Limit state design and verification

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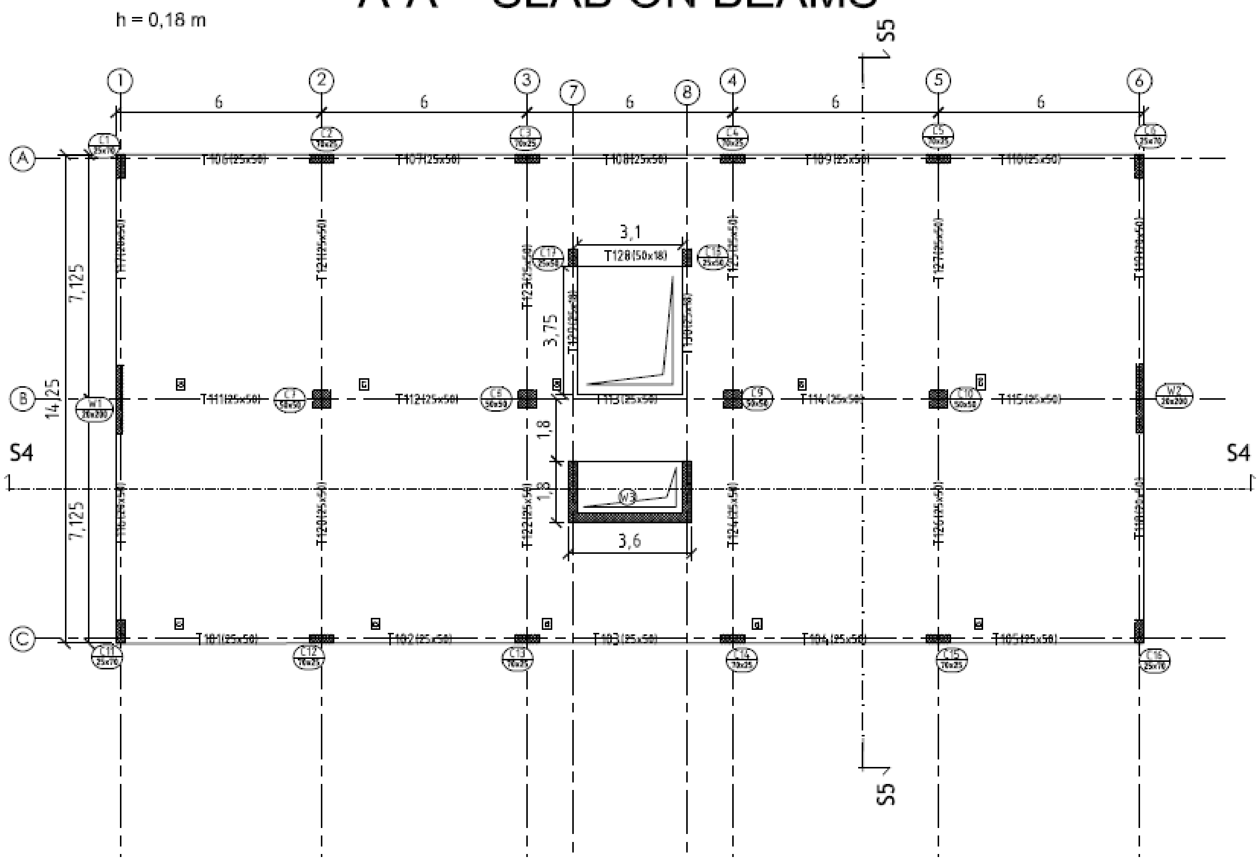
25 October 2011

Delft University of Technology

Flat slab on beams

A-A SLAB ON BEAMS



To be considered: beam axis 2

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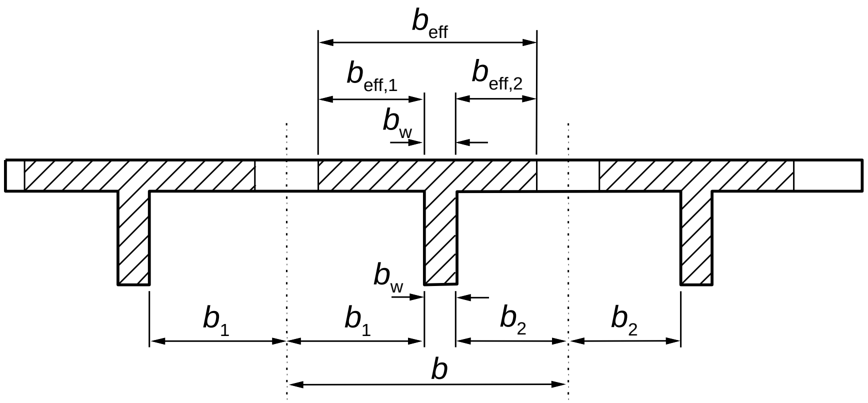
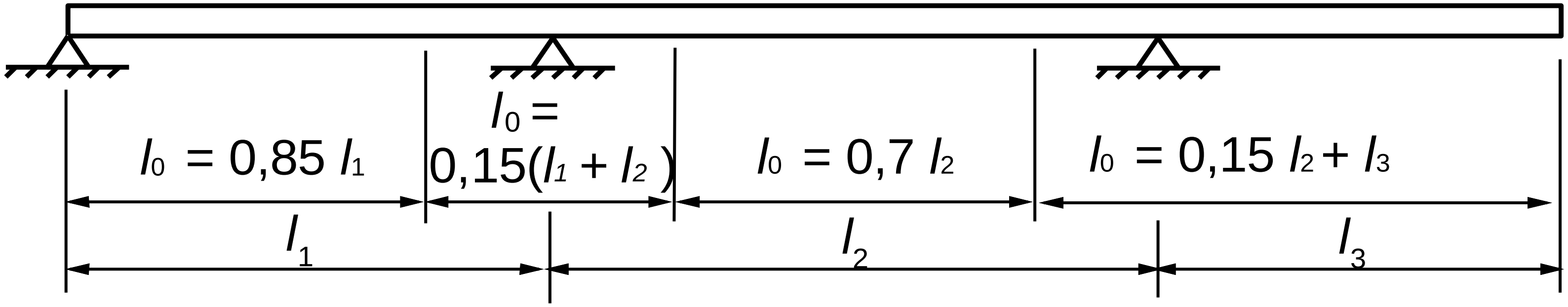
Determination of effective width (5.3.2.1)

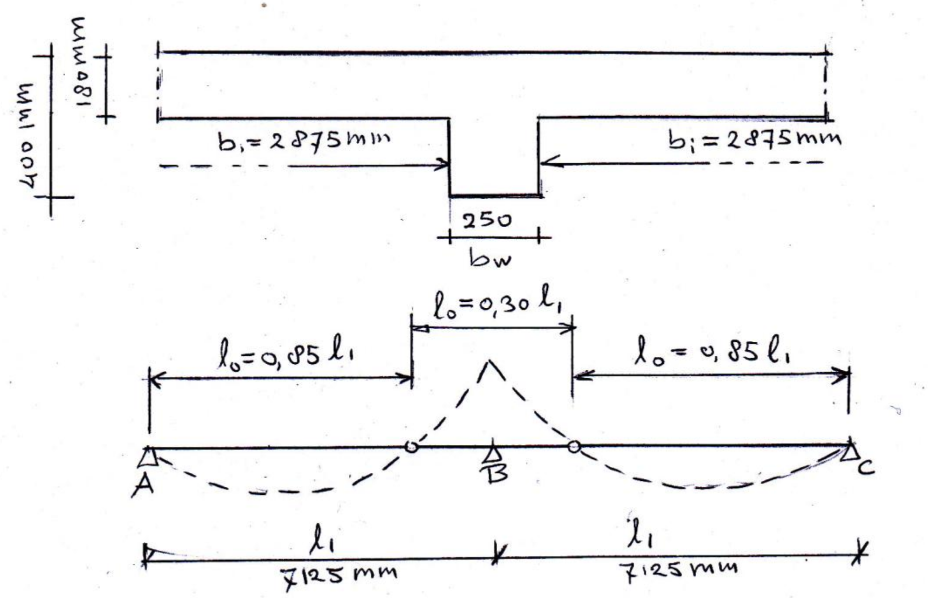
$$b_{eff} = \Sigma\, b_{eff,i} + b_w \leq b$$

where $b_{eff,i} = 0{,}2b_i + 0{,}1l_0 \leq 0{,}2l_0$ and $b_{eff,I} \leq b_i$





Cross-section of beam with slab



$b_{\text{eff,i}} = 0{,}2b_{\text{i}} + 0{,}1l_0 \;\leq\; 0{,}2l_0$ and $b_{\text{eff,I}} \leq\; b_{\text{i}}$

$b_{eff,i} = 0{,}2 \cdot 2875 + 0{,}1 \cdot (0{,}85 \cdot 7125) = 1180\ mm\ (<2875mm)$

$b_{\text{eff}} = \Sigma\, b_{\text{eff,i}} + b_{\text{w}} = 2 \cdot 1180 + 250 = 2610\text{mm}$

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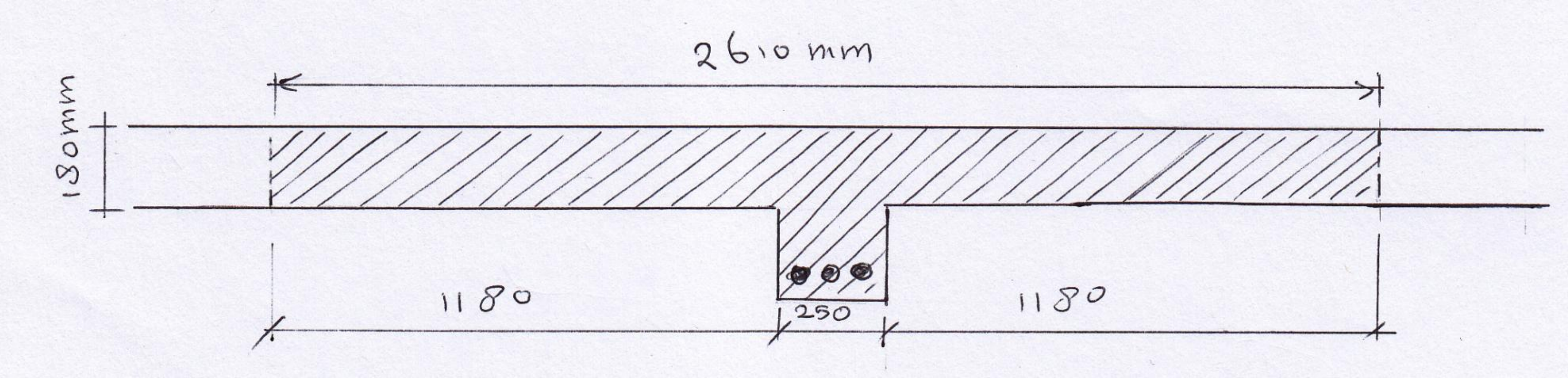
Beam with effective width



Cross-section at mid-span

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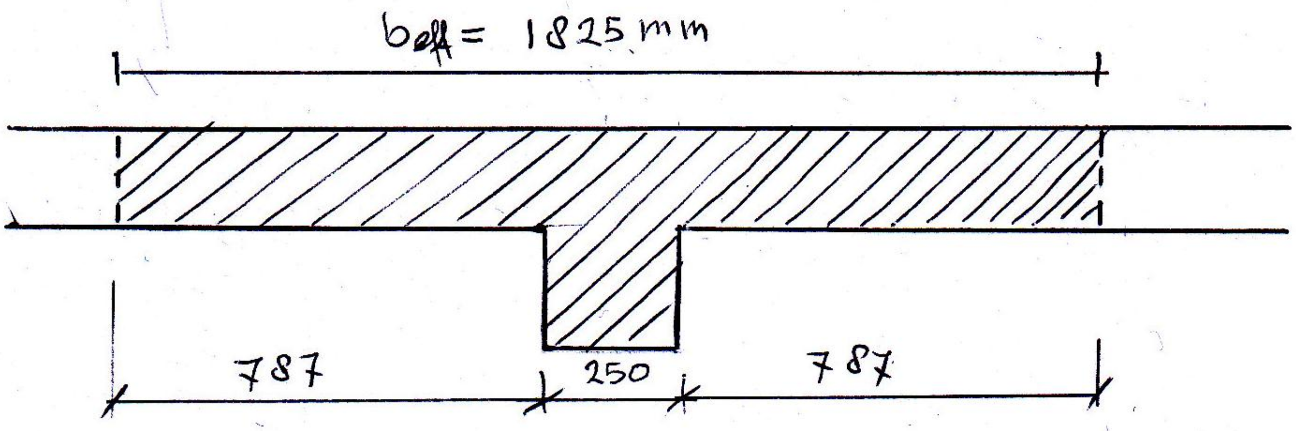
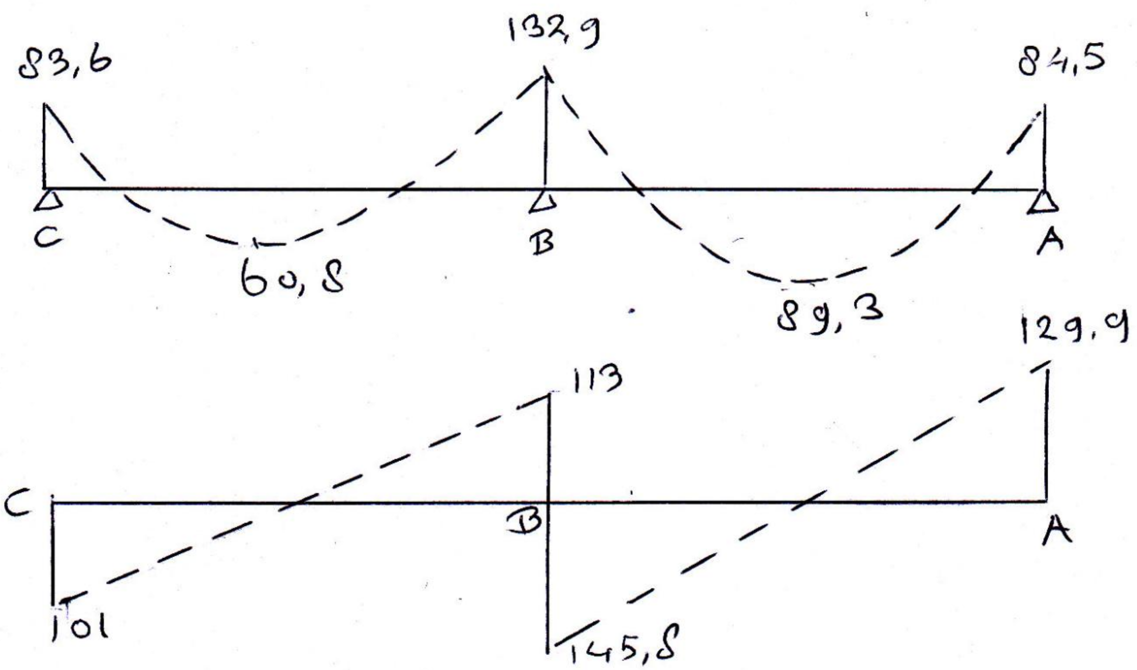
Beam with effective width



Cross-section at intermediate support

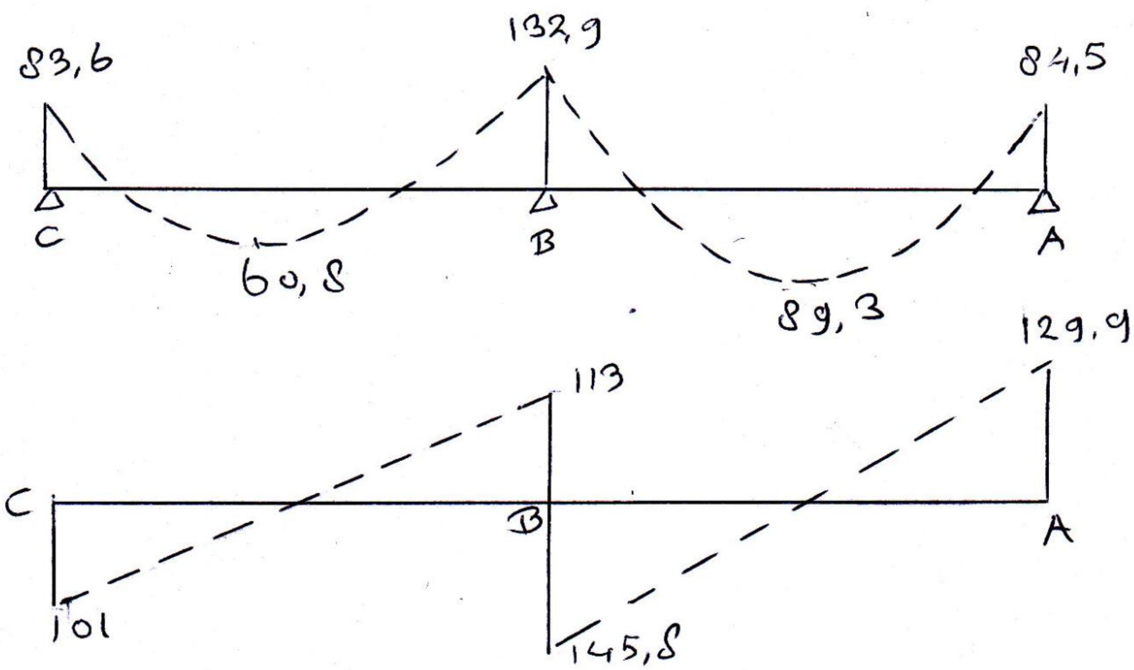
Maximum design bending moments and shear forces



Maximum design moments M_{ed} in kNm (values for different load cases)

Maximum shear forces V_{ed} in kN (values for different load cases)

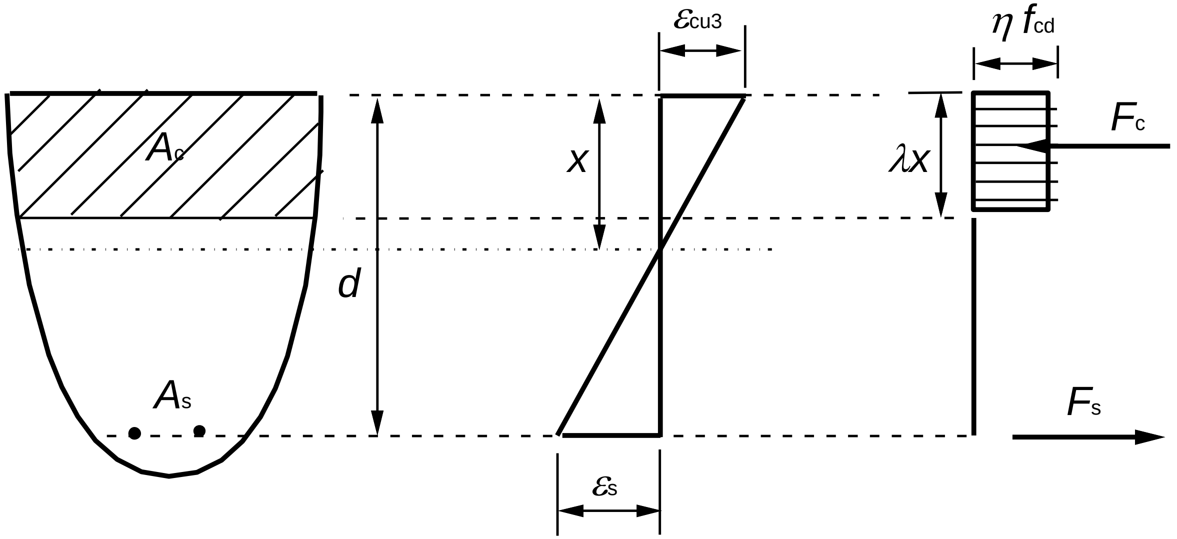
Maximum design bending moments and shear forces



Maximum design moments M_{ed} in kNm (values for different load cases)

Maximum shear forces V_{ed} in kN (values for different load cases)

Determination of bending reinforcement using method with simplified concrete design stress block (3.1.7)



$\lambda = 0{,}8 \quad$ for $f_{ck} \leq 50$ MPa

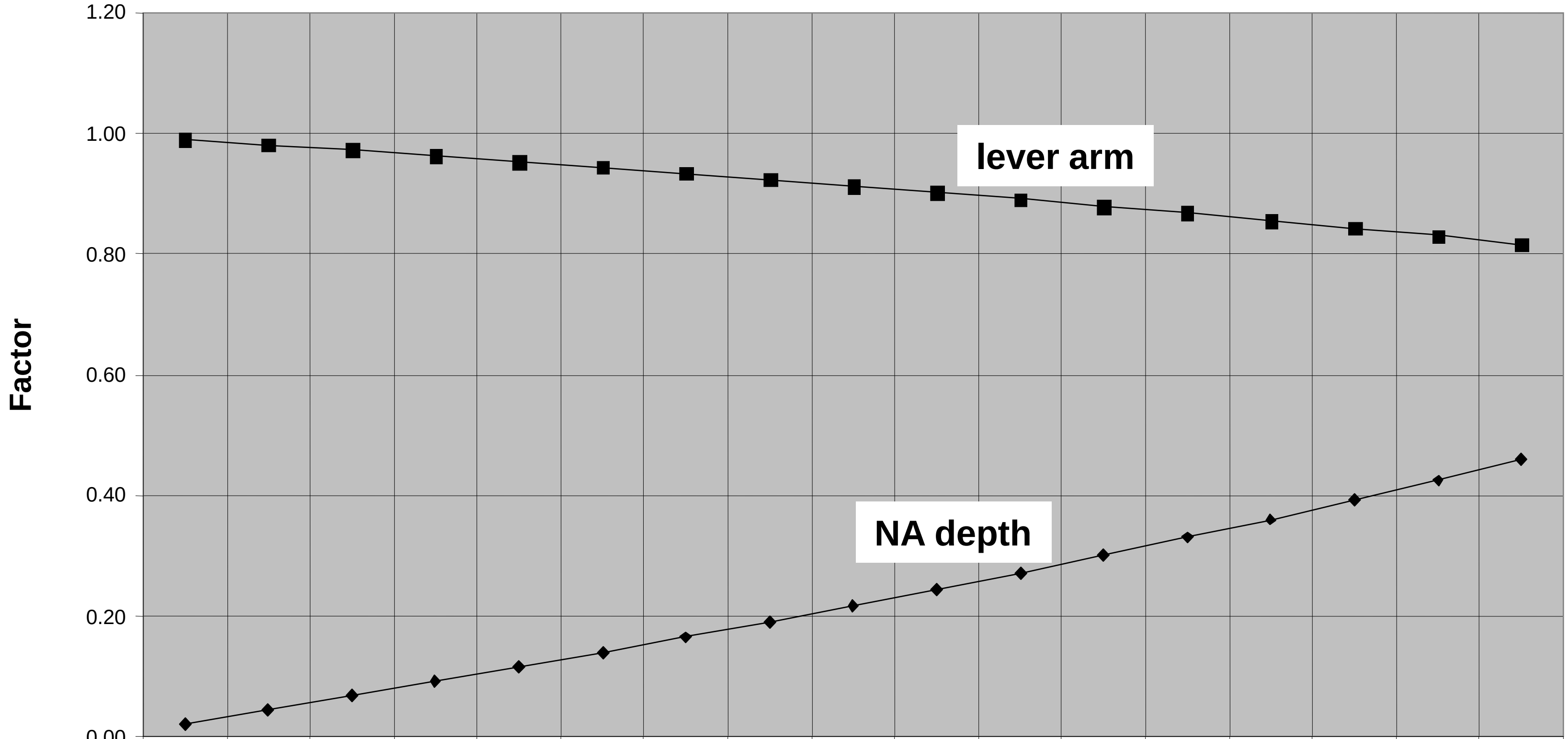
$$= 0{,}8 - \frac{(f_{ck} - 50)}{400}$$ for $50 < f_{ck} \leq 90$ MPa

