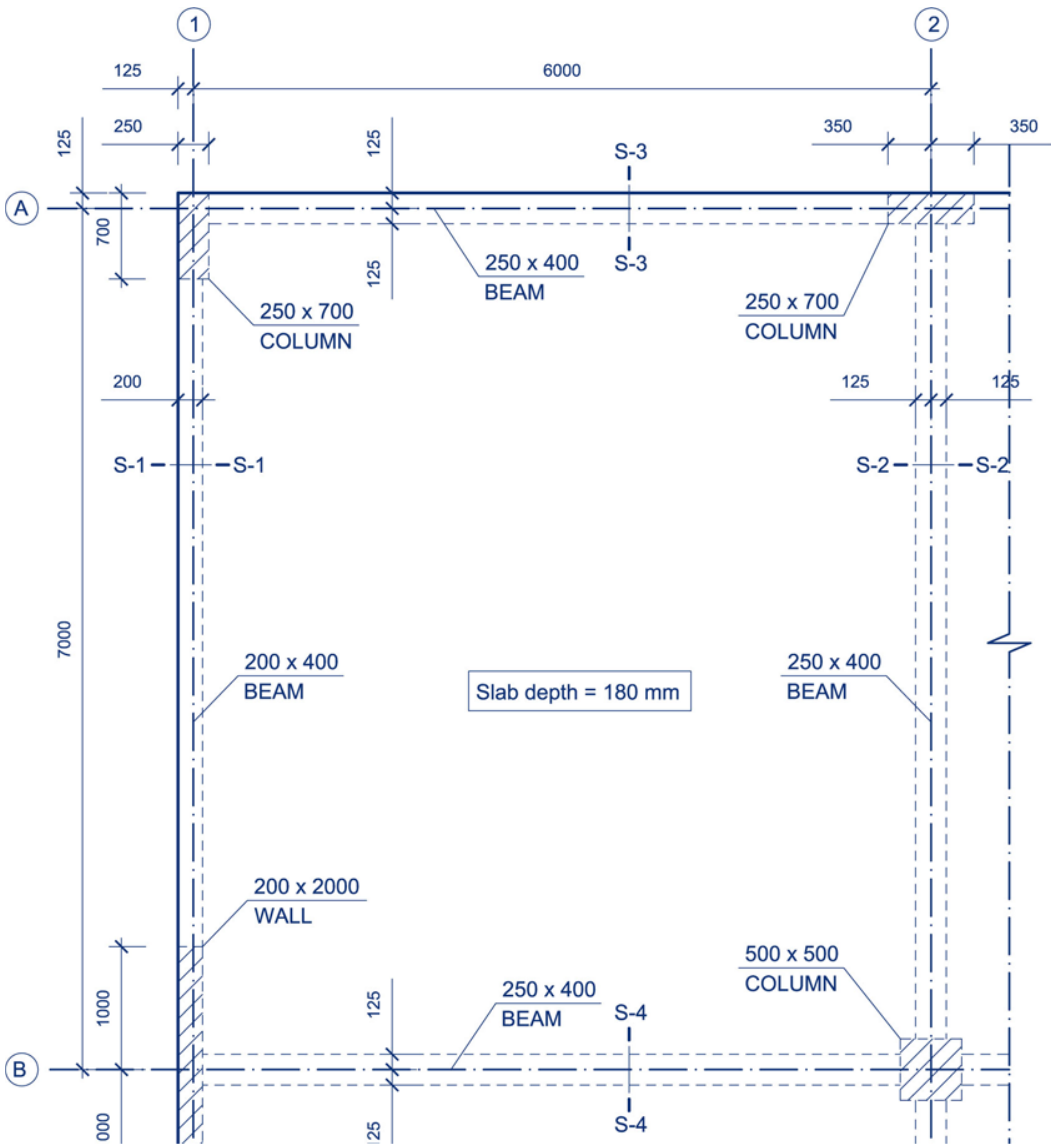
$$s_{l,max} = 0{,}75 \cdot d = 266 \ mm$$

$$s_{t,max} = 0{,}75 \cdot d = 266 \ mm$$



SLAB AB12 – Case 1

Same material properties than beams



Detailing provisions [9.3]

$$d = h - c_{nom} - \phi - \frac{\phi}{2} = 180 - 30 - 12 - 6 = 132 \text{ mm} \qquad (\phi = 12 \text{ mm})$$

$$b_t = 1 \text{mm}$$

$$A_{s,min} = 0{,}00133 \cdot b_t \cdot d = 0{,}18 \frac{\text{mm}^2}{\text{mm}}$$