$\eta = 1{,}0$ for $f_{ck} \leq 50$ MPa

$= 1{,}0 - (f_{ck} - 50)/200$ for $50 < f_{ck} \leq 90$ MPa

Simplified factors for flexure (1)

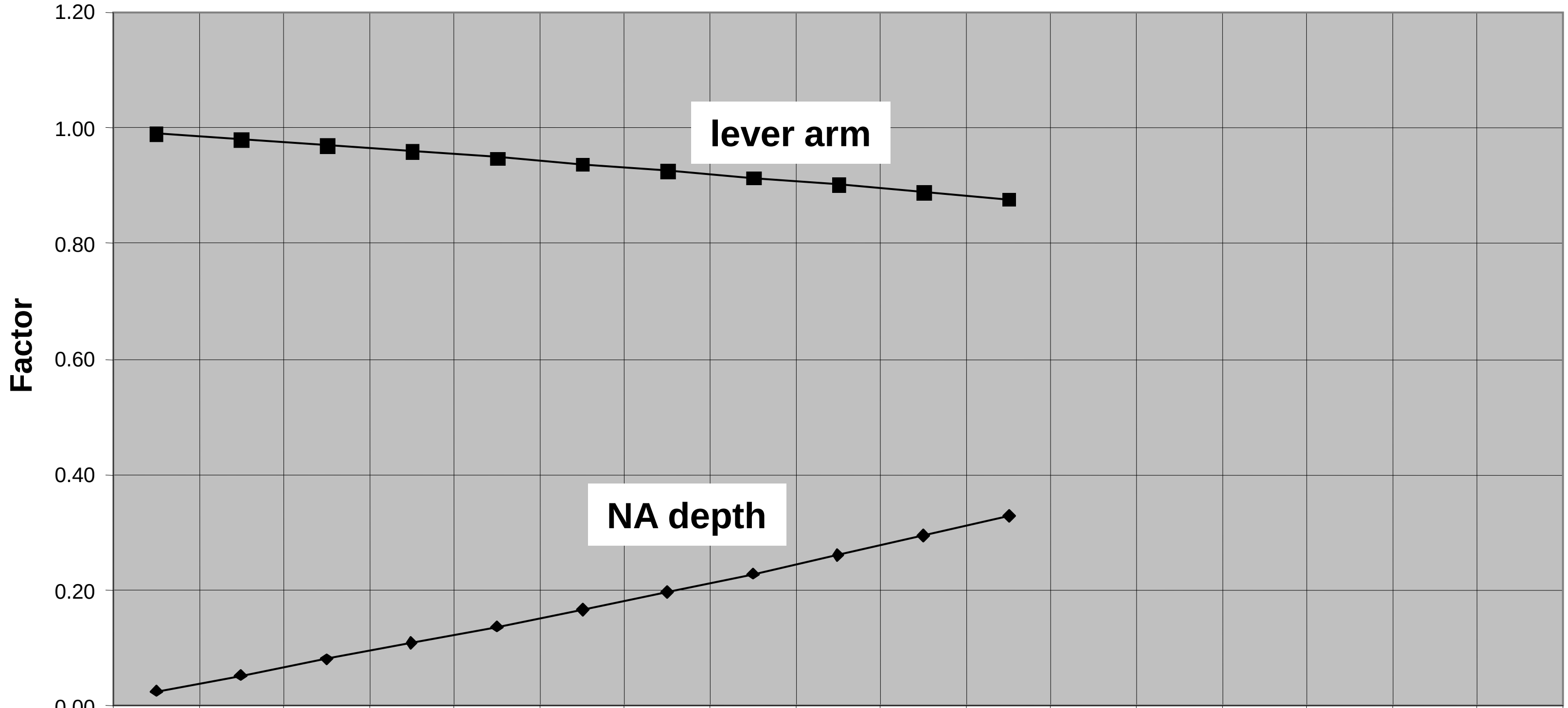
Factors for NA depth (n) and lever arm (=z) for concrete grade ≤ 50 MPa



M/bd^2f_{ck}	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17
n	0.02	0.04	0.07	0.09	0.12	0.14	0.17	0.19	0.22	0.24	0.27	0.30	0.33	0.36	0.39	0.43	0.46
z	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.84	0.83	0.82

Simplified factors for flexure (2)

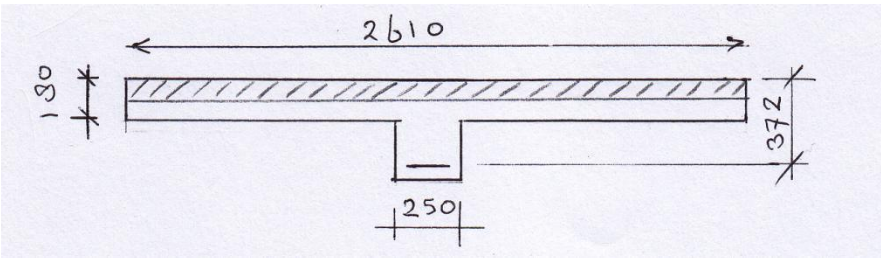
Factors for NA depth (=n) and lever arm (=z) for concrete grade 70 MPa



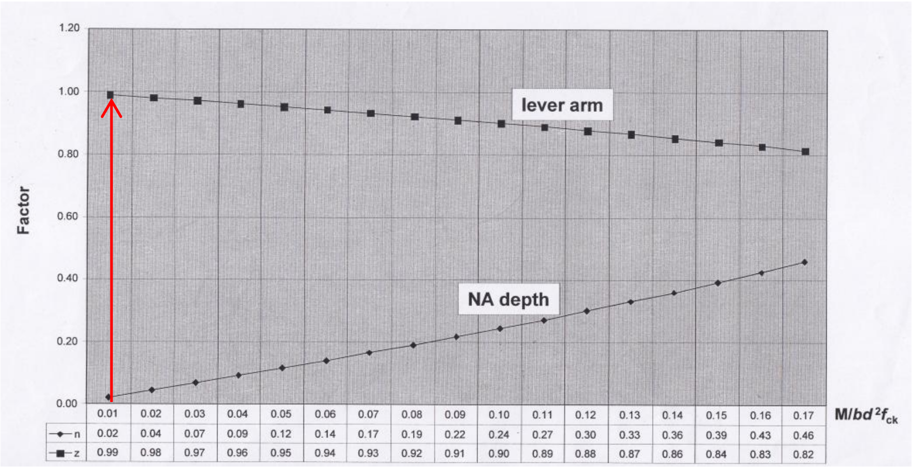
M/bd^2f_{ck}	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17
n	0.03	0.05	0.08	0.11	0.14	0.17	0.20	0.23	0.26	0.29	0.33						
z	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.91	0.90	0.89	0.88						

Determination of bending reinforcement (span AB)

Example: largest bending moment in span AB: M_{ed} = 89,3 kNm



$$\frac{M_{Ed}}{bd^2 f_{ck}} = \frac{89{,}3 \cdot 10^6}{2610 \cdot 372^2 \cdot 25} = 0{,}001$$



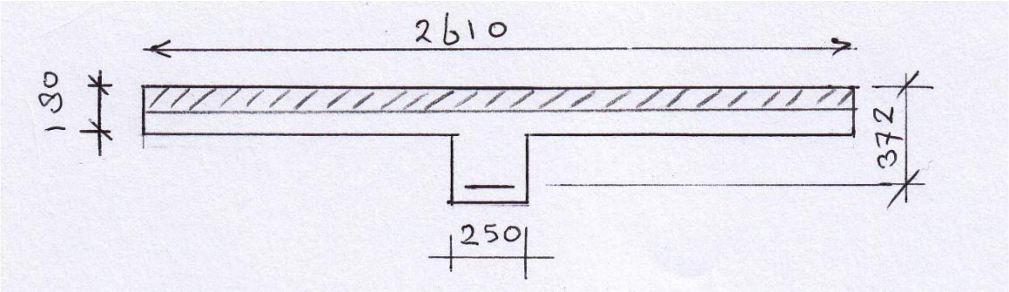
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17
n	0.02	0.04	0.07	0.09	0.12	0.14	0.17	0.19	0.22	0.24	0.27	0.30	0.33	0.36	0.39	0.43	0.46
z	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.84	0.83	0.82

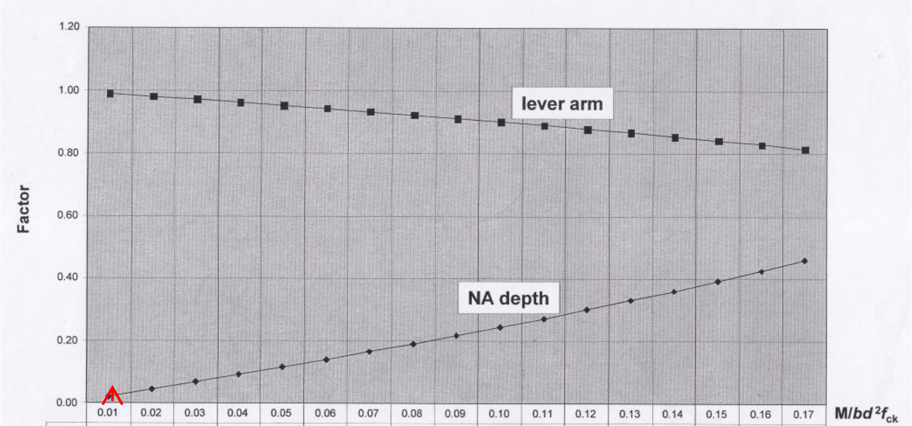
Read in diagram: lever arm factor = 0,99, so:

$$A_{sl,req} = \frac{M_{Ed}}{z \cdot f_{yd}} = \frac{89{,}3 \cdot 10^6}{0{,}98 \cdot 372 \cdot 435} = 563 mm^2$$

Determination of bending reinforcement (span AB)

Example: largest bending moment in span AB: M_{ed} = 89,3 kNm





M/bd^2f_{ck}	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17
n	0.02	0.04	0.07	0.09	0.12	0.14	0.17	0.19	0.22	0.24	0.27	0.30	0.33	0.36	0.39	0.43	0.46
z	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.84	0.83	0.82

Moreover, from diagram: neutral axis depth factor is 0,02, so x_u = 0,02·180 = 4 mm. So height of compression zone < flange thickness (180 mm), OK

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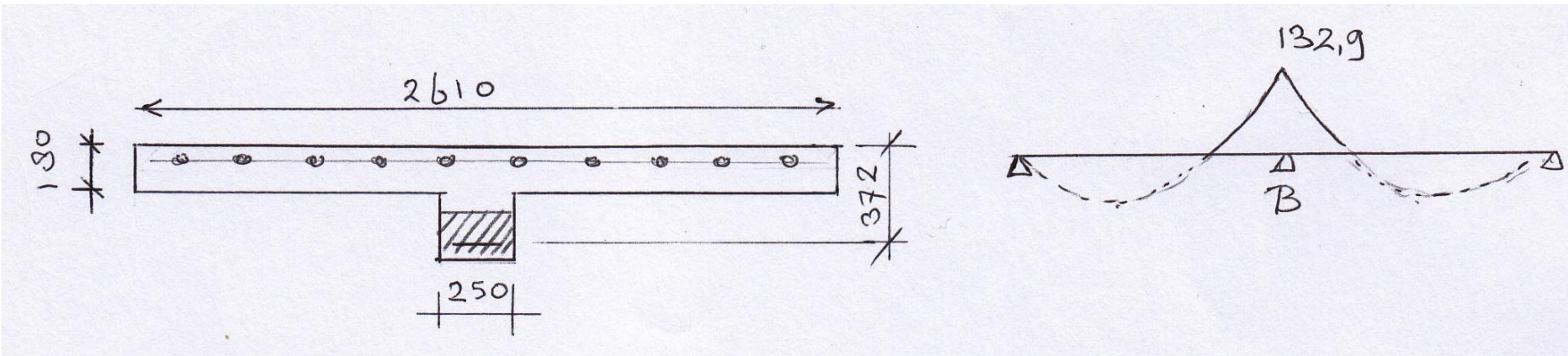
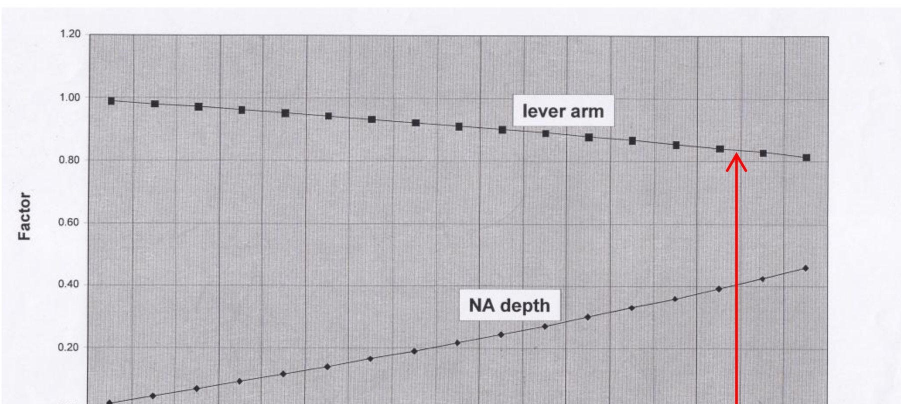
Determination of bending reinforcement (intermediate support B

Bending moment at support B: M_{ed} = 132,9 kNm





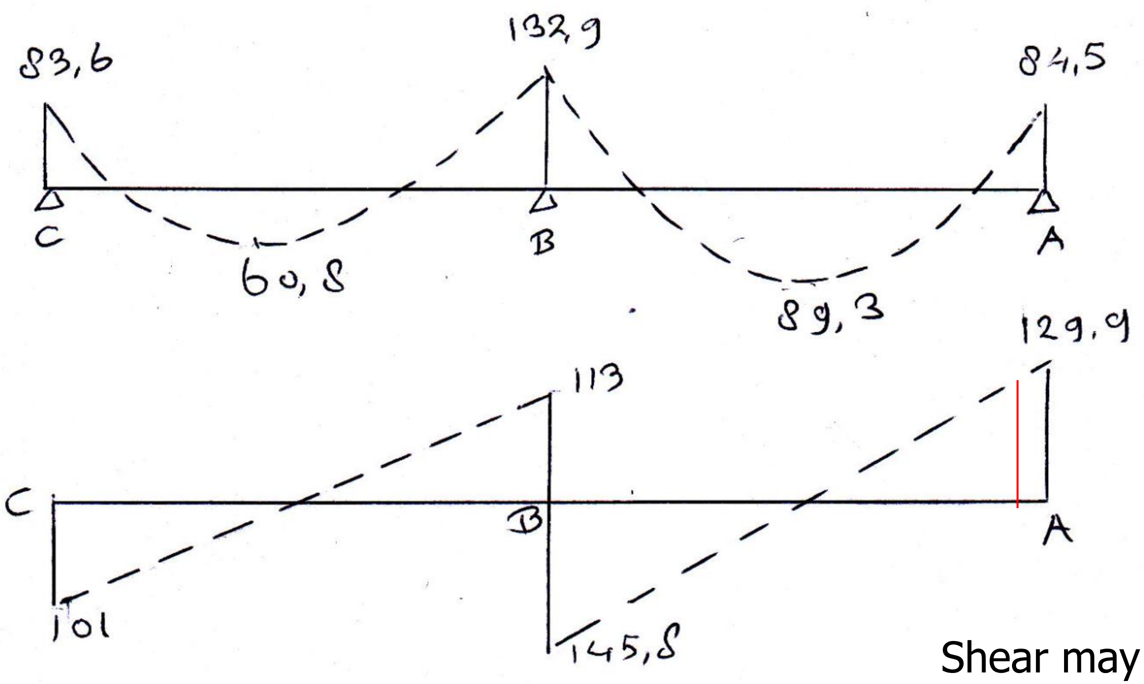
M/bd^2f_{ck}	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17
n	0.02	0.04	0.07	0.09	0.12	0.14	0.17	0.19	0.22	0.24	0.27	0.30	0.33	0.36	0.39	0.43	0.46
z	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.84	0.83	0.82

$$\frac{M_{Ed}}{bd^2 f_{ck}} = \frac{132{,}0 \cdot 10^6}{250 \cdot 372^2 \cdot 25} = 0{,}154$$

Read: lever arm factor 0,81

$$A_{sl} = \frac{M_{Ed}}{z \cdot f_{yd}} = \frac{132{,}9 \cdot 10^6}{0{,}81 \cdot 372 \cdot 435} = 1014 mm^2$$

Maximum design bending moments and shear forces



Maximum design moments M_{ed} in kNm (values for different load cases)

Maximum shear forces V_{ed} in kN (values for different load cases)

Shear may be determined at distance *d* from support, so $V_{ed} \cong 115$ kN

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Design of beams for shear (6.2.2)

First check (6.2.2): if $V_{Ed} \geq V_{Rd,c}$ then shear reinforcement is required:

$$V_{Rd,c} = (0{,}18/\gamma_c)k(100\rho_l f_{ck})^{1/3}bd$$

where: f_{ck} in Mpa

k $= 1+\sqrt{\frac{200}{d}} \leq 2{,}0$ with d in mm

ρ_l $= \frac{A_{sl}}{b_w d} \leq 0{,}02$

with d = 372mm, b_w = 250mm, ρ_l = 0,61%, f_{ck} = 25MPa

$$V_{Rd,c} = (0{,}18/1{,}5)\cdot 1{,}73\cdot(0{,}61\cdot 25)^{1/3}\cdot 250\cdot 372\cdot 10^{-3} = 47{,}8kN < 115kN$$

so shear reinforcement is required

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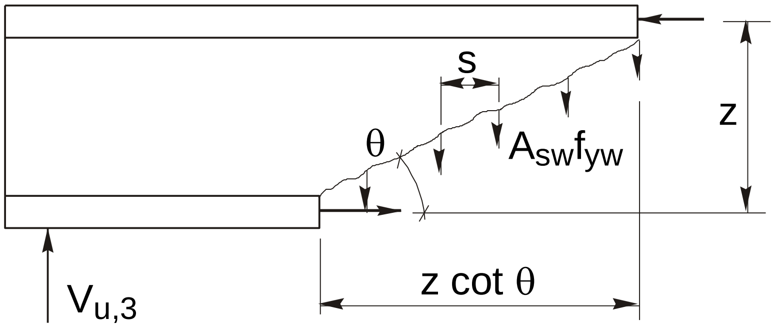
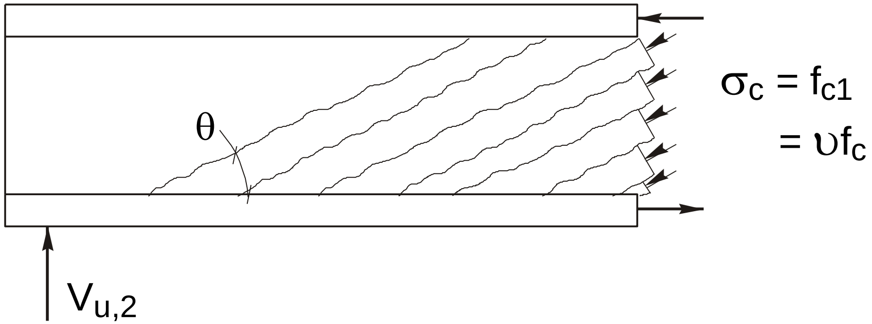
Expressions for shear capacity at stirrup yielding ($V_{Rd,s}$) and web crushing ($V_{Rd,max}$)





For yielding shear reinforcement:
$V_{Rd,s} = (A_{sw}/s)\ z\ f_{ywd} \cot\theta$

with θ between 45^0 and $21{,}8^0$
$(1 \leq \cot\theta \leq 2{,}5)$

At web crushing:
$V_{Rd,max} = b_w\, z\, \upsilon\, f_{cd} /(\cot\theta + \tan\theta)$

with θ between 45^0 and $21{,}8^0$
$(1 \leq \cot\theta \leq 2{,}5)$

$\nu = 0.6\ (1 - f_{ck}/250)$

Design of beams for shear

Basic equation for determination of shear reinforcement:

$$V_{Ed,s} = (A_{sw}/s)\, z\, f_{ywd} \cot\theta$$

With $V_{ed,s}$ = 115000 N, f_{ywd} = 435 Mpa, $z = 0{,}9d$, d = 372 mm and cot θ = 2,5 it is found that

$A_{sw}/s \geq 0{,}32$ e.g. stirrups ∅6mm – 175mm

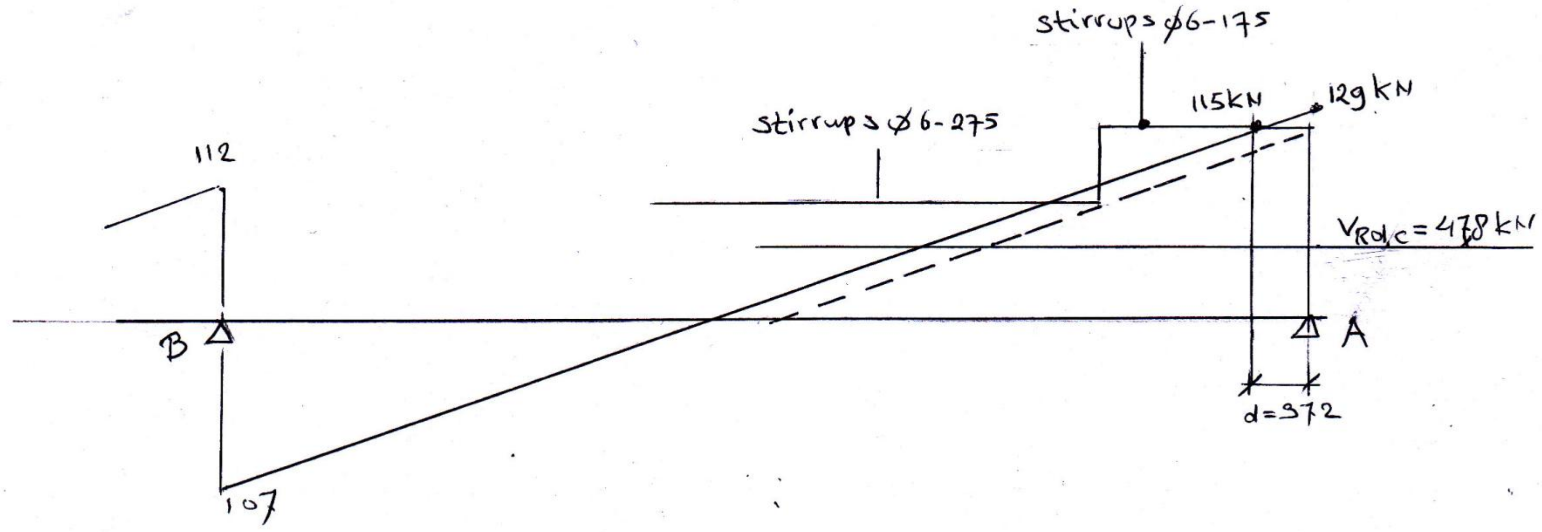
Check upper value of shear capacity (web crushing criterion)

$$V_{Rd,max} = b_w\, z\, \upsilon\, f_{cd} /(\cot\theta + \tan\theta)$$

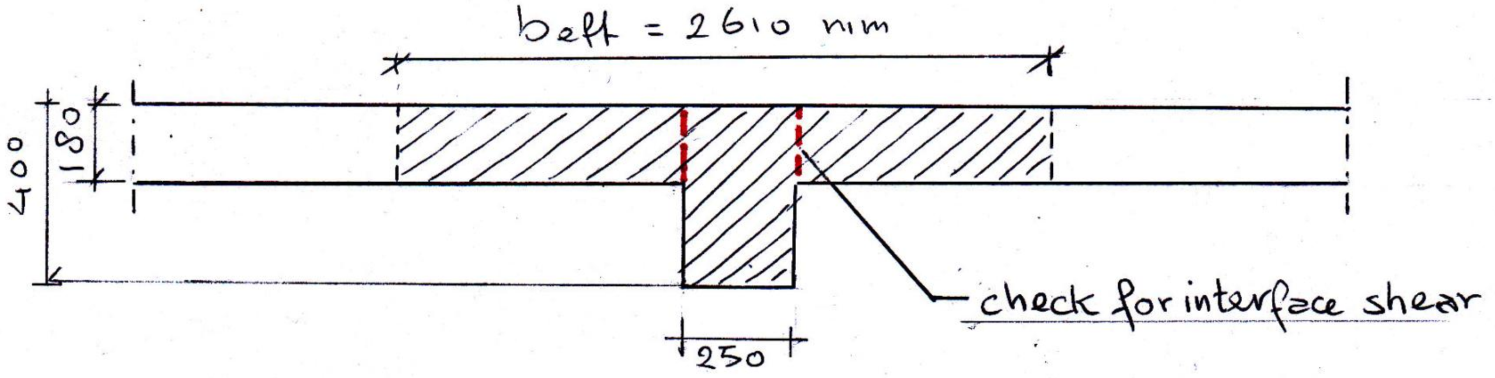
with b_w = 250mm, d = 372mm, z = 0,9d, $\nu = 0{,}6(1-f_{ck}/250) = 0{,}54$, $f_{cd} = 25/1{,}5$ = 13,3 Mpa and cot θ = 2,5 it is found that

$V_{Rd,max}$ = 1774 kN which is much larger than the design shear force of 115 kN

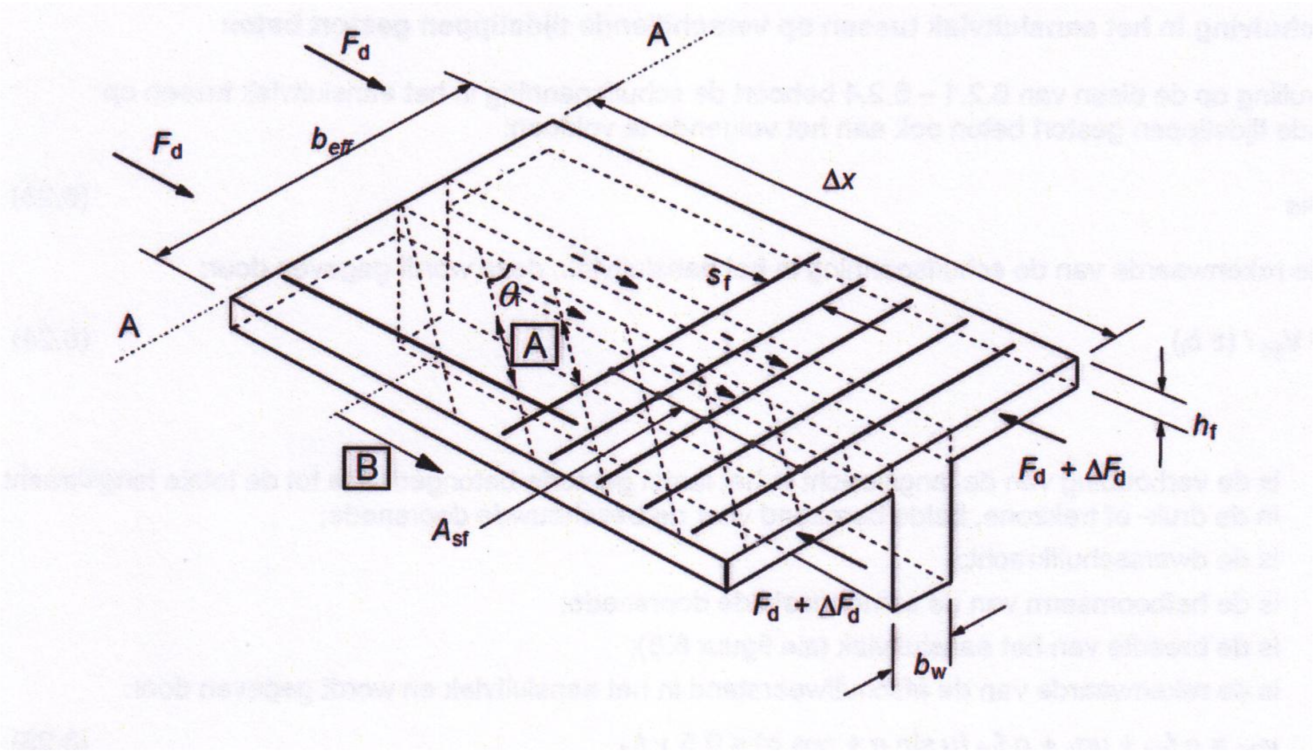
Stirrup configuration near to support A



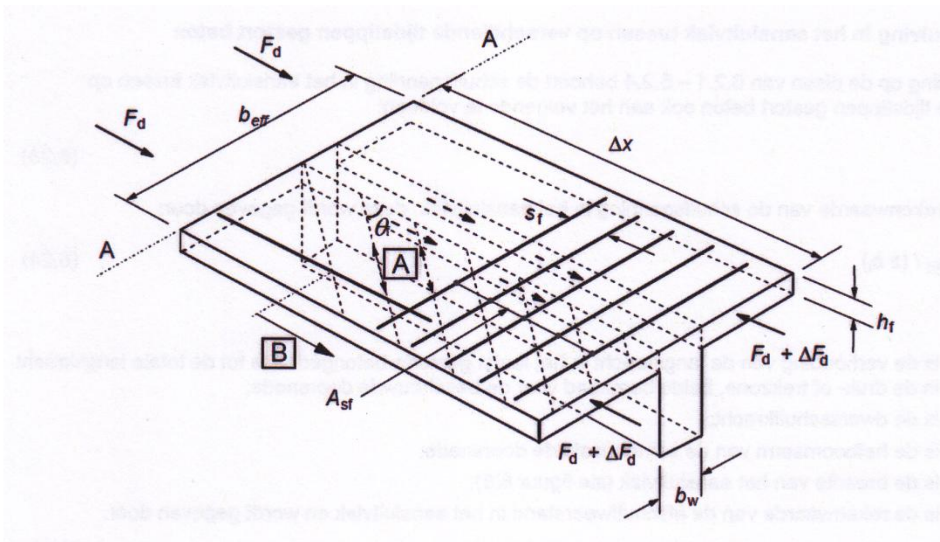
Transverse shear in web-flange interface



Shear between web and flanges of T-sections



Shear between web and flanges of T-sections



Strut angle θ:

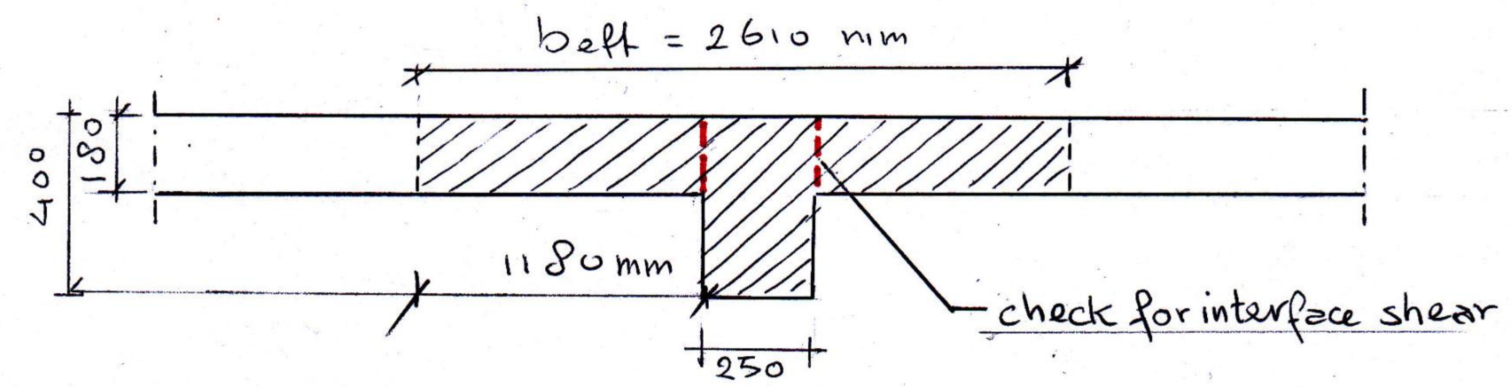
$1{,}0 \leq \cot \theta_f \leq 2{,}0$ for compression flanges ($45^0 \geq \theta_f \geq 26{,}5^0$

$1{,}0 \leq \cot \theta f \leq 1{,}25$ for tension flanges ($45^0 \geq \theta_f \geq 38{,}6^0$)

No transverse tension ties required if shear stress in interface

$v_{Ed} = \Delta F_d/(h_f \cdot \Delta x) \leq kf_{ctd}$ (recommended $k = 0{,}4$)

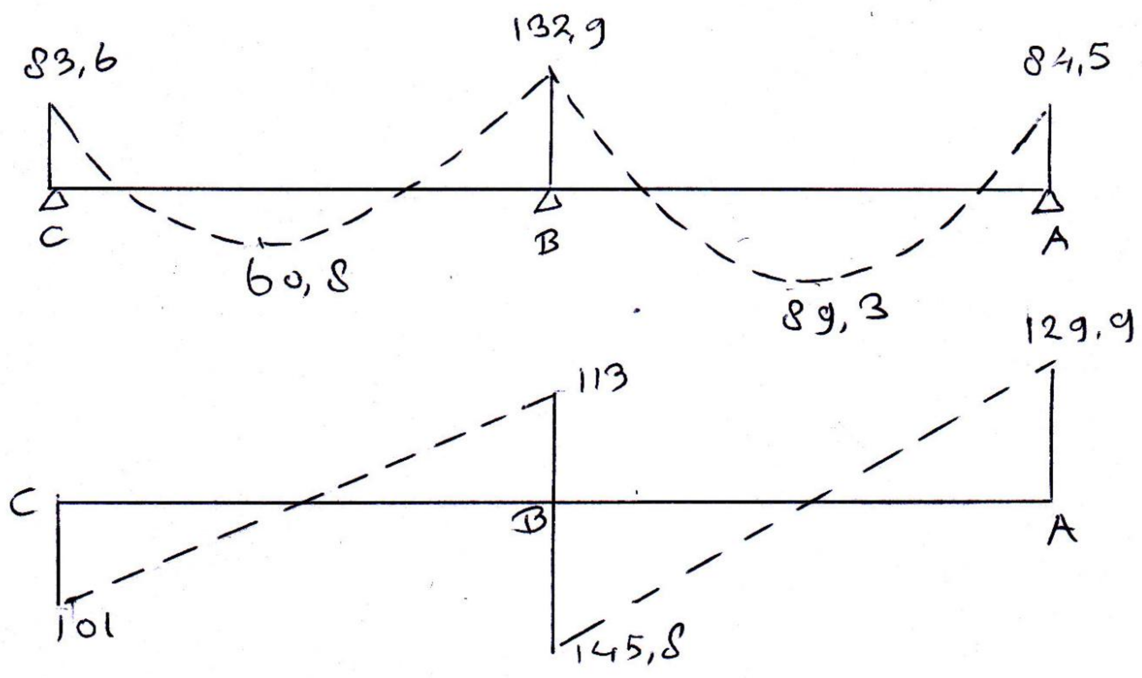
Check necessity of transverse reinforcement



$$v_{Ed} = \frac{b_f}{b_{eff}} \cdot \frac{V_{Ed}}{z \cdot h_f} = \frac{1180}{2610} \cdot \frac{115000}{0{,}9 \cdot 372 \cdot 180} = 0{,}86 MPa$$

No transverse reinforcement required if $v_{Ed} \leq 0{,}4 f_{ctd}$
For C25/30 $f_{ctd} = f_{ctk}/\gamma_c$ =1,8/1,5 = 1,38 Mpa, so the limit value for interface shear is $0{,}4 f_{ctk}$ = 0,4·1,38 = 0,55 MPa.
Transverse shear reinforcement is required at the end of the beam.

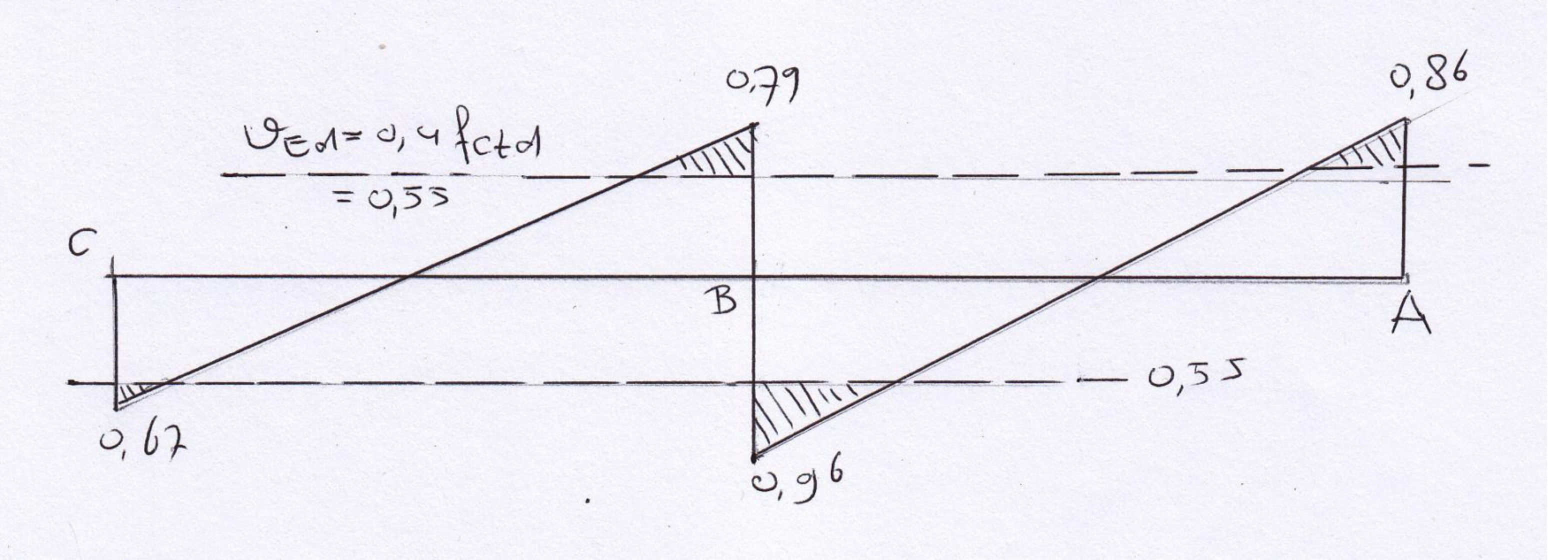
Maximum design bending moments and shear forces



Maximum design moments M_{ed} in kNm (values for different load cases)

Maximum shear forces V_{ed} in kN (values for different load cases)

Areas in beam axis 2 where transverse reinforcement is required



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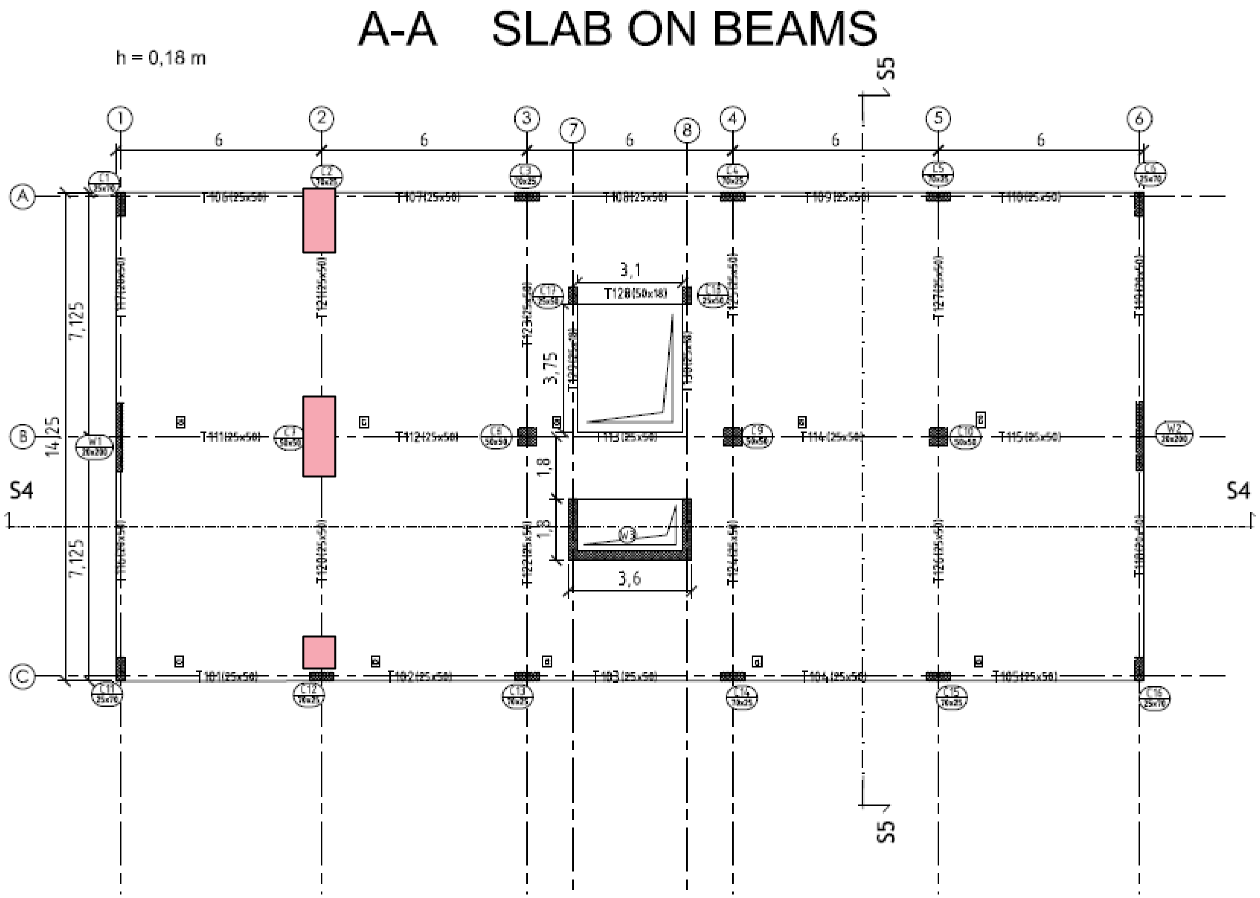
Areas in beam axis 2 where transverse reinforcement is required



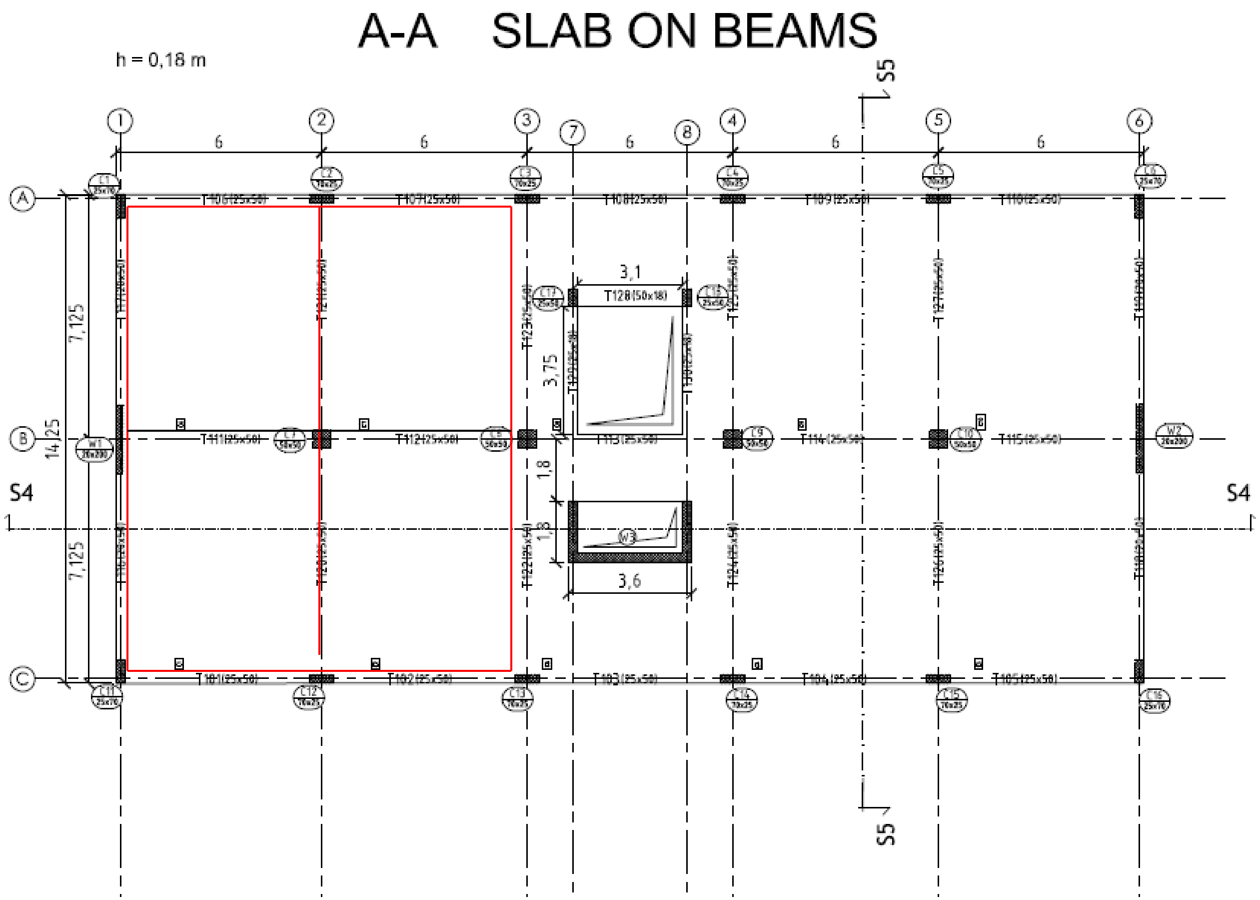
Example: transverse reinforcement near to support A