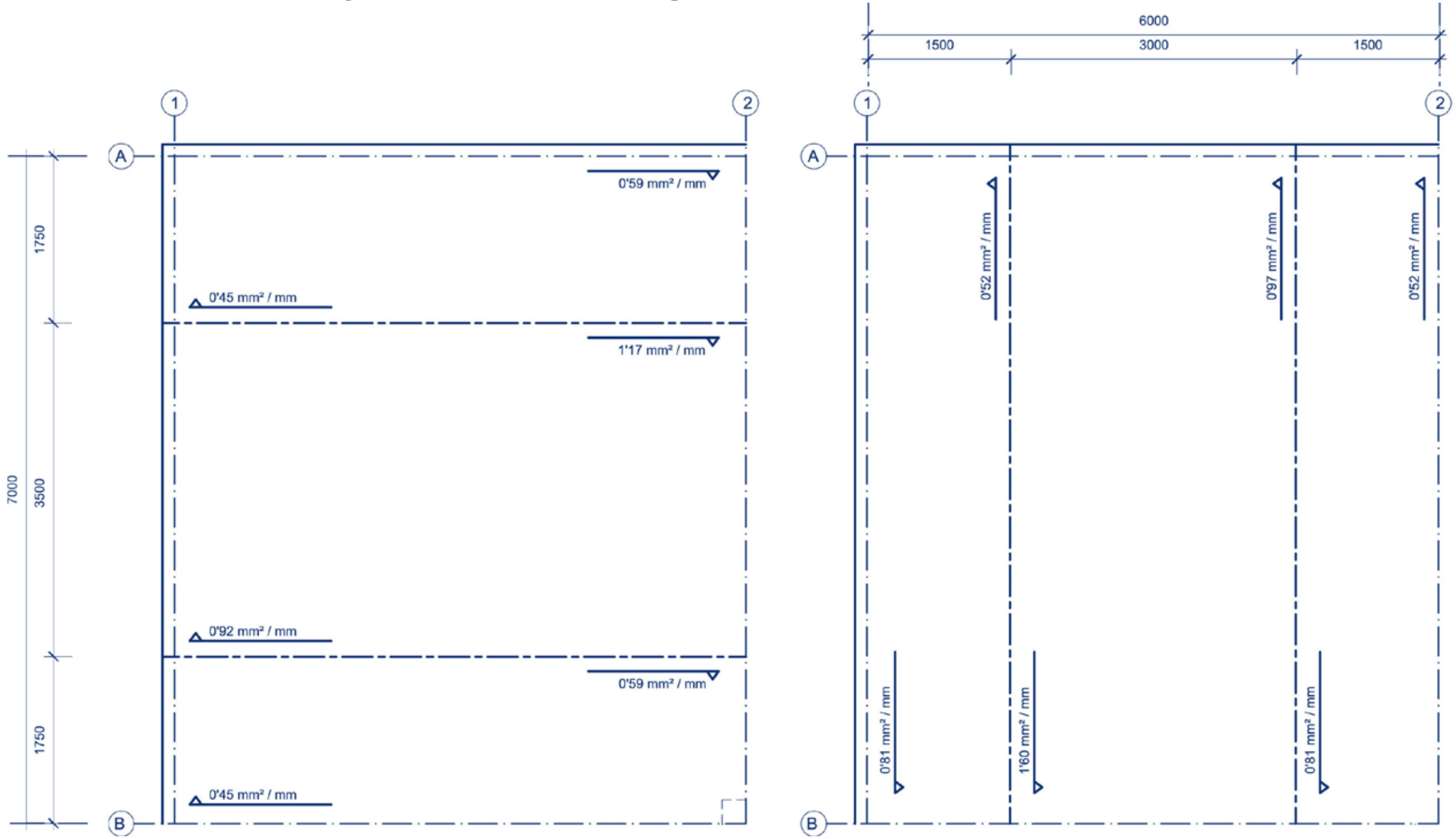
$$A_{s,max} = 0{,}04 \cdot A_c = 7{,}20 \frac{\text{mm}^2}{\text{mm}}$$

$$s_{max,slabs} = 2{,}0 \cdot h \not> 250 \text{ mm} \quad \Rightarrow \quad s_{max,slabs} = 250 \text{ mm}$$