Required transverse reinforcement for V_{ed} = 115 kN

$$\frac{A_{st}}{s} = \frac{b_f}{b_{eff}} \cdot \frac{V_{Ed}}{z f_{yd}} \cdot \frac{1}{\cot\theta_f} = \frac{1180}{2610} \cdot \frac{115000}{335 \cdot 435} \cdot \frac{1}{2{,}0} = 0{,}18 mm^2 / mm$$

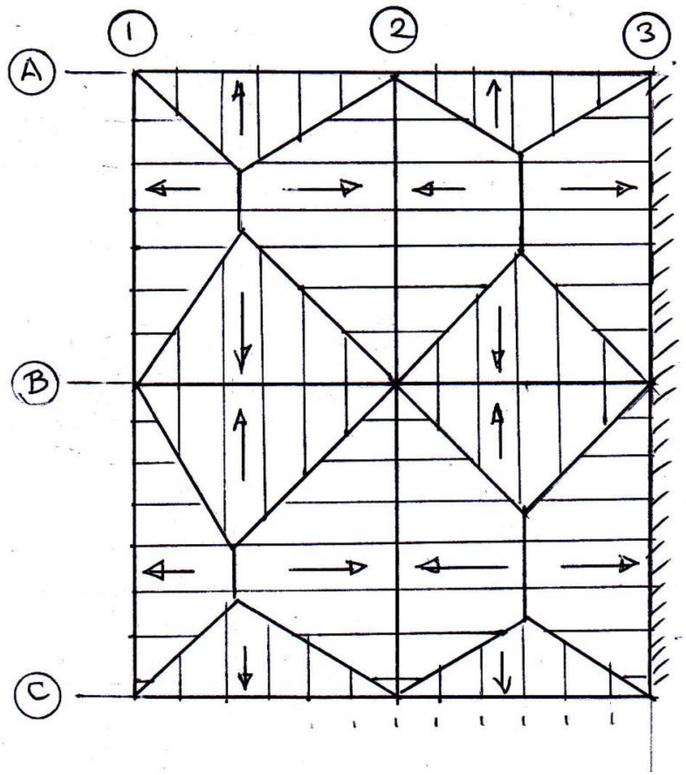
e.g. ∅8 – 250 (=0,20 mm^2/mm)

Design of slabs supported by beams



Design of slabs supported by beams

Load transmission from slabs to beams



Simplified load transmission model

Dead load G_1 = 0,18·25 = 4,5 kN/m^2
Partitions, etc. G_2 = 3,0 kN/m^2

Variable load Q = 2,0 kN/m^2

G_{ed} = 1,3(4,5 + 3,0) = 9,75 kN/m^2
Q_{ed} = 1,5·2,0 = 3,0 kN/m^2

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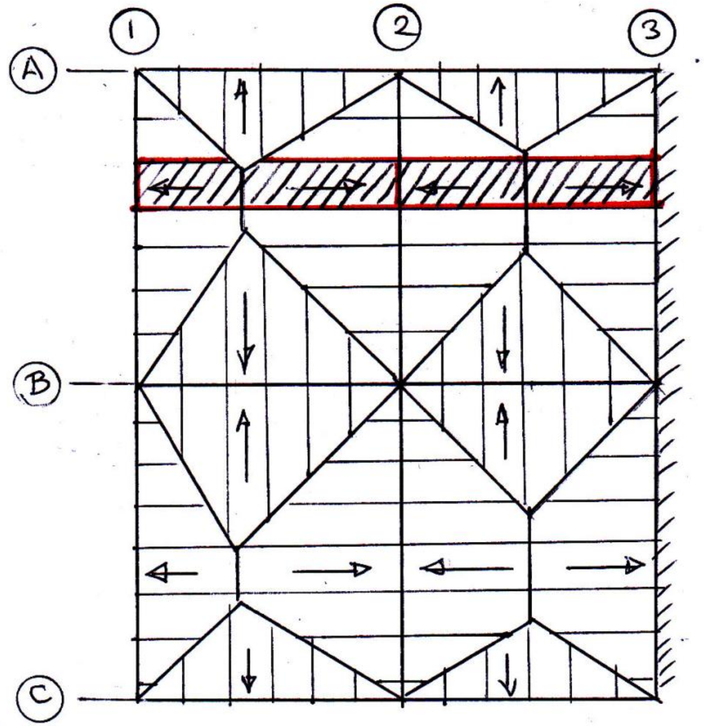
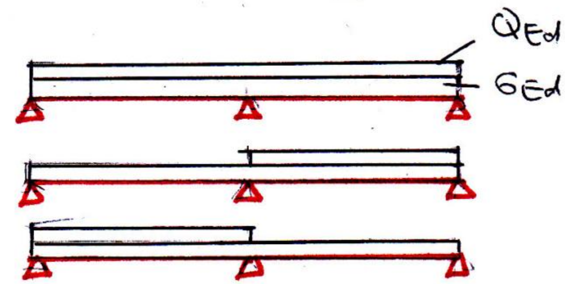
Load transfer from slabs to beams





Loading cases on arbitrary strip (dashed in left figure)

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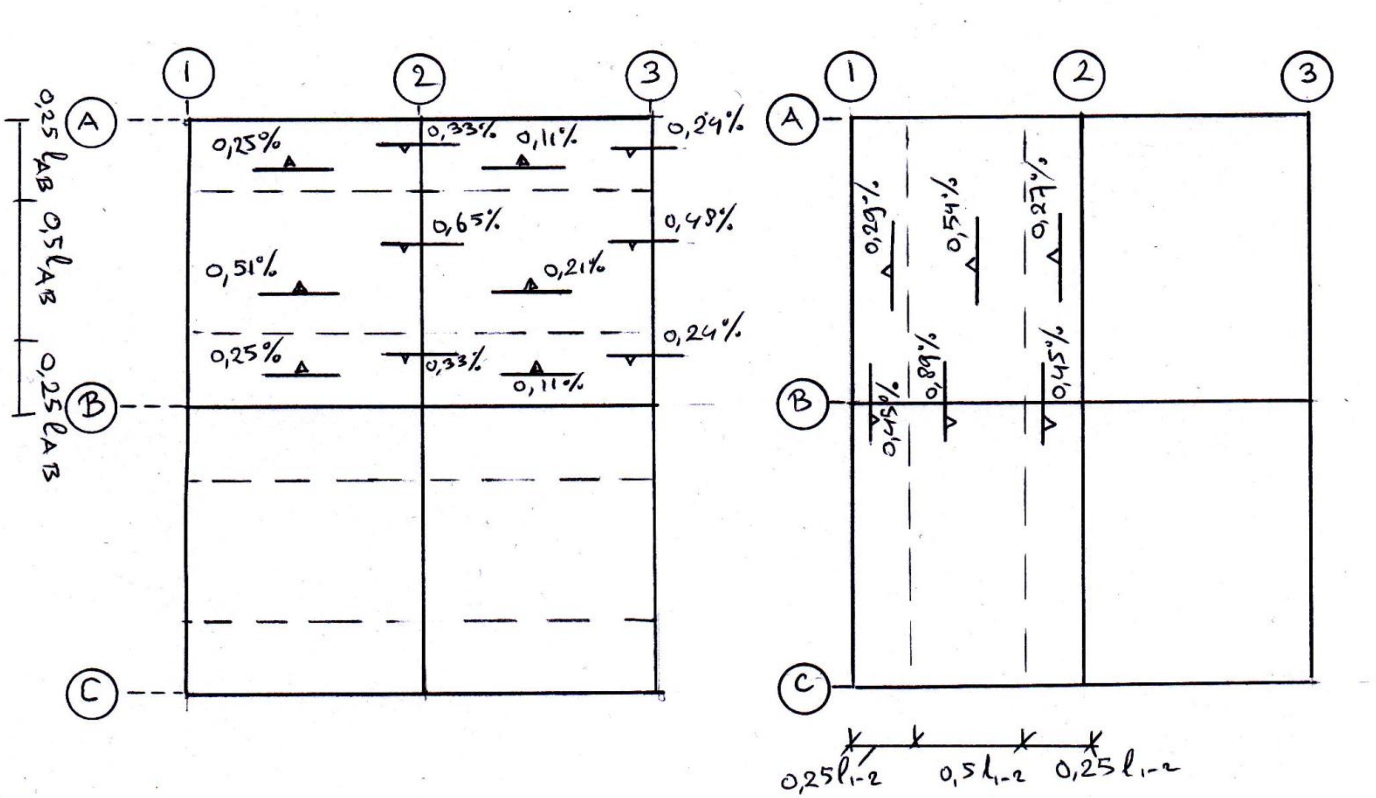
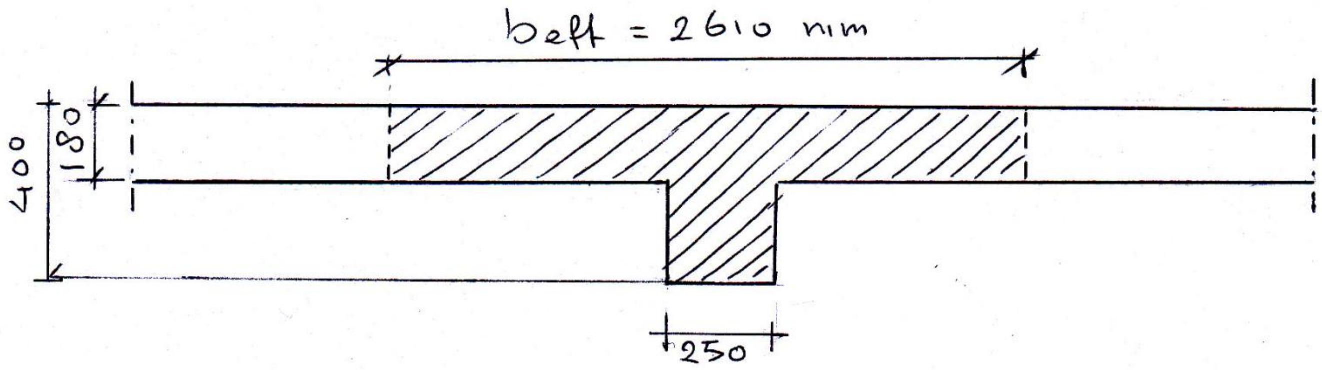
Longitudinal reinforcement in slabs on beams



Examples of reinforced areas

Floor type 2: flat slab d = 210 mm

From floor on beams to flat slab: replace beams by strips with the same bearing capacity



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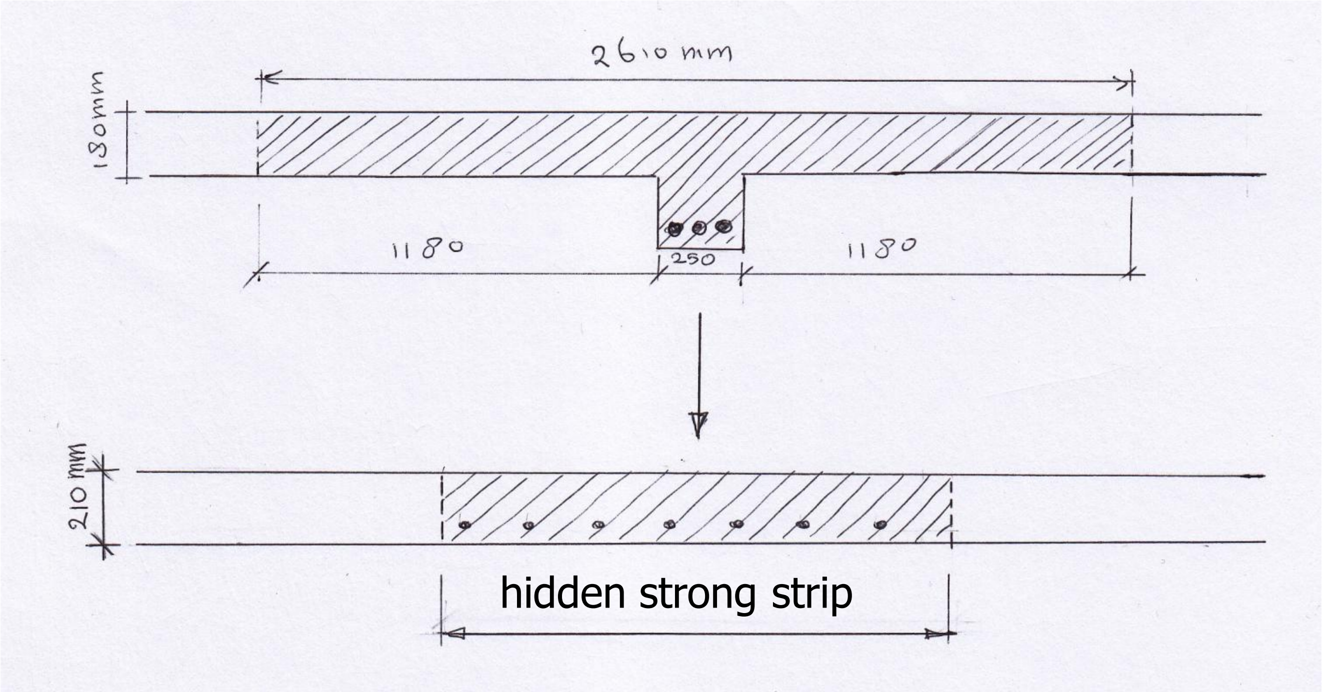
From slab on beams to flat slab



-Strips with small width and large reinforcement ratio favourable for punching resistance

- Strips not so small that compression reinforcement is necessary

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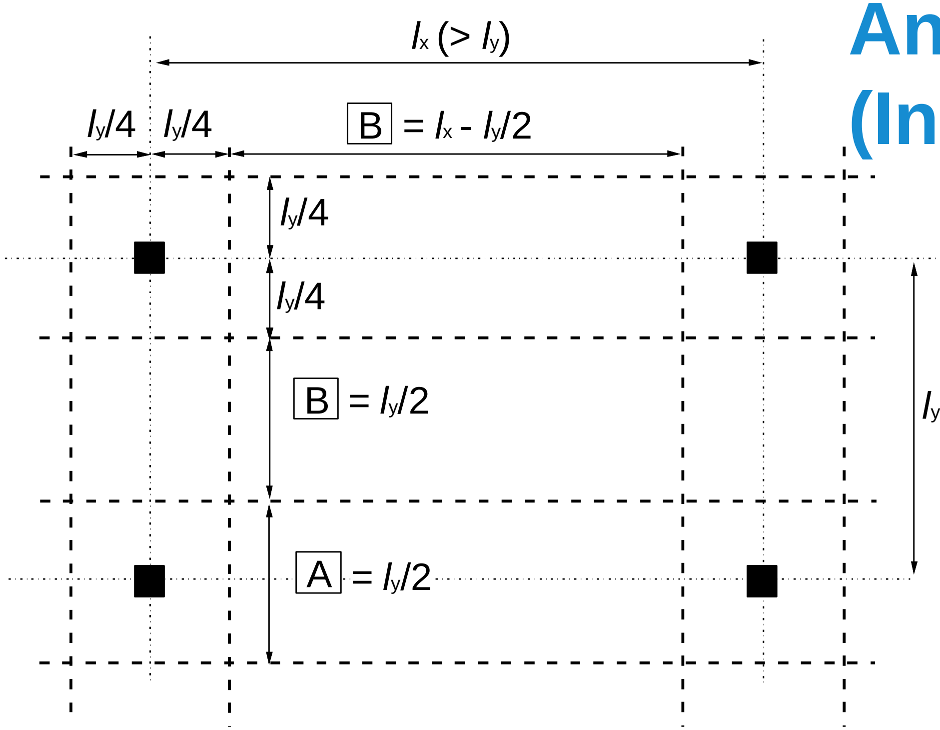
Methods of analysis: Equivalent Frame Analysis – Annex I (Informative)



A – Column strip

B – Middle strip

	Negative moments	Positive moments
Column Strip	60 - 80%	50 - 70%
Middle Strip	40 - 20%	50 - 30%
Note: Total negative and positive moments to be resisted by the column and middle strips together should always add up to 100%.		

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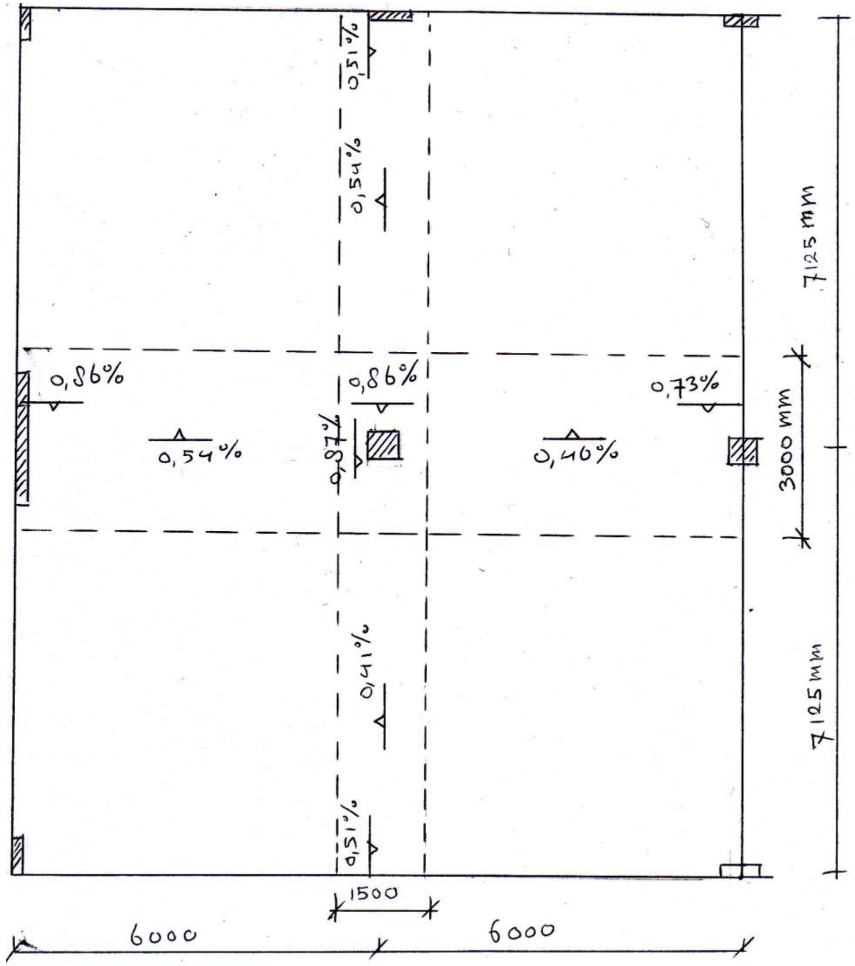
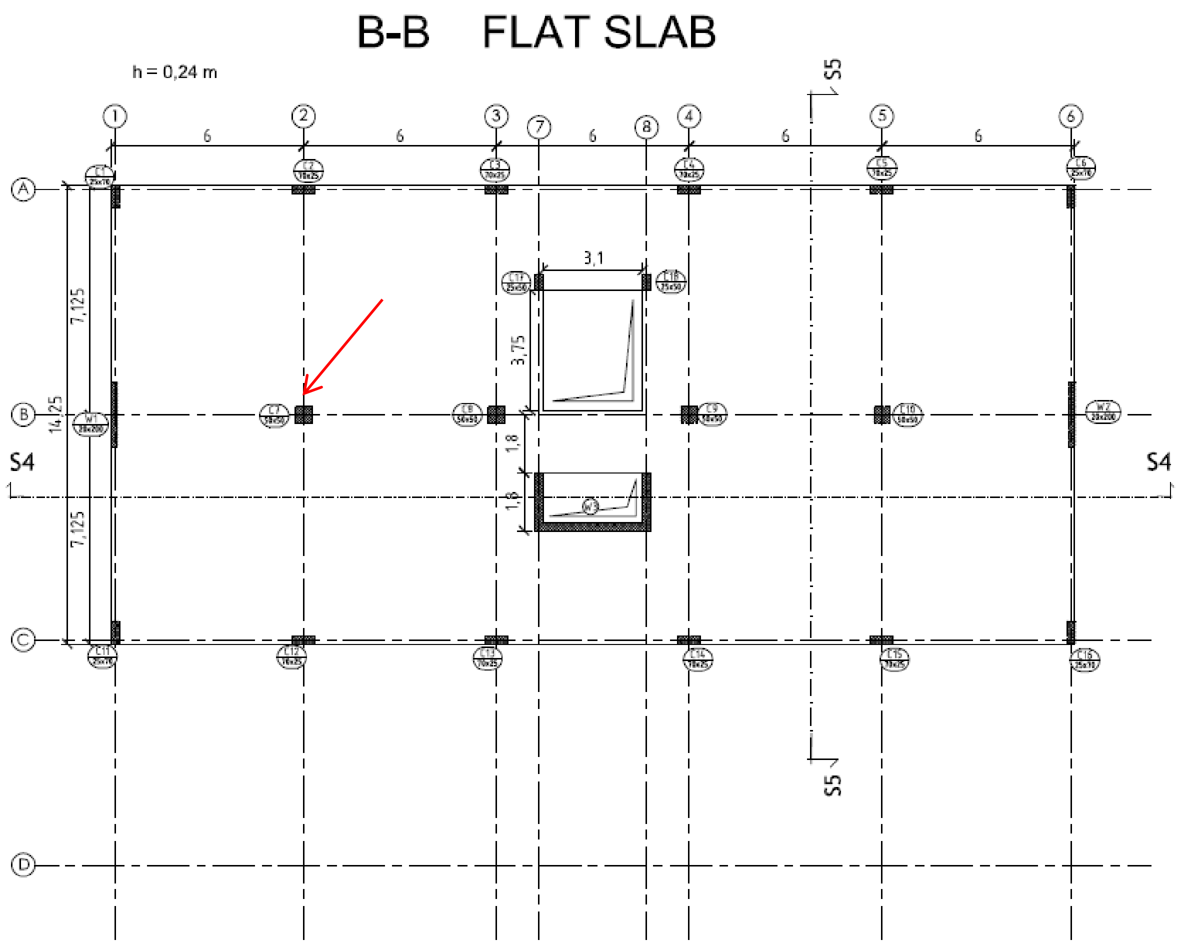
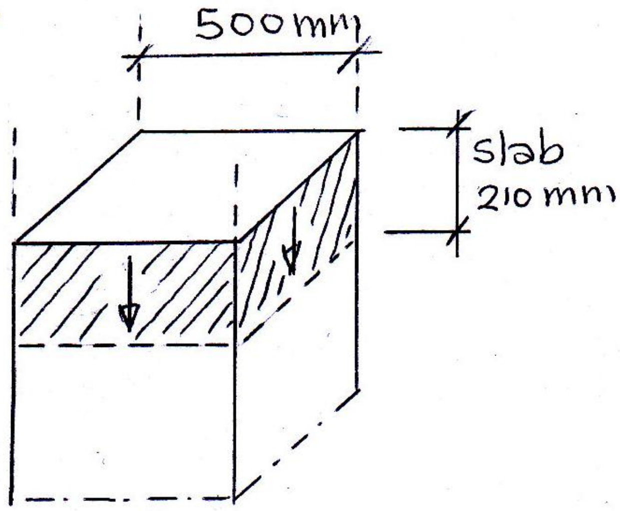
Flat slab with "hidden strong strips"



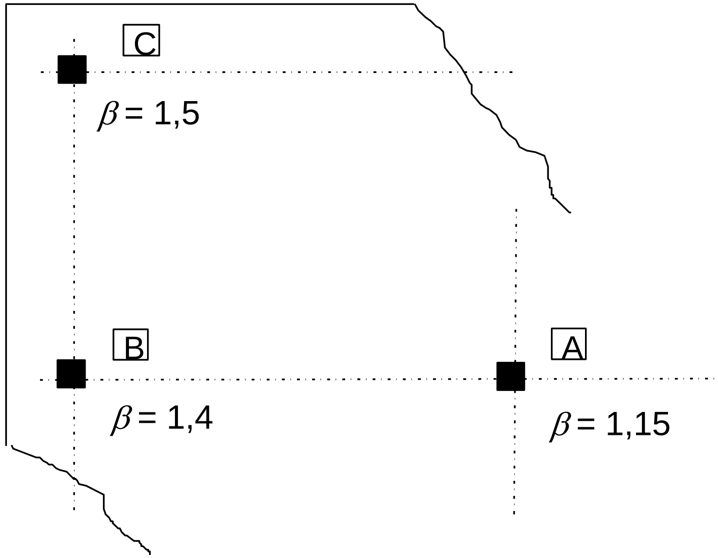
Punching shear control column B2



Punching column B2



Junction column to slab
Vertical load from slab to column V_{ed} = 705 kN



Simplified assumptions for eccentricity factor β according to EN 1992-1-1 Cl. 6.4.3

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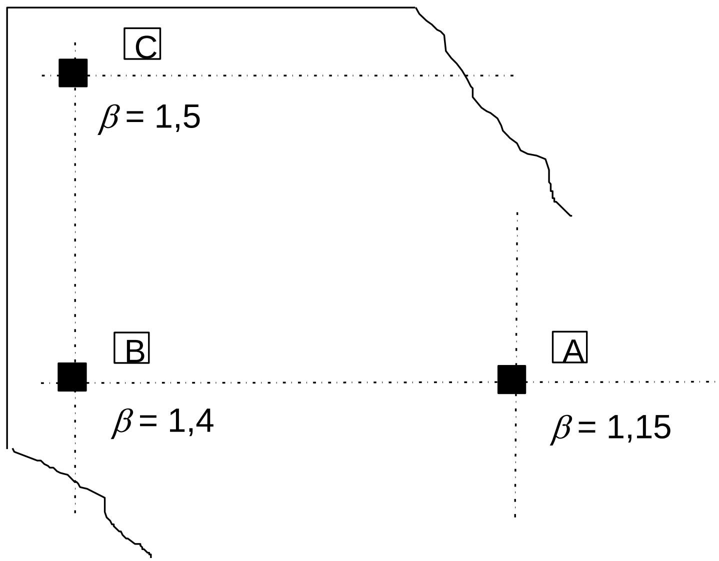
How to take account of eccentricity (simplified case)

Or, how to determine β in equation

$$v_{Ed} = \beta \frac{V_{Ed}}{u_i d}$$

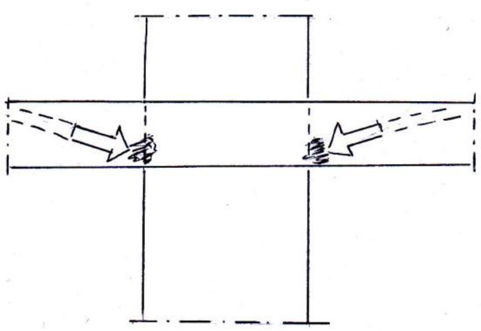


Only for structures where lateral stability does not depend on frame action and where adjacent spans do not differ by more than 25% the approximate values for β shown left may be used:

Upper limit value for design punching shear stress in design

At the perimeter *of the loaded area* the maximum punching shear stress should satisfy the following criterion:

$$v_{Ed} = \frac{\beta V_{Ed}}{u_0 d} \leq v_{Rd,\max} = 0{,}4 \nu f_{cd}$$



where:

u_0 = perimeter of loaded area
υ = $0{,}6[1 - f_{ck}/250]$

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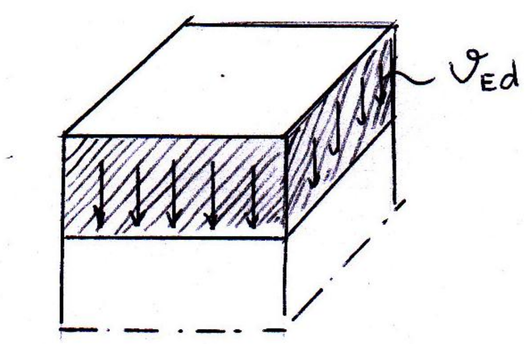
Punching shear column B2

1. Check of upper limit value of punching shear capacity



Further data: $d_y = 210 - 30 - 16/2 = 172\text{mm}$

$d_z = 210 - 30 - 16 - 16/2 = 156 \text{ mm}$

Mean effective depth $0{,}5(172 + 156) = 164\text{mm}$

$\nu = 0{,}6(1 + f_{ck}/250) = 0{,}54$

$v_{Rd,max} = 0{,}4\nu f_{cd} = 0{,}4 \cdot 0{,}54 \cdot (25/1{,}5) =$ 3,60 Mpa

$v_{Ed} = \beta V_{ed}/(u_0 \cdot d) = 1{,}15 \cdot 705000/(4 \cdot 500 \cdot 164)$

$=$ 2,47 Mpa $<$ 3,60 Mpa

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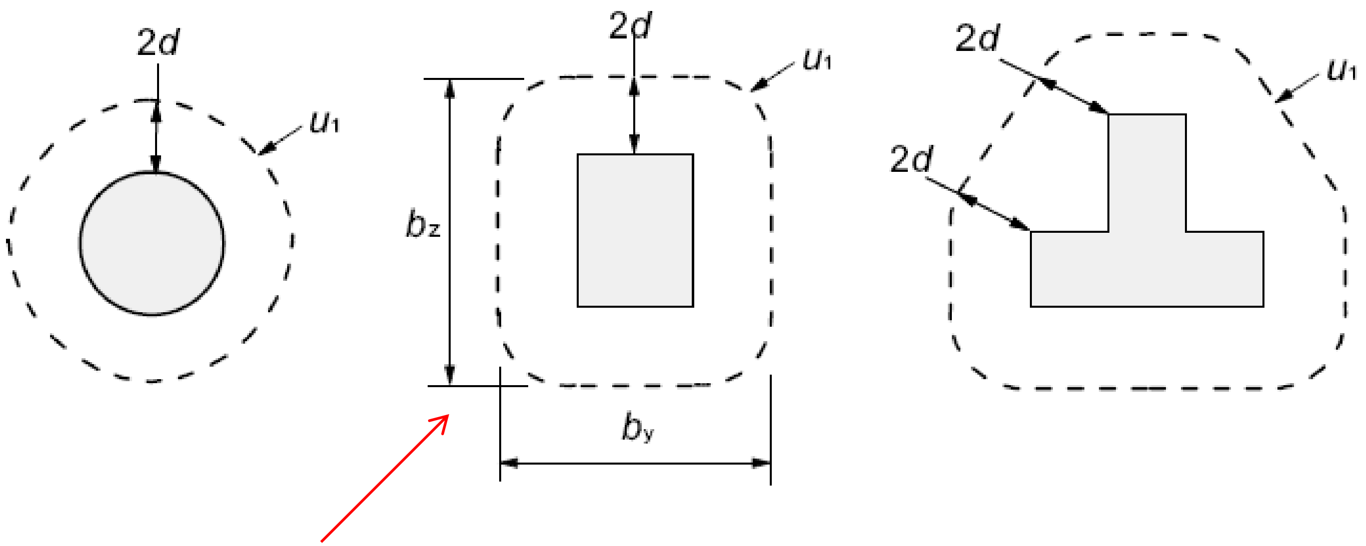
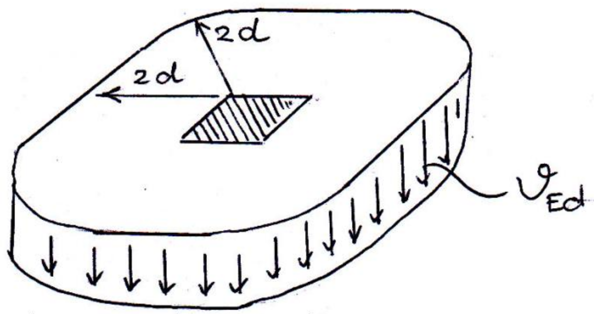
Definition of control perimeters

The basic control perimeter u_1 is taken at a distance 2,0d from the loaded area and should be constructed as to minimise its length



Length of control perimeter of column 500x500mm: $u = 4 \cdot 500 + 2 \cdot \pi \cdot 2 \cdot 164 = 4060$ mm

Punching shear capacity column B2



Punching shear stress at perimeter:

$$v_{Ed} = \frac{\beta V_{Ed}}{u_1 d} = \frac{1{,}15 \cdot 705000}{4060 \cdot 164} = 1{,}22 MPa$$

No punching shear reinforcement required if:

$$v_{Ed} < v_{Rd,c}$$

Limit values for design punching shear stress in design

The following limit values for the punching shear stress are used in design:

If $v_{Ed} \leq v_{Rd,c}$ no punching shear reinforcement required

where:

$$v_{Rd,c} = C_{Rd,c} k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp} \geq (v_{\min} + k_1 \sigma_{cp})$$

where: k_1 = 0,10 (advisory value)

Punching shear capacity of column B2

No punching shear reinforcement required if $v_{Ed} < v_{Rd,c}$

$$v_{Rd,c} = C_{Rd,c} k (100 \rho_l f_{ck})^{1/3}$$

With $C_{Rd,c}$ = 0,12

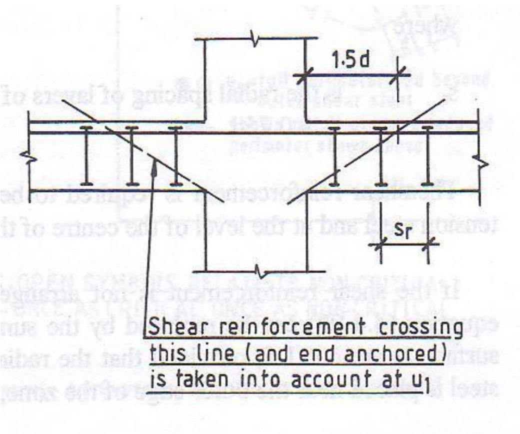
$k = 1 + \sqrt{(200/d)} = 1 + \sqrt{(200/164)} < 2$, so $k = 2,0$

$\rho = \sqrt{(\rho_x \cdot \rho_y)} = \sqrt{(0,86 \cdot 0,87)} = 0,865\%$

f_{ck} = 25 Mpa

It is found that $v_{Rd,c}$ = 0,67 Mpa

Since v_{Ed} = 1,22 MPa> 0,67 MPa punching shear reinforcement should be applied.



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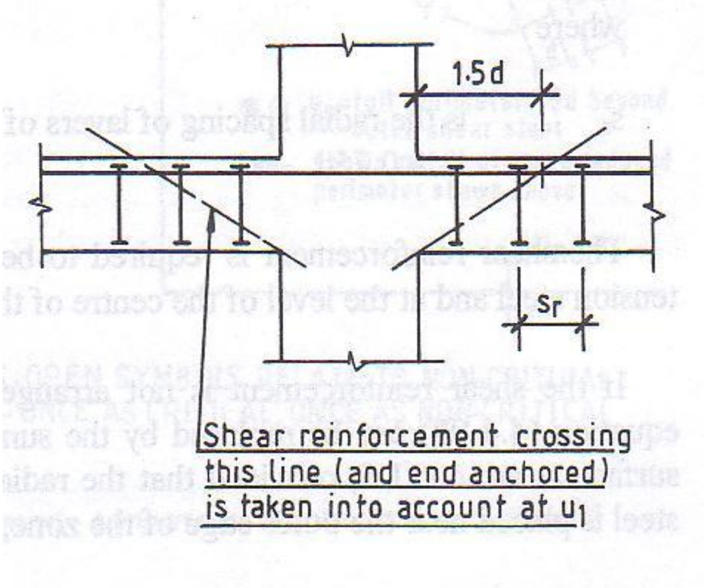
Punching shear reinforcement

Capacity with punching shear reinforcement

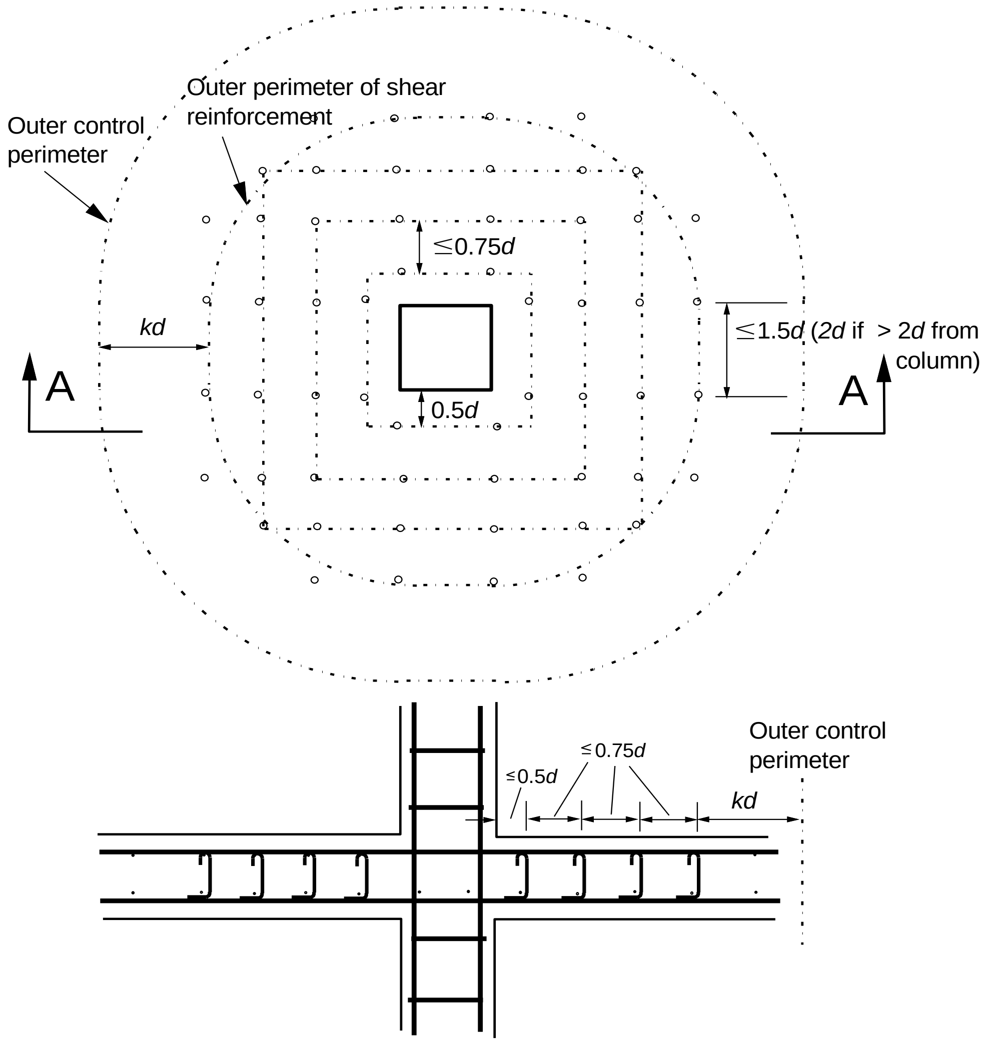
$$V_u = 0{,}75V_{Rd,c} + V_S$$

Shear reinforcement within 1,5d from column is accounted for with

$f_{y,red} = 250 + 0{,}25d(mm) \leq f_{ywd}$



Punching shear reinforcement



Section A - A

The outer control perimeter at which shear reinforcement is not required, should be calculated from:

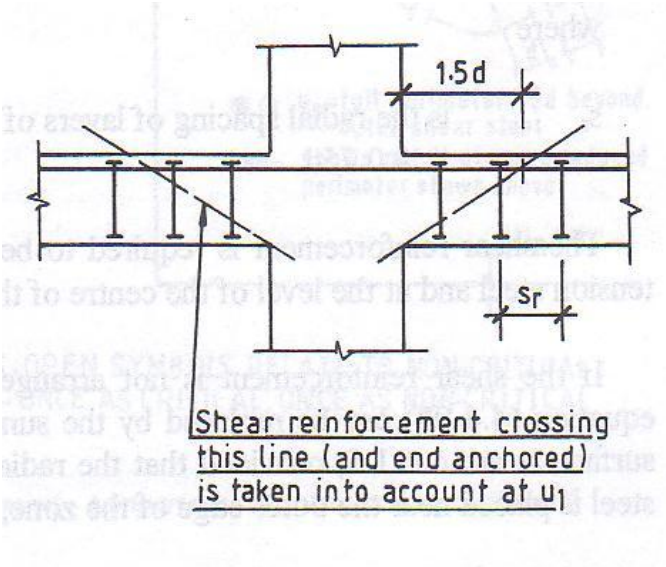
$$u_{out,ef} = V_{Ed} / (v_{Rd,c}\, d)$$

The outermost perimeter of shear reinforcement should be placed at a distance not greater than *kd* (*k* = 1.5) within the outer control perimeter.

Design of punching shear reinforcement

The necessary punching shear reinforcement per perimeter is found from:

$$A_{sw} = \frac{u_1 s_r (v_{Ed} - 0,75 v_{Rd,c})}{1,5 f_{ywd,ef}}$$



with:

v_{Ed} = 1,22 N/mm^2

$v_{Rd,c}$= 0,67 N/mm^2

u_1 = 4060 mm

$f_{yd,ef}$ = 250 + 0,25 · 164 = 291 N/mm^2

s_r = 0,75 · 164 = 123 mm → 120 mm

It is found that: A_{sw} = 800 mm^2 per reinforcement perimeter

Design of column B2 for punching shear

Determination of the outer perimeter for which $v_{Ed} = v_{Rd,c}$

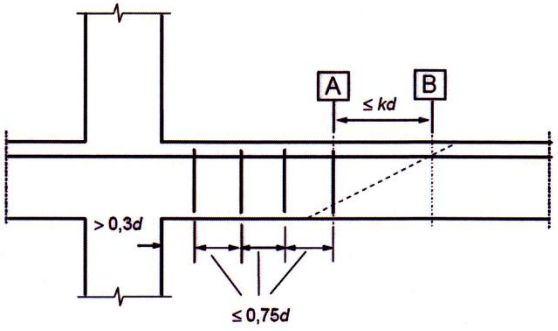
$$u_{out} = \beta V_{Ed} /(v_{Rd,c} \cdot d) = (1{,}15 \cdot 705000)/(0{,}67 \cdot 164 = 7378 mm$$

The distance from this perimeter to the edge of the column follows from:

$$a = (u_{out} - 4h)/2\pi = (7378 - 4 \cdot 500)/(2\pi) = 856 mm = 5{,}22d$$

The outer punching shera reinforcement should be at a distance of not more than 1,5d from the outer perimeter. This is at a distance 5,22d – 1,5d = 3,72d = 610 mm.

The distance between the punching shear reinforcement perimeters should not be larger than 0,75d = 0,75·164 = 123mm.