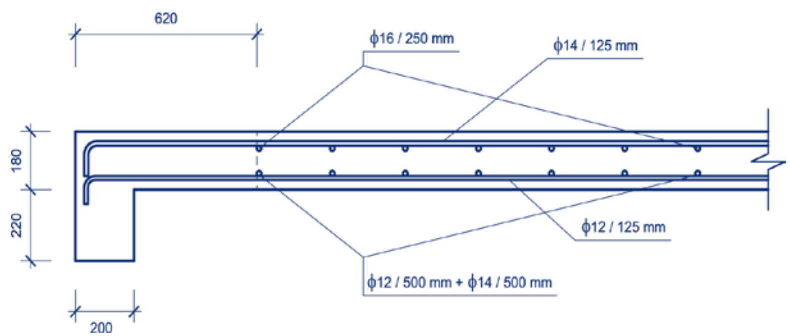
$$a_l = d = 132 \text{mm}$$

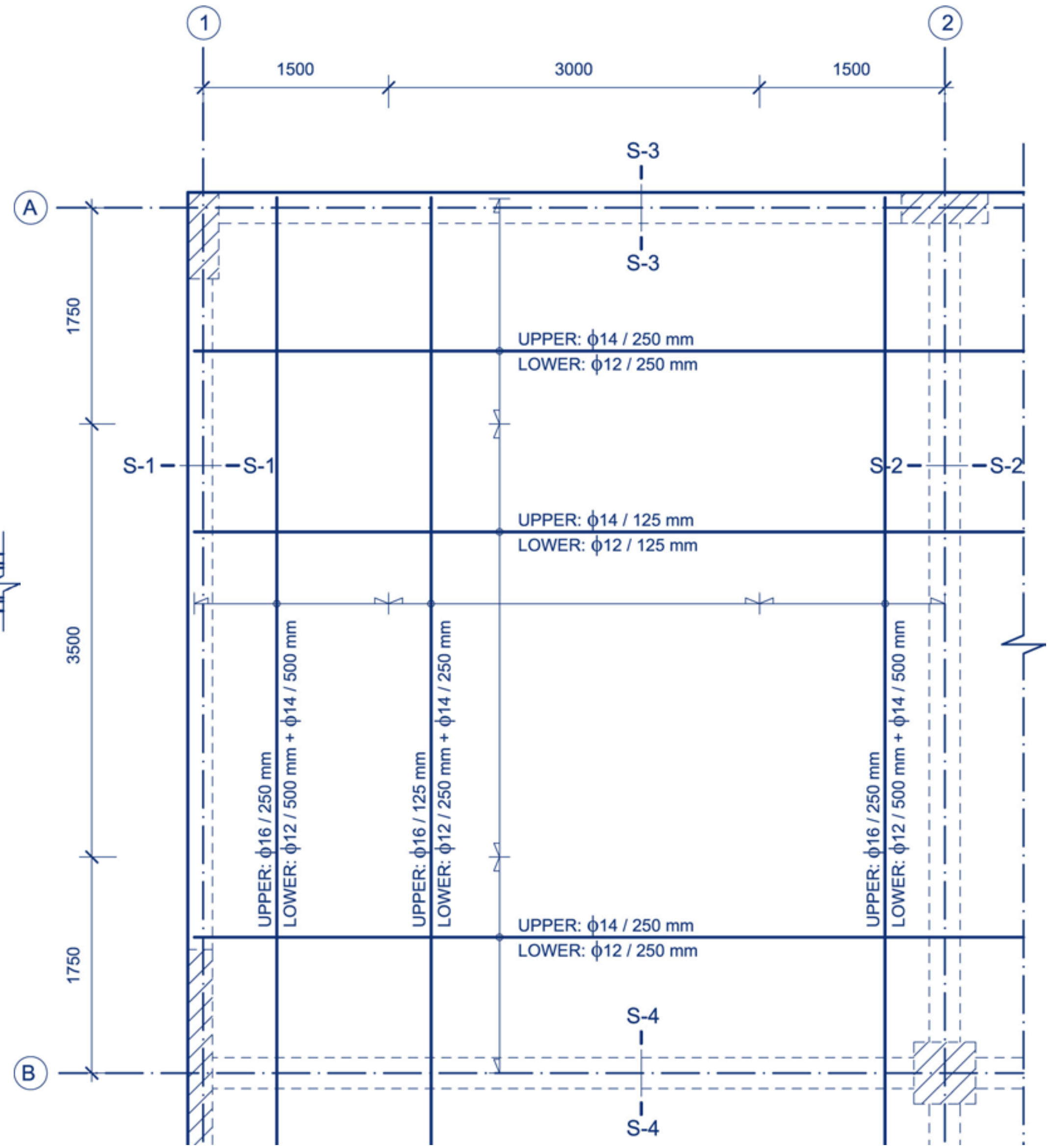
Design reinforcement

Structural analysis → Bending moments → Reinforcement



Reinforcement





COLUMNS [9.5]