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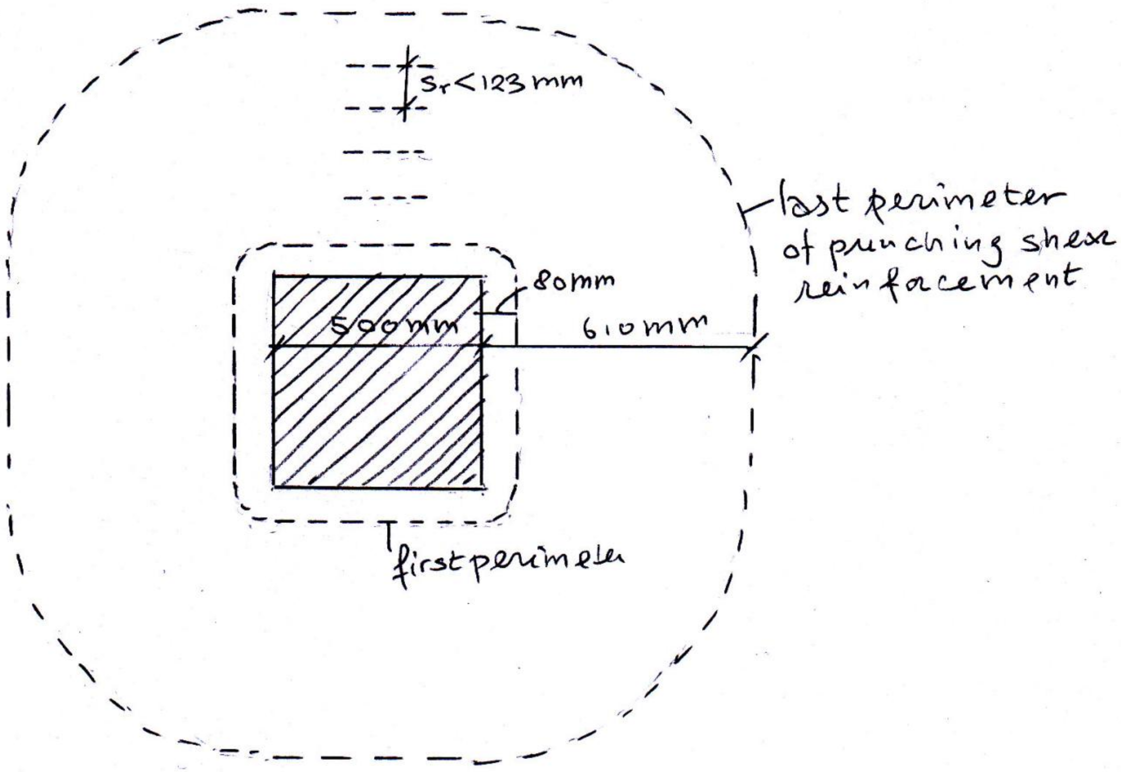
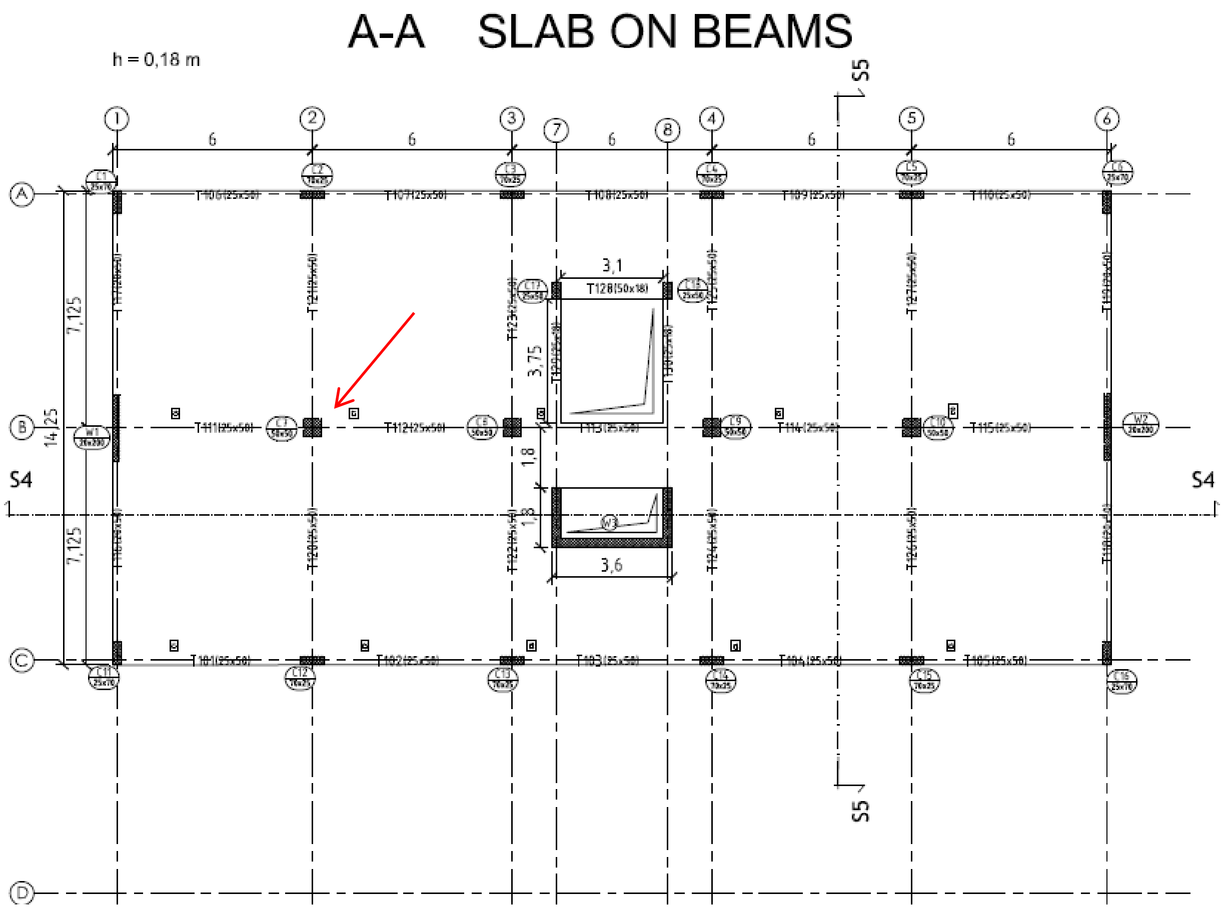
Punching shear design of slab at column B2



Perimeters of shear reinforcement

Design of column B2



General background: Second order effects at axial loading (EC2, 5.8.2, 5.8.3.1 & 5.8.3.3

- Second-order effects may be ignored if they are smaller than 10% of the corresponding 1th order effects

- "Slenderness": is defined as $\lambda = l_0/i$ where $i = \sqrt{(I/A)}$
so for rectangular cross-section $\lambda = 3{,}46\ l_0/h$
and for circular cross section $\lambda = 4l_0/h$

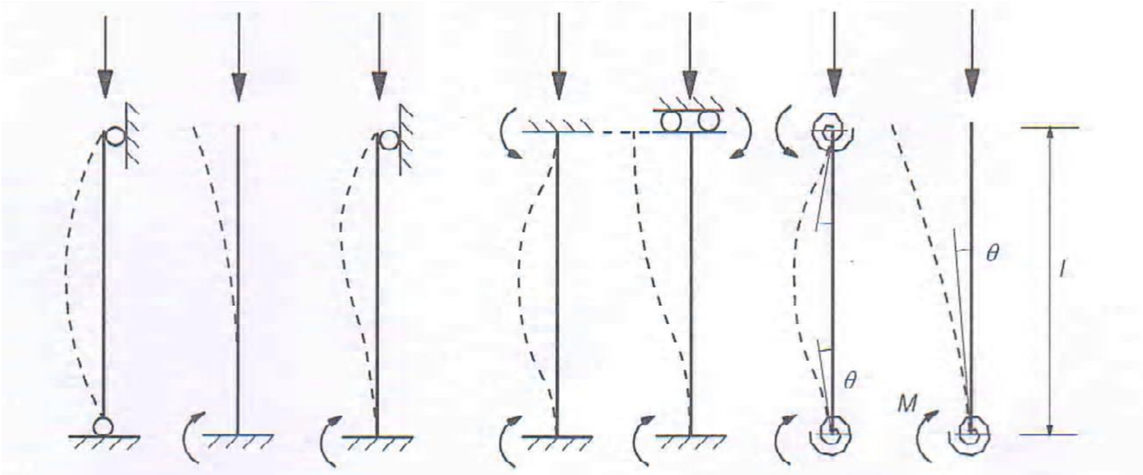
- Second order effects may be ignored if the slenderness is smaller than the limit value λ_{lim}

- In case of biaxial bending the slenderness should be calculated for any direction; second order effects need only to be considered in the direction(s) in which λ_{lim} is exceeded.

General background: "Slender" versus "short" columns

Definition of slenderness

$$\lambda = \frac{l_0}{i} = \frac{l_0}{\sqrt{(I/A)}}$$



l_0	effective height of the column
i	radius of gyration of the uncracked concrete section
I	moment of inertia around the axis considered
A	cross-sectional area of column

EC2 fig. 5.7

Basic cases

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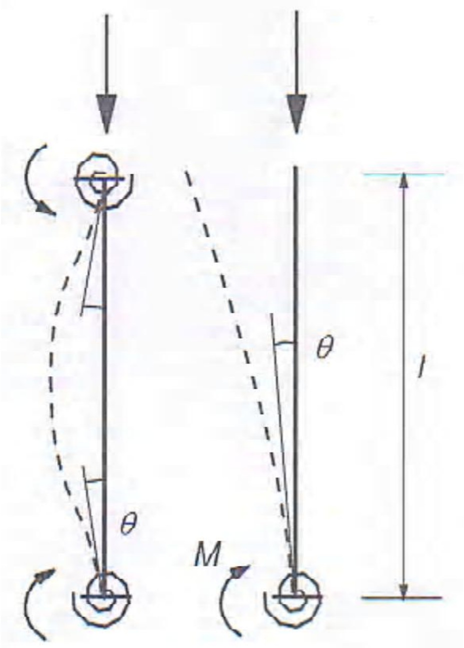
General background: when is a column slender?

Relative **flexibilities of** rotation-springs
at the column ends 1 en 2

$k = (\theta/M)\cdot(EI/l)$

where

θ = rotation of restraining members for a bending moment M
EI = bending stiffness of compression member
l = height of column between rotation-springs



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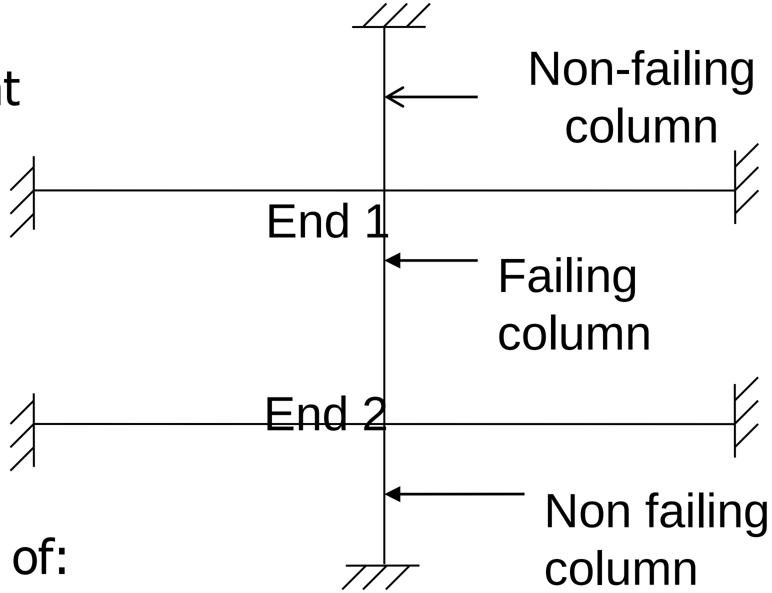
General background: when is a column slender?

Determination of effective column height in a frame



For **braced frames**:

$$l_0 = 0{,}5l\sqrt{(1+\frac{k_1}{0{,}45+k_1})(1+\frac{k_2}{0{,}45+k_2})}$$

For **unbraced frames**: the largest value of:

$$l_0 = l\sqrt{(1+10\frac{k_1 \cdot k_2}{k_1+k_2})}$$ and $$l_0 = l\left(1+\frac{k_1}{1+k_1}\right)\left(1+\frac{k_2}{1+k_2}\right)$$

where k_1 and k_2 are the relative spring stiffnesses at the ends of the column, and l is the clear height of the column between the end restraints

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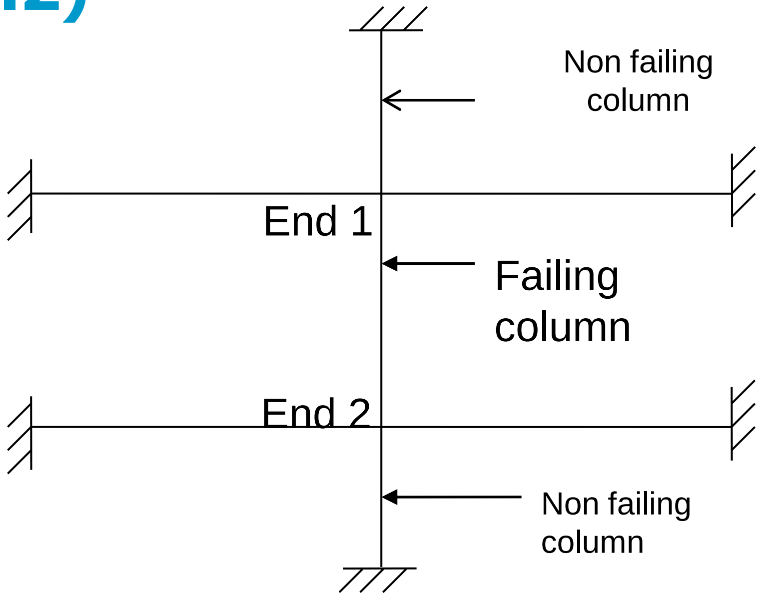
General background: determination of effective column length (1)   (5.8, 5.8.3.2)

Simplifying assumption:

* The contribution of the adjacent “non failing ” **columns** to the spring stiffness is ignored (if this contributes in a positive sense to the restraint)

* for **beams** for θ/M the value l/2EI may be assumed (taking account of loss of beam stiffness due to cracking)



Assuming that the beams are symmetric with regard to the column and that their dimensions are the same for the two stories, the following relations are found:

k1 = k2 = [EI/l]column / [ΣEI/l]beams = [EI/l]column / [2·2EI/l]beams = 0,25 χ

where: χ = [EI/l]column / [EI/l]beams

General background: Determination of effective column length (2) (5.8, 5.8.3.2)

The effective column length l_0 can, for this situation be read from the table as a function of χ

$$\lambda = \frac{l_0}{i} = \frac{l_0}{\sqrt{(I/A)}}$$

χ	0 (fixed end)	0.25	0.5	1.0	2.0	∞ (pinned end)
or $k_1 = k_2$	0	0.0625	0.125	0.25	0.50	1.0
l_0 for braced column	0.5 l	0.56 l	0.61 l	0.68 l	0.76 l	1.0 l
l_0 for unbraced column:	1.0 l	1.14 l	1.27 l	1.50 l	1.87 l	∞
Larger of the values in the two rows	1.0 l	1.12 l	1.13 l	1.44 l	1.78 l	∞

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General background: when is a column slender ?

A column is qualified as "slender", which implies that second order effects should be taken into account, if $\lambda \geq \lambda$lim. The limit value is defined as:

$$\lambda_{\lim} = 20 \cdot A \cdot B \cdot C / \sqrt{n}$$

where:

$$A = 1/(1 + 0{,}2\phi_{ef})$$

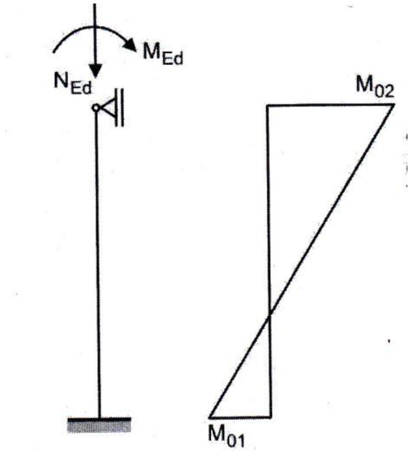
$$B = \sqrt{1 + 2\omega}$$

$$C = 1{,}7 - r_m$$

φ_{ef} = effective creep factor: if unknown it can be assumed that A = 0,7

ω = $A_s f_{yd}/(A_c f_{cd})$: mech. reinforcement ratio, if unknown B = 1,1 can be adopted

n = $N_{Ed}/(A_c f_{cd})$;



r_m = M_{01}/M_{02}: ratio between end-moments in column, with $|M_{02}| \geq |M_{01}|$

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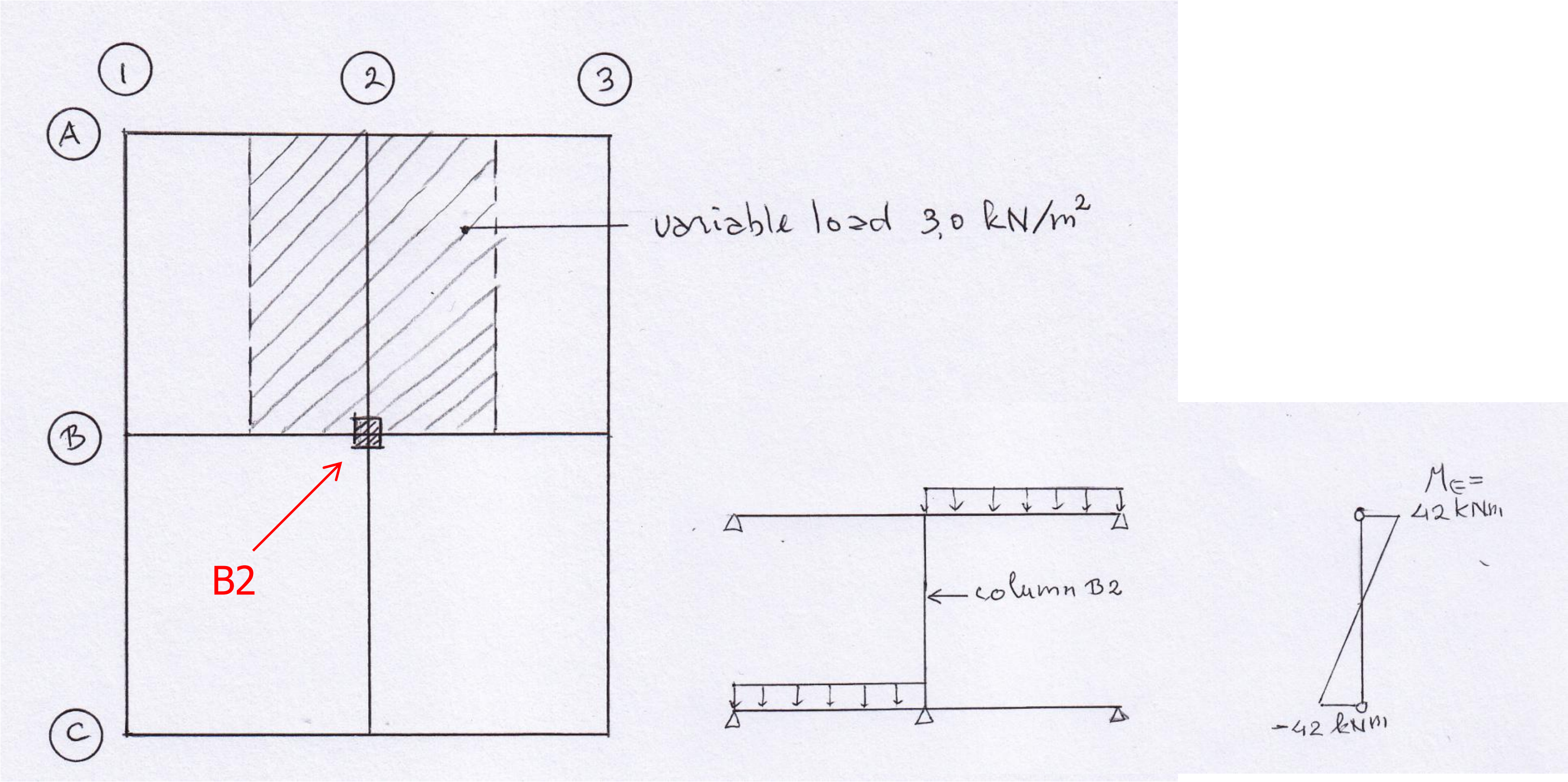
Design of column B2



Configuration of variable load on slab

Determination of columns slenderness λ

First step: determination of rotational spring stiffness at end of column:

Column: $EI/l = 0{,}043 \cdot 10^6$ kNm^2
Beam: $EI/l = 0{,}052 \cdot 10^6$ kNm^2

$K_1 = [EI/l]_{col}/[\Sigma EI]_{beams} = 0{,}043/(2 \cdot 0{,}052) = 0{,}41$

$$l_0 = 0{,}5l\sqrt{(1+\frac{k_1}{0{,}45+k_1})(1+\frac{k_2}{0{,}45+k_2})} = 0{,}5l\sqrt{(1+\frac{0{,}41}{1{,}02})^2} = 0{,}70l$$

If the beam would be cracked a value of 1,5 k_1 would be more realistic. This would result in $l_0 = 0{,}80l = 3{,}2$m.

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Verification of column slenderness

Actual slenderness of column: $$\lambda = \frac{3{,}46 l_0}{h} = \frac{3{,}46 \cdot 3{,}2}{0{,}5} = 22{,}1$$

Limit slenderness according to EC2, Cl. 5.8.3.1: $$\lambda_{\lim} = \frac{20 A \cdot B \cdot C}{\sqrt{n}}$$

With the default values A = 0,7 B=1,1 C = 0,7 whereas the value *n* follows from *n*= $N_{ed}/(A_c f_{cd})$ = 438400/(500^2·20) = 0,88, the value of λ_{lim} becomes:

$$\lambda_{\lim} = \frac{20 \cdot 0{,}7 \cdot 1{,}1 \cdot 0{,}7}{\sqrt{0{,}88}} = 11{,}5$$

Because the actual slenderness of the column is larger than the limit slenderness second order effects have to be taken into account.

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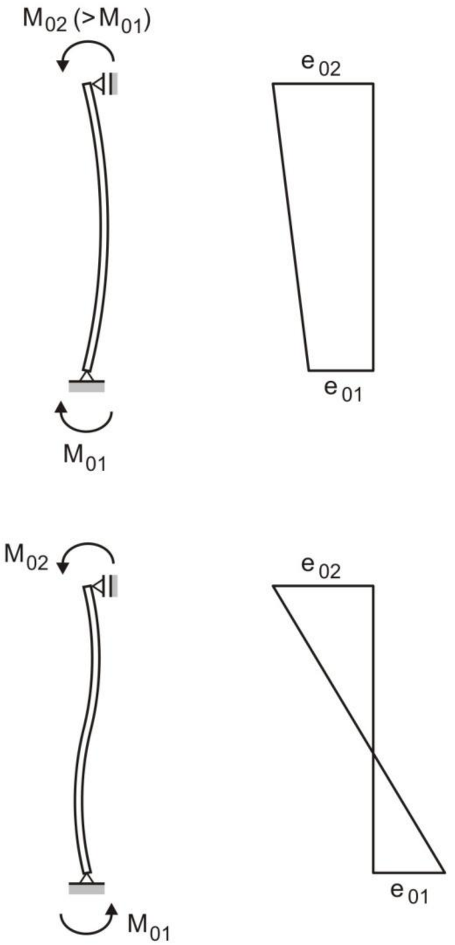
General : Method based on nominal curvature

$Mt = N_{Ed} (e_0 + e_i + e_2)$

Different first order eccentricities e_{01} en e_{02}
At the end of the column can be replaced by
an equivalent eccentricity e_0 defined as:

$e_0 = 0{,}6e_{02} + 0{,}4e_{01} \geq 0{,}4e_{02}$

e_{01} and e_{02} have the same sign if they lead to tension at the same side, otherwise different signs.
Moreover $|e_{02}| \geq |e_{01}|$



General : Method based on the nominal curvature

$M_t = N_{Ed} (e_0 + e_i + e_2)$

The eccentricity e_i by imperfection follows from (5.2(7)):

$$e_i = v \frac{l_0}{2}$$

where l_0 = effective column height around the axis regarded

$$v = \frac{1}{100\sqrt{l}} \geq \frac{1}{200}$$

where l = the height of the column in meters

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General: Method based on nominal curvature

$M_t = N_{Ed} (e_0 + e_i + e_2)$

The second order eccentricity e_2 follows from:

$$e_2 = K_\varphi K_r \frac{l_0^2}{\pi^2} \frac{\varepsilon_{yd}}{0,45d}$$

where

$$K_\varphi = 1 + (0{,}35 + \frac{f_{ck}}{200} - \frac{\lambda}{150})\varphi_{ef} \geq 1$$

and

$$K_r = \frac{N_{ud} - N_{Ed}}{N_{ud} - N_{bal}} \leq 1{,}0$$

Calculation of bending moment including second order effects

The bending moment on the column follows from:

$$M_t = M_{Ed}(e_0 + e_i + e_2)$$

e_0 = M_{ed}/N_{ed} = 42/4384 = 0,010m = 10mm . However, at least the maximum value of {$l_0/20$, b/20 or 20mm} should be taken. So, e_0 =b/20 = 500/20 = 25mm.

$e_i = \theta_i(l_0/2)$ where $\theta_i = \theta_0\alpha_h\alpha_m$ θ_0 = 1/200 rad, $\alpha_h = 2/\sqrt{l}$ = 1 and

$$\alpha_m = \sqrt{0{,}5(1+\frac{1}{m})} = \sqrt{0{,}5(1+\frac{1}{1})} = 1$$ so that e_i =(1/200)·(4000/2) = 10mm

$$e_2 = K_\varphi K_r \frac{l_0^2}{\pi^2}\frac{\varepsilon_{yd}}{0{,}45d}$$ where $$K_\varphi = 1 + (0{,}35 + \frac{f_{ck}}{200} - \frac{\lambda}{150})\varphi_{eff}$$ and

$$\varphi_{eff} = (M_{0Eqp}/M_{0Ed})\varphi_{\infty,t}$$ and finaly $$K_r = \frac{n_u - n_{Ed}}{n_u - n_{bal}}$$

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Calculation of bending moment including second order effects

$$\varphi_{eff} = \frac{M_{0Eqp}}{M_{0Ed}}\varphi_{\infty} = \frac{0,3 \cdot 2}{1,5 \cdot 2} \cdot 2 = 0,4$$

$$K_{\varphi} = 1 + (0,35 + \frac{f_{ck}}{200} - \frac{\lambda}{150})\varphi_{eff} = 1 + (0,35 + \frac{30}{200} - \frac{22,9}{150})0,4 = 1,14$$

$$K_r = \frac{n_u - n_{Ed}}{n_u - n_{bal}}$$ where $$n_u = 1 + \frac{\rho f_{yd}}{f_{cd}} = 1 + \frac{0,03 \cdot 435}{20} = 1,65$$ (estimated value ρ = 0,03)

$$n_{Ed} = \frac{N_{Ed}}{A_c f_{cd}} = 0,88$$ $$n_{bal} \cong 0,4$$ so K_r = 0,62 and finaly:

$$e_2 = K_{\varphi} K_r \frac{l_0^2}{\pi^2} \frac{\varepsilon_{yd}}{0,45d} = 1,15 \cdot 0,62 \cdot \frac{3200^2}{\pi^2} \frac{2,17 \cdot 10^{-3}}{0,25 \cdot 454} = 14mm$$

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Calculation of bending moment including second order effects and reinforcement

$$M_{tot} = N_{Ed}(e_0 + e_1 + e_2) = 4384 \cdot (25 + 10 + 14) \cdot 10^{-3} = 215 kNm$$

Determination of reinforcement

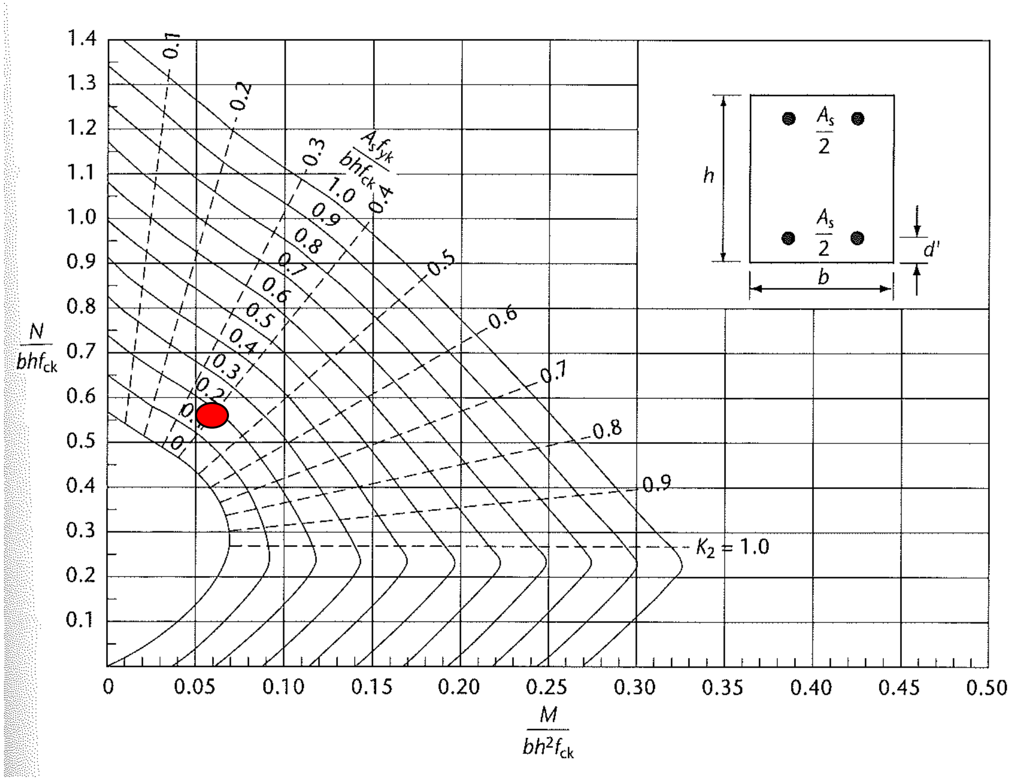
$$\frac{N_{Ed}}{bhf_{ck}} = \frac{4384000}{500^2 \cdot 30} = 0,58$$

$$\frac{M_{Ed}}{bh^2 f_{cd}} = \frac{215000}{500^3 \cdot 30} = 0,06$$

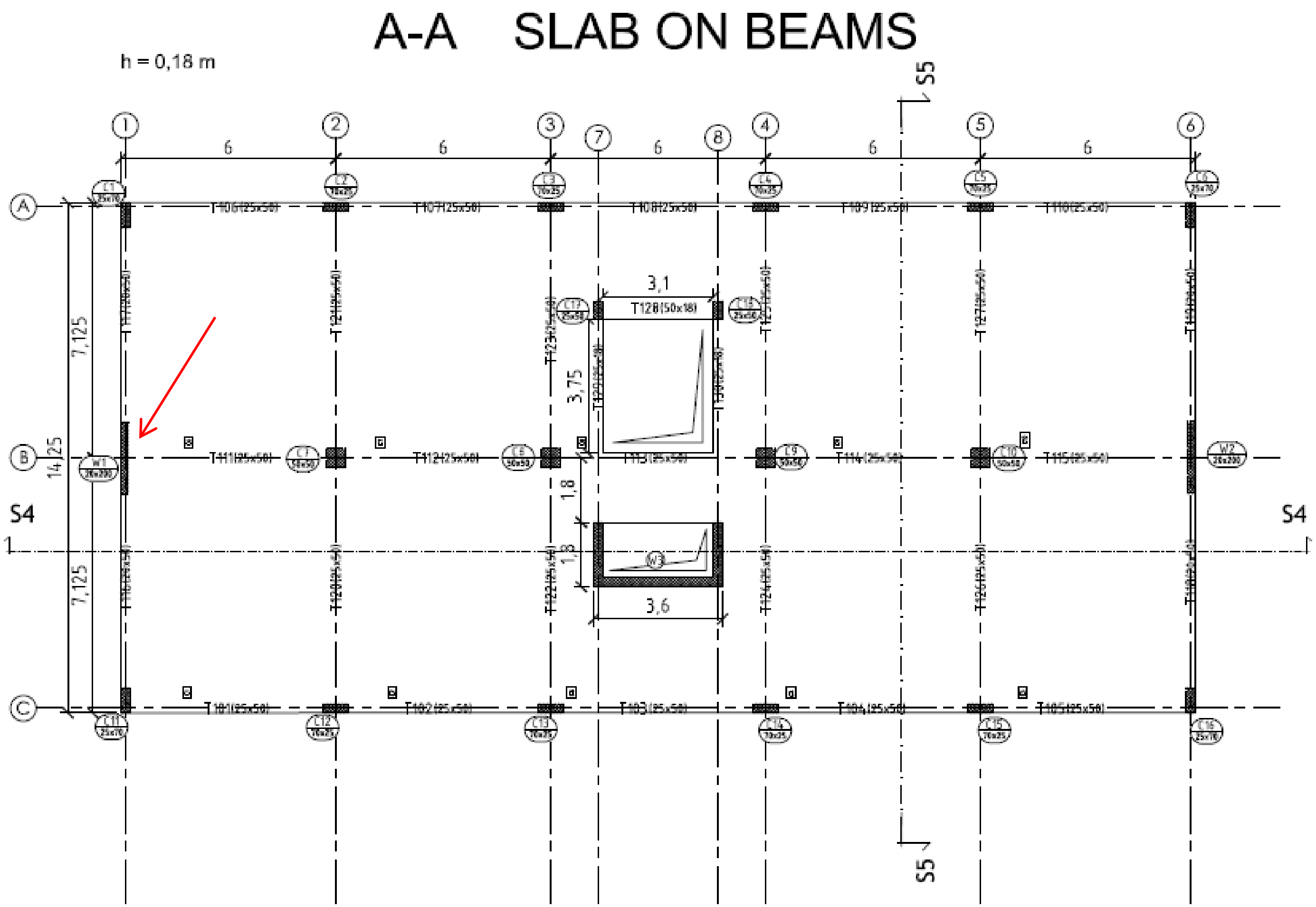
From diagram: $\dfrac{A_s f_{yk}}{bhf_{ck}} = 0,15$

So: $A_s = \dfrac{0,20 \cdot 500^2 \cdot 30}{435} = 3448 mm^2$

(1,4%)



Design of shear wall



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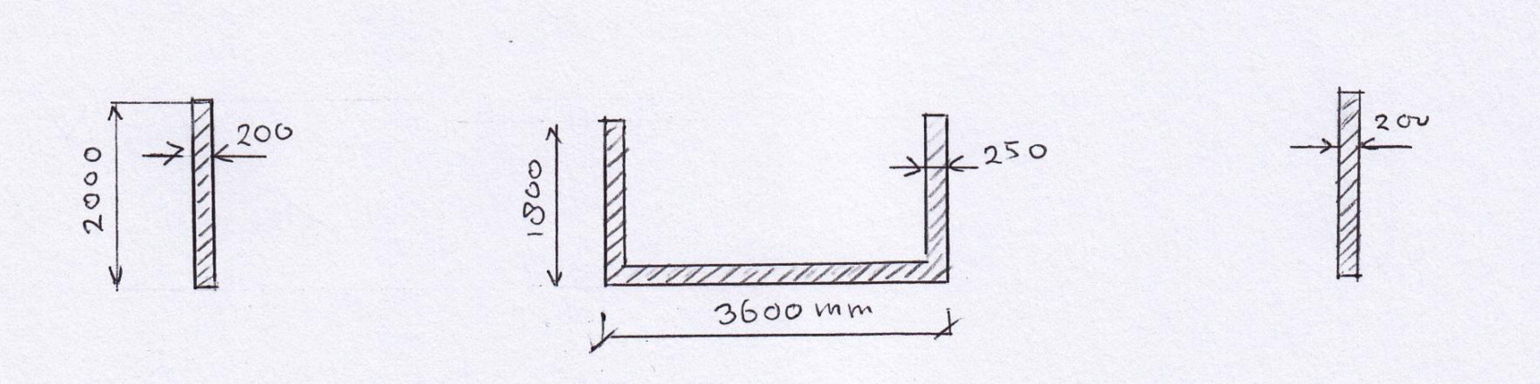
Design of shear wall

The stability of the building is ensured by two shear walls (one at any end of the building) and one central core

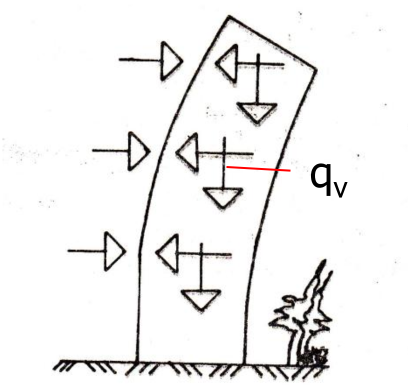


shear wall 1	core	shear wall 2
$I = 0{,}133\ m^4$	$I = 0{,}514\ m^4$	$I = 0{,}133\ m^4$

Contribution of shear wall 1: $0{,}133/(2\cdot 0{,}133 + 0{,}514) = 0{,}17$ (17%)

Second order effects to be regarded?

"If second order effects are smaller than 10% of the first order moments they can be neglected".



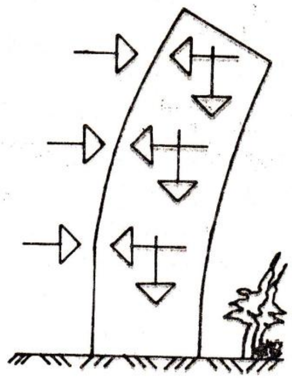
Moment magnification factor:

$$M_{Ed} = M_{0Ed}[1 + \frac{\beta}{N_B / N_{Ed} - 1}]$$

$$N_B = \frac{\pi^2 EI}{(1{,}12l)^2} \qquad N_{Ed} = q_v l$$

N_B is the buckling load of the system sketched, l = height of building, q_v = uniformely distributed load in vertical direction, contributing to 2nd order deformation.

Second order effects to be regarded?



The moment magnification factor is:

$$f = \frac{n}{n-1} \qquad \text{where } n = N_B/N_{Ed}$$

Requiring f < 1,1 and substituting the corresponding values in the equation above gives the condition:

$$l\sqrt{\frac{q_{vEd}\,l}{EI}} \leq 0{,}84 \qquad \text{(Eq.1)}$$

Assuming 30% of the variable load as permanent, the load per story is 30·14,25·10,65 = 4553 kN. Since the storey height is 3m, this corresponds with q_v=1553 kN/m’ height.
With l = 19m, E = 33.000/1,2 = 27.500 MPA, I = 0,78 m^4

$$\frac{19}{10^3}\sqrt{\frac{1518 \cdot 19}{27{,}5 \cdot 0{,}78}} = 0{,}70 \leq 0{,}84$$

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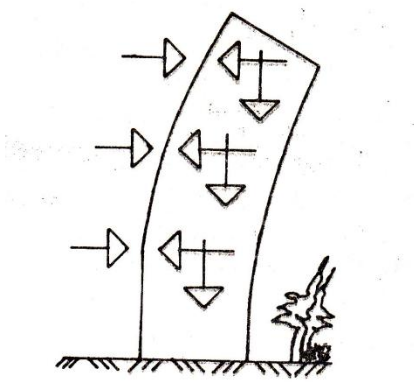
Second order effects to be regarded?



However, in the calculation it was assumed that the stabilizing elements were not cracked. In that case a lower stiffness should be used.

For the shear wall the following actions apply:
Max M_y = 66,59 kNm = 0,0666 MNm
Corresp. N = -2392,6 kN = 2,392 MN/m²

$$\sigma_N = \frac{-2392}{2 \cdot 0{,}25} = -4{,}78 MN/m^2$$

$$\sigma_M = \frac{M}{W} = \frac{0{,}0666}{0{,}01667} = +3{,}99 MN/m^2$$

So the shear wall remains indeed uncracked and 2nd order effects may be ignored.

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Alternative check by Eq. 5.18 in EC2

According to Cl. 5.8.3.3 of EC-22nd order effects may be ignored if:

$$F_{V,Ed} \leq k_1 \cdot \frac{n_s}{n_s + 1{,}6} \frac{\Sigma E_{cd} I_c}{L^2}$$

Where

$F_{V,Ed}$	total vertical load (both on braced and unbraced elements)
n_s	number of storeys
L	total height of building above fixed foundation
E_{cd}	design E-modulus of the concrete
I_c	moment of intertia of stabilizing elements

The advisory value of the factor k_1 is 0,31. If it can be shown that the stabilizing elements remain uncracked k_1 may be taken 0,62

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Alternative check by Eq. 5.18 in EC2

Verification for the building considered:

$$F_{V,Ed} \leq k_1 \cdot \frac{n_s}{n_s + 1{,}6} \frac{\Sigma E_{cd} I_c}{L^2}$$

Condition:

$$6 \cdot 4553 \leq 0{,}62 \cdot \frac{6}{6 + 1{,}6} \cdot \frac{27{,}5 \cdot 10^6 \cdot 0{,}78}{19^2}$$

or: $27.318 \leq 29.084$

so the condition is indeed fullfilled

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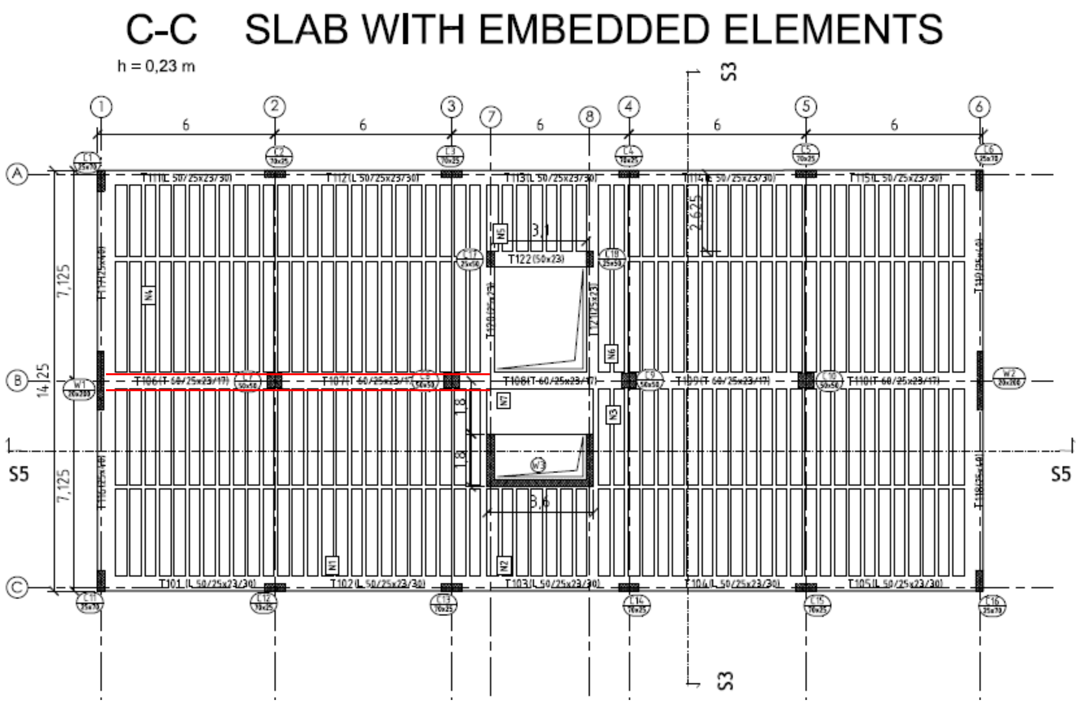
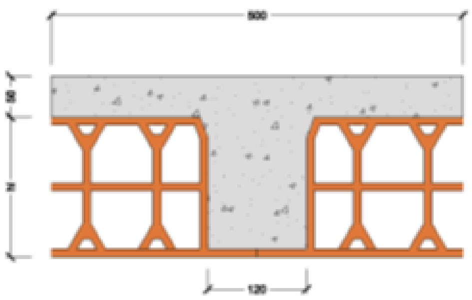
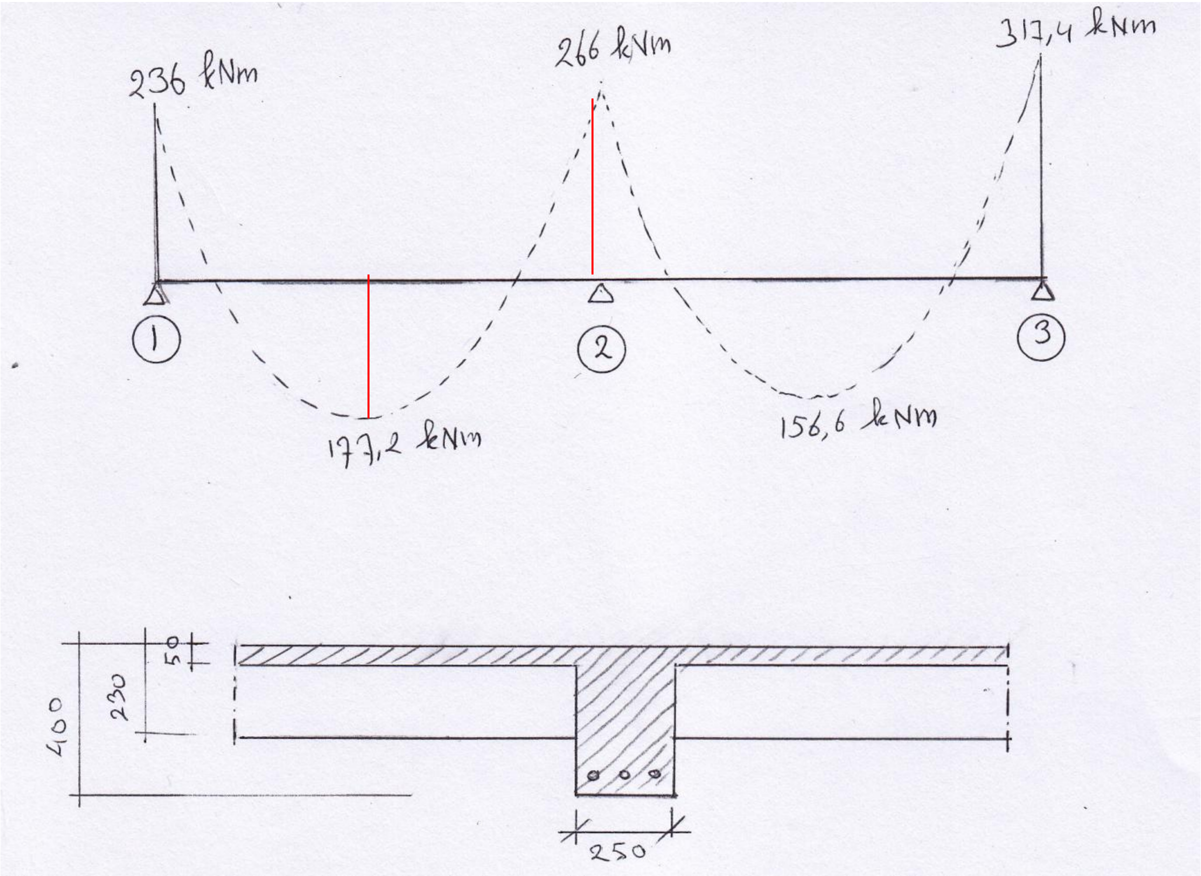
Monodirectional slab with embedded lighting elements