Longitudinal reinforcement

$$\phi_{min} = 8 \text{ mm}$$

$$A_{s,min} = \max\left[\frac{0{,}10 \cdot N_{Ed}}{f_{yd}}; 0{,}002 \cdot Ac\right]$$

$$A_{s,max} = 0{,}04 \cdot A_c$$

Transverse reinforcement

$$\phi_{t,min} = \max\left[6 \text{ mm}; \frac{1}{4} \cdot \phi_{long}\right]$$

$$s_{t,max} = \min\left\lfloor 20 \cdot \phi_{long}; b_{min}; 400 \text{ mm}\right\rfloor$$

Factor 0,60:

- Near beams/slab (h)
- Lap length if ϕ>14 mm

Compression bars:

- Not farer than 150 mm from a restrained bar

COLUMN B-2 – Case 2

Materials:

- Concrete: f_{ck} = 25 N/mm^2 γ_c = 1,50
- Steel: f_{yk} = 500 N/mm^2 γ_s = 1,15

Longitudinal reinforcement

$$\phi_{min} = 8\ \text{mm}$$

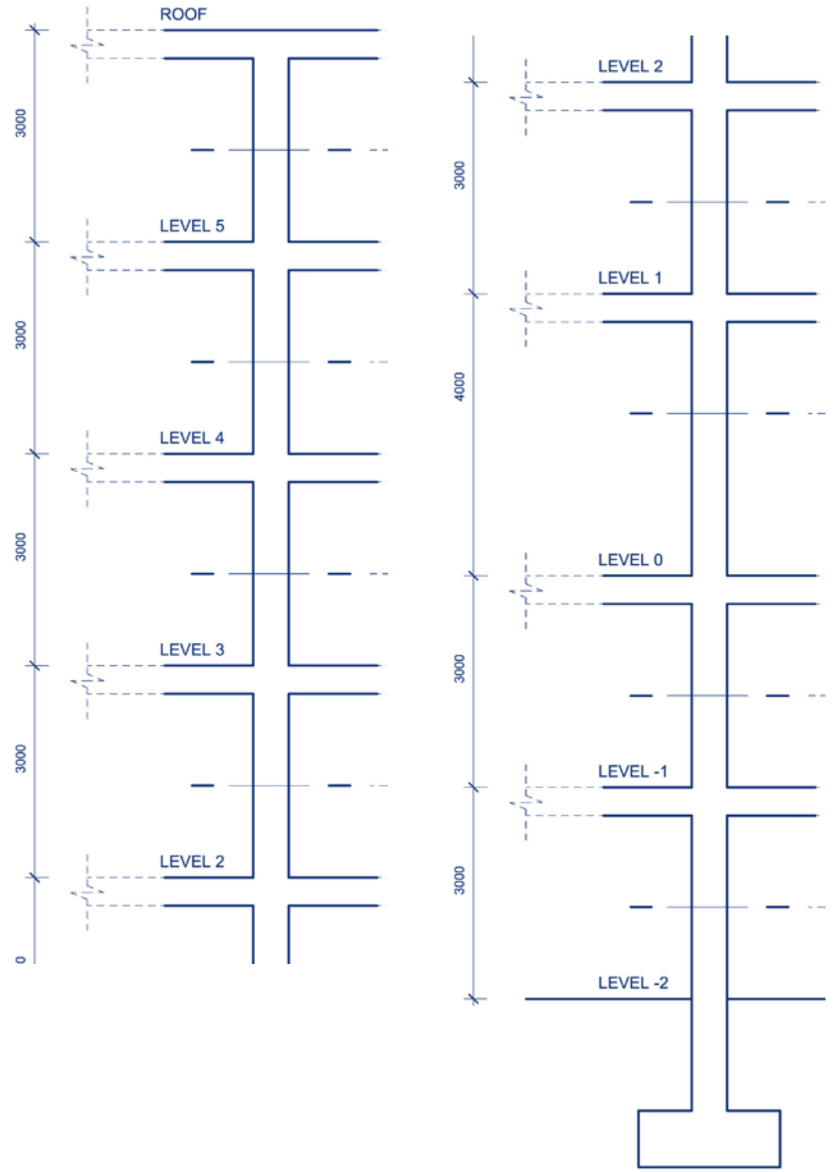
$$A_{s,min} = \max\left[0{,}23 \cdot N_{Ed};\ 500\ \text{mm}^2\right]$$

$$A_{s,max} = 10000\ \text{mm}^2$$

Transverse reinforcement

$$\phi_{t,min} = \begin{cases} 6\ \text{mm} & \text{if } \phi_{long} \le 24\ \text{mm} \\ \dfrac{\phi_{long}}{4} & \text{if } \phi_{long} > 24\ \text{mm} \end{cases}$$

$$s_{t,max} = \min\left[20 \cdot \phi_{long};\ 400\ \text{mm}\right]$$



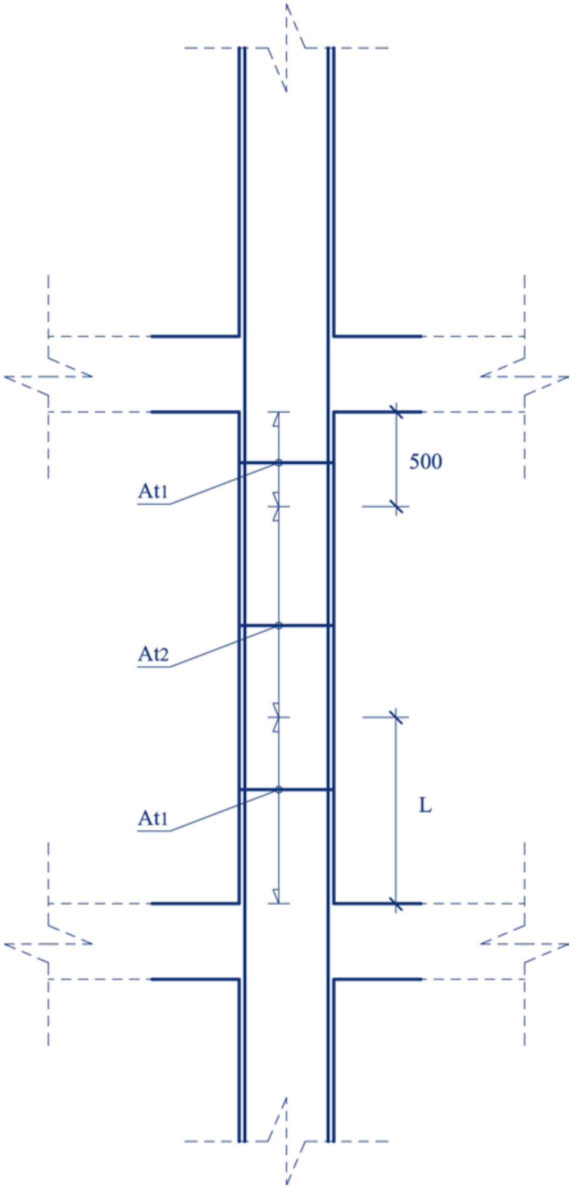
Longitudinal reinforcement

Structural analysis $\rightarrow A_{s,rqd} \rightarrow A_{s,disp}$

	[mm²]	[mm²]	[mm²]	[mm²]	
Floor	$A_{s,rqd}$	$A_{s,min1}$	$A_{s,min2}$	$A_{s,disp}$	
L-2/L-1	5581	1305	500	5892	12 ϕ 25
L-1/L0	3551	1177	500	3768	12 ϕ 20
L0/L1	1082	1012	500	1232	8 ϕ 14
L1/L2	0	838	500	904	8 ϕ 12
L2/L3	0	670	500	904	8 ϕ 12
L3/L4	0	504	500	628	8 ϕ 10
L4/L5	0	344	500	628	8 ϕ 10
L5/Roof	0	216	500	628	8 ϕ 10

Transverse reinforcement

Floor	[mm] $\phi_{t,min}$	[mm] $s_{t,max}$	[mm] L	Links A_{t1}	Links A_{t2}
L-2/L-1	8	400	1340	ϕ 8 - 240	ϕ 8 - 400
L-1/L0	6	400	1340	ϕ 6 - 240	ϕ 6 - 400
L0/L1	6	280	1072	2 ϕ 6 - 160	2 ϕ 6 - 280
L1/L2	6	240	751	2 ϕ 6 - 140	2 ϕ 6 - 240
L2/L3	6	240	643	2 ϕ 6 - 140	2 ϕ 6 - 240
L3/L4	6	200	643	2 ϕ 6 - 120	2 ϕ 6 - 200
L4/L5	6	200	536	2 ϕ 6 - 120	2 ϕ 6 - 200
L5/Roof	6	200	536	2 ϕ 6 - 120	2 ϕ 6 - 200



DETAILING OF COLUMN B2 – CASE 2

Reinforcement