Bearing beams in floor with embedded elements



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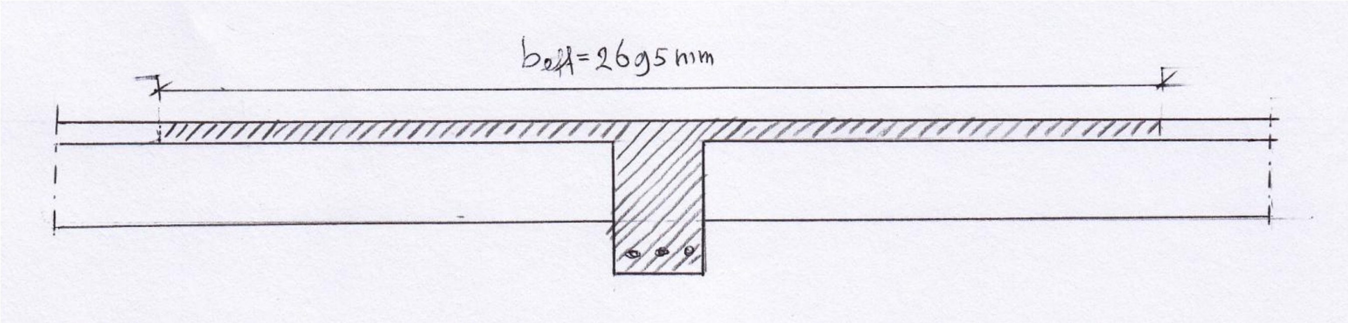
Design for bending of main bearing beam in span 1-2

M_{ed} = 177,2 kNm

Effective width: $b_{eff} = \Sigma b_{eff,i} + b_w \leq b$ $b_{eff,i} = 0{,}2b_i + 0{,}1l_0$

Midspan: b_{eff} = 2695 mm



$$\frac{M_{Ed}}{bd^2 f_{ck}} = \frac{172{,}2 \cdot 10^6}{2695 \cdot 372^2 25} = 0{,}02$$

from diagram z = 0,98d = 365mm

$$A_{sl} = \frac{M_{Ed}}{z \cdot f_{yd}} = \frac{172{,}2 \cdot 10^6}{365 \cdot 435} = 1367mm^2$$

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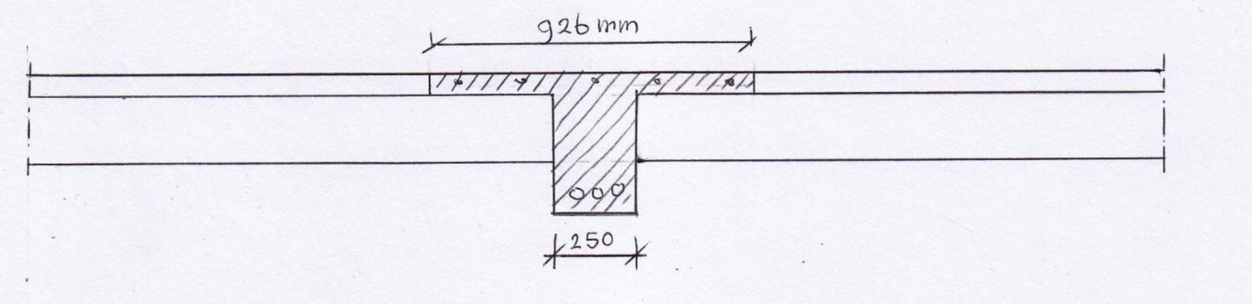
Design for bending of main bearing beam in span 1-2 (intermediate support)

M_{ed} = 266 kNm

Effective width: $b_{eff} = \Sigma b_{eff,i} + b_w \leq b$ $\quad b_{eff,i} = 0{,}2b_i + 0{,}1l_0$

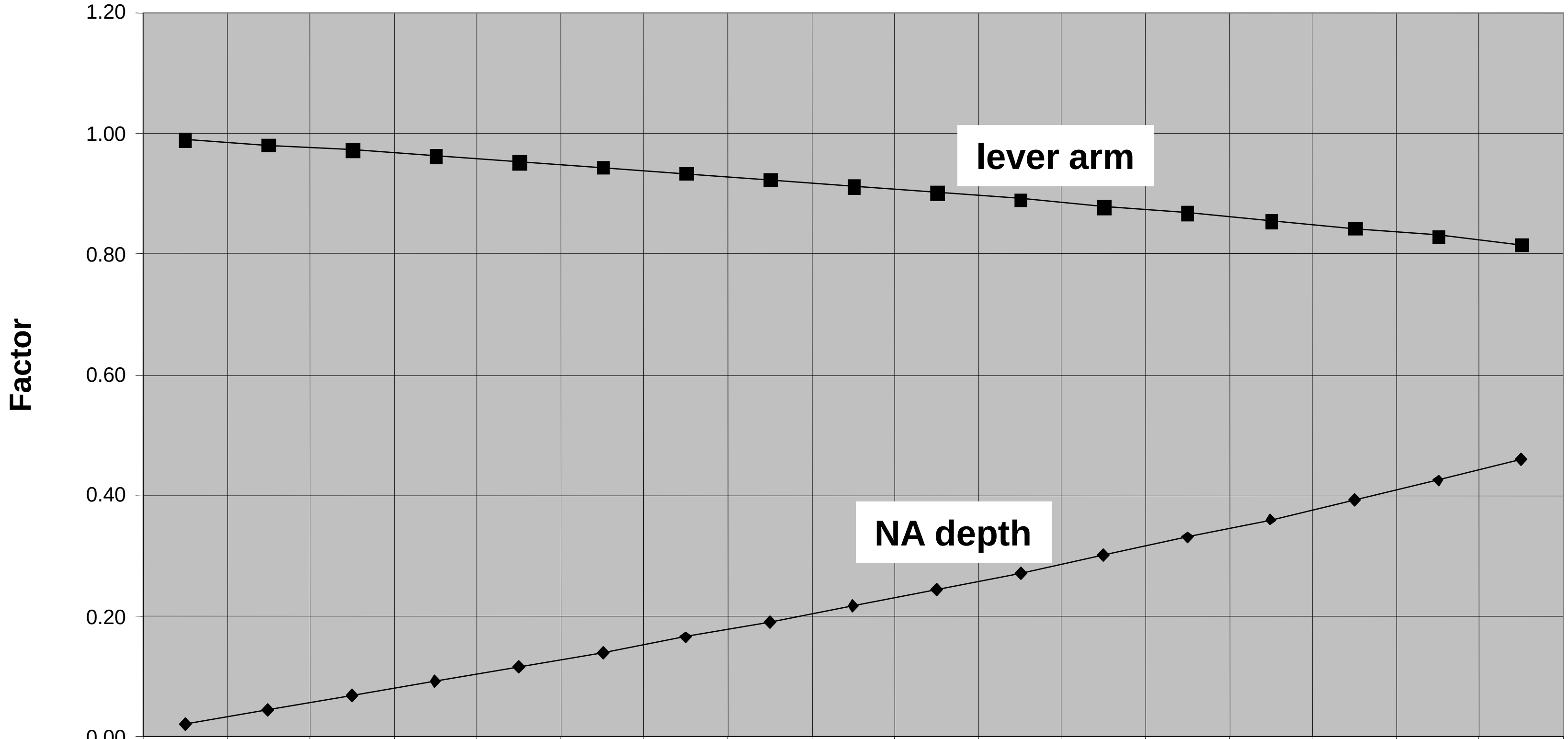
Internal support: b_{eff} = 926 mm



At intermediate support: $\dfrac{M_{Ed}}{bd^2 f_{ck}} = \dfrac{266 \cdot 10^6}{250 \cdot 372^2 \cdot 25} = 0{,}31$!?

Simplified factors for flexure (1)

Factors for NA depth (n) and lever arm (=z) for concrete grade ≤ 50 MPa



M/bd^2f_{ck}	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17
n	0.02	0.04	0.07	0.09	0.12	0.14	0.17	0.19	0.22	0.24	0.27	0.30	0.33	0.36	0.39	0.43	0.46
z	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.84	0.83	0.82

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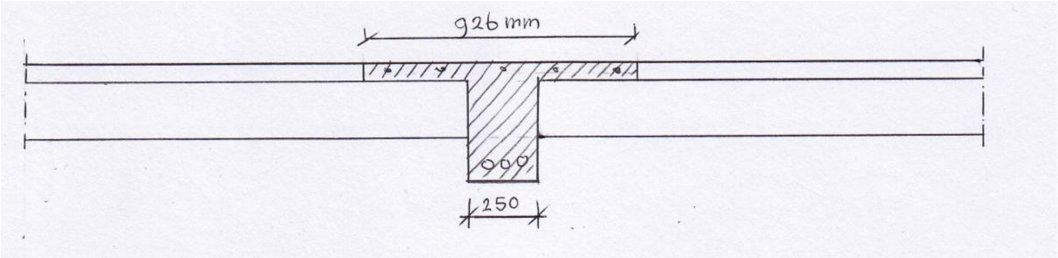
Design for bending of main bearing beam in span 1-2 (intermediate support)

M_{ed} = 266 kNm

Effective width: $b_{eff} = \Sigma b_{eff,i} + b_w \leq b$ $\quad b_{eff,i} = 0{,}2b_i + 0{,}1l_0$

Internal support: b_{eff} = 926 mm



At intermediate support compression reinforcement required:

$$A_{sc} = \frac{(K-K')f_{ck}bd^2}{f_{yd}(d-d')} = \frac{(0{,}31-0{,}167)25\cdot 250\cdot 372^2}{435\cdot(372-35)} = 826mm^2$$

e.g. 3∅20

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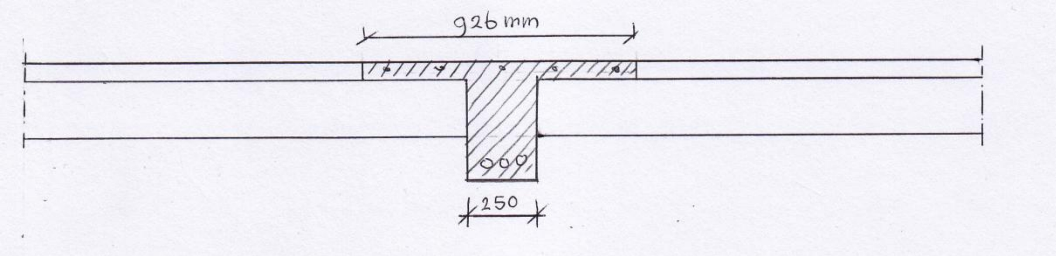
Design for bending of main bearing beam in span 1-2 (intermediate support)

M_{ed} = 266 kNm

Effective width: $b_{eff} = \Sigma b_{eff,i} + b_w \leq b$ $\quad b_{eff,i} = 0{,}2b_i + 0{,}1l_0$

Internal support: b_{eff} = 926 mm



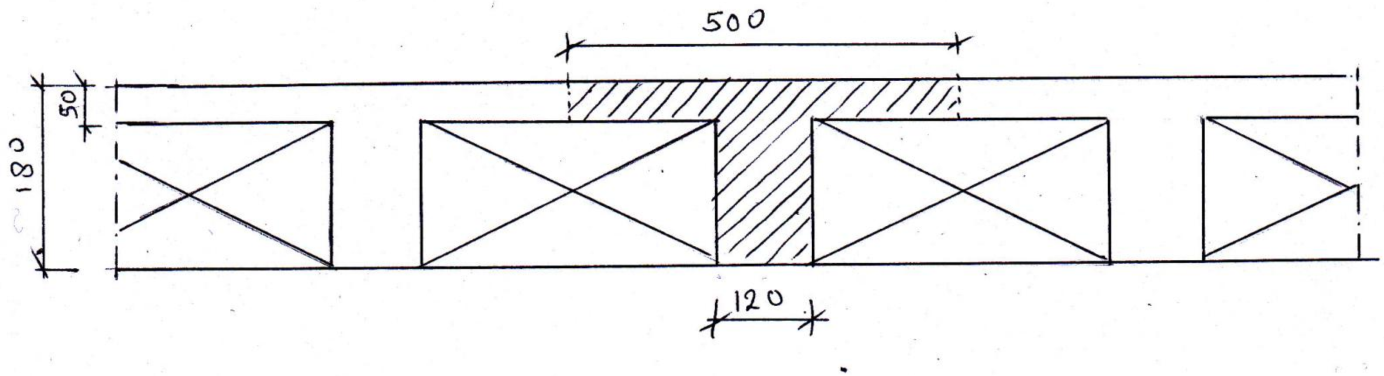
Calculation of tensile reinforcement:

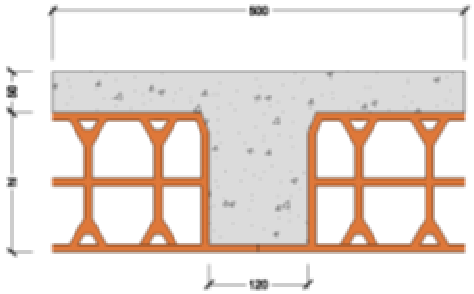
For K = 0,167 z = 0,81·372=301 mm

$$A_{sl} = \frac{M_{Ed}}{z \cdot f_{yd}} = \frac{266 \cdot 10^6}{301 \cdot 435} = 2031 mm^2$$

e.g. 7∅20 = 2198 mm²

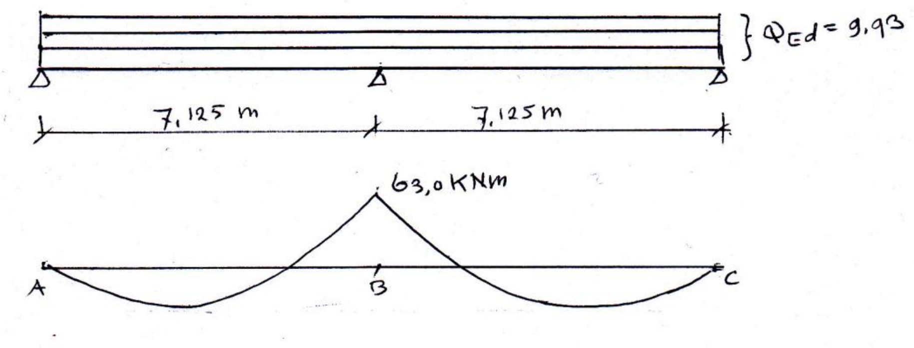
Design of one-way beams with embedded elements





Loads:
G_1 = 2,33 kN
G_2 = 3,0
Q = 2,0
Q_{ed} = 1,3(2,33+3,0) + 1,5·2,0=9,93 kN/m^2

Beams with embedded elements: design for bending at intermediate support



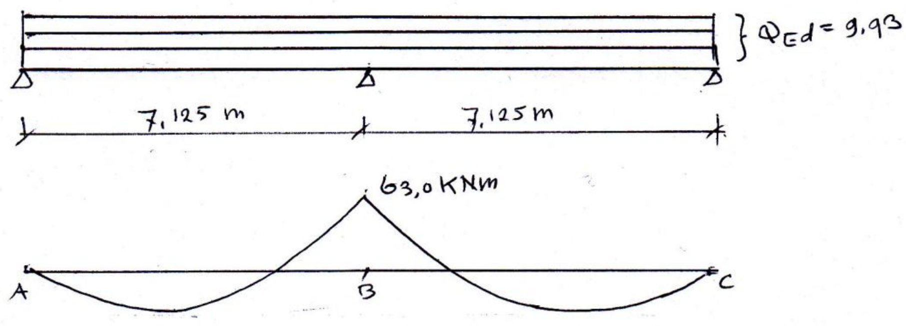
$$k = \frac{M_{Ed}}{bd^2 f_{ck}} \frac{63 \cdot 10^6}{240 \cdot 189^2 \cdot 25} = 0{,}294 > 0{,}167$$

Compression reinforcement required

$$A_{sc} = \frac{(K - K') f_{ck} bd^2}{f_{yd}(d - d')} = \frac{(0{,}294 - 0{,}167) \cdot 25 \cdot 240 \cdot 189^2}{435 \cdot (189 - 35)} = 406 mm^2$$

In any rib 203 mm^2

Beams with embedded elements: design for bending at intermediate support

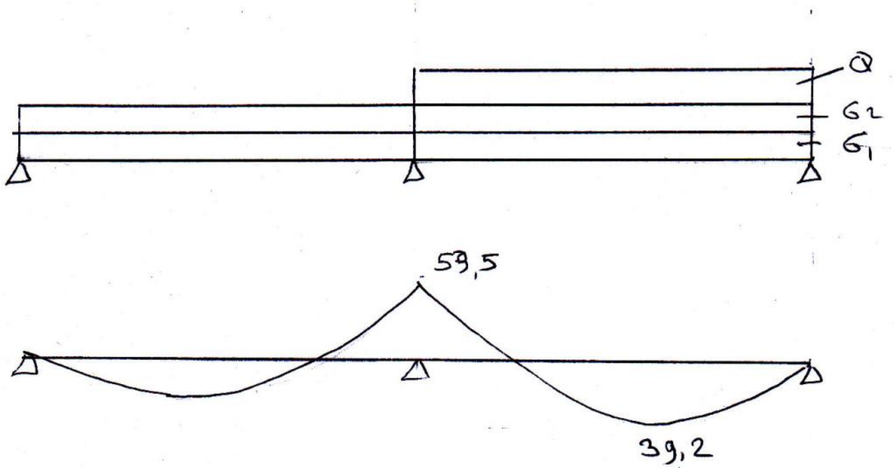


Tensile reinforcement: for K = 0,167 z = 0,8·189=151 mm

$$A_{sl} = \frac{M_{Ed}}{z \cdot f_{yd}} = \frac{63 \cdot 10^6}{151 \cdot 435} = 959 mm^2$$

e.g. ∅12-100 = 1130 mm^2 or
∅10-75 = 1040 mm^2

Beams with embedded elements: design for bending at midspan



$$K = \frac{M_{Ed}}{bd^2 f_{ck}} = \frac{39{,}2 \cdot 10^6}{1000 \cdot 189^2 \cdot 25} = 0{,}044$$

From diagram z = 0,95d = 0,95·189 = 180 mm

$$A_{sl} = \frac{M_{Ed}}{z \cdot f_{yd}} = \frac{39{,}2 \cdot 10^6}{180 \cdot 435} = 501 mm^2$$

251 mm² per rib

Deflection control by slenderness limitation

For span-depth ratios below the following limits no further checks is needed

$$\frac{l}{d} = K\left[11 + 1{,}5\sqrt{f_{ck}}\,\frac{\rho_0}{\rho} + 3{,}2\sqrt{f_{ck}}\left(\frac{\rho_0}{\rho} - 1\right)^{3/2}\right] \quad \text{if } \rho \le \rho_0 \qquad (7.16.a)$$

$$\frac{l}{d} = K\left[11 + 1{,}5\sqrt{f_{ck}}\,\frac{\rho_0}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}}\sqrt{\frac{\rho'}{\rho_0}}\right] \quad \text{if } \rho > \rho_0 \qquad (7.16.b)$$

l/d	is the limit span/depth
K	is the factor to take into account the different structural systems
ρ_0	is the reference reinforcement ratio = $\sqrt{f_{ck}}\ 10^{-3}$
ρ	is the required tension reinforcement ratio at mid-span to resist the moment due to the design loads (at support for cantilevers)
ρ'	is the required compression reinforcement ratio at mid-span to resist the moment due to design loads (at support for cantilevers)

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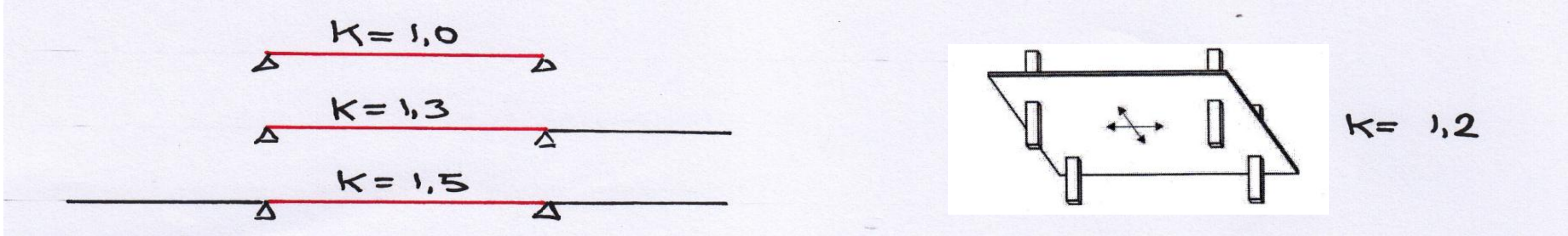
Deflection control by slenderness limitation

The expressions given before (Eq. 7.6.a/b) are derived based on many different assumptions (age of loading, time of removal of formwork, temperature and humidity effects) and represent a conservative approach.

The coefficient K follows from the static system:



The expressions have been derived for an assumed stress of 310 Mpa under the quasi permanent load. If another stress level applies, or if more reinforcement than required is provided, the values obtained by Eq. 7.16a/b can be multiplied with the factor

$$\frac{\sigma_s}{310} = \frac{500}{f_{yk} \cdot \left(A_{s,req} / A_{s,prov} \right)}$$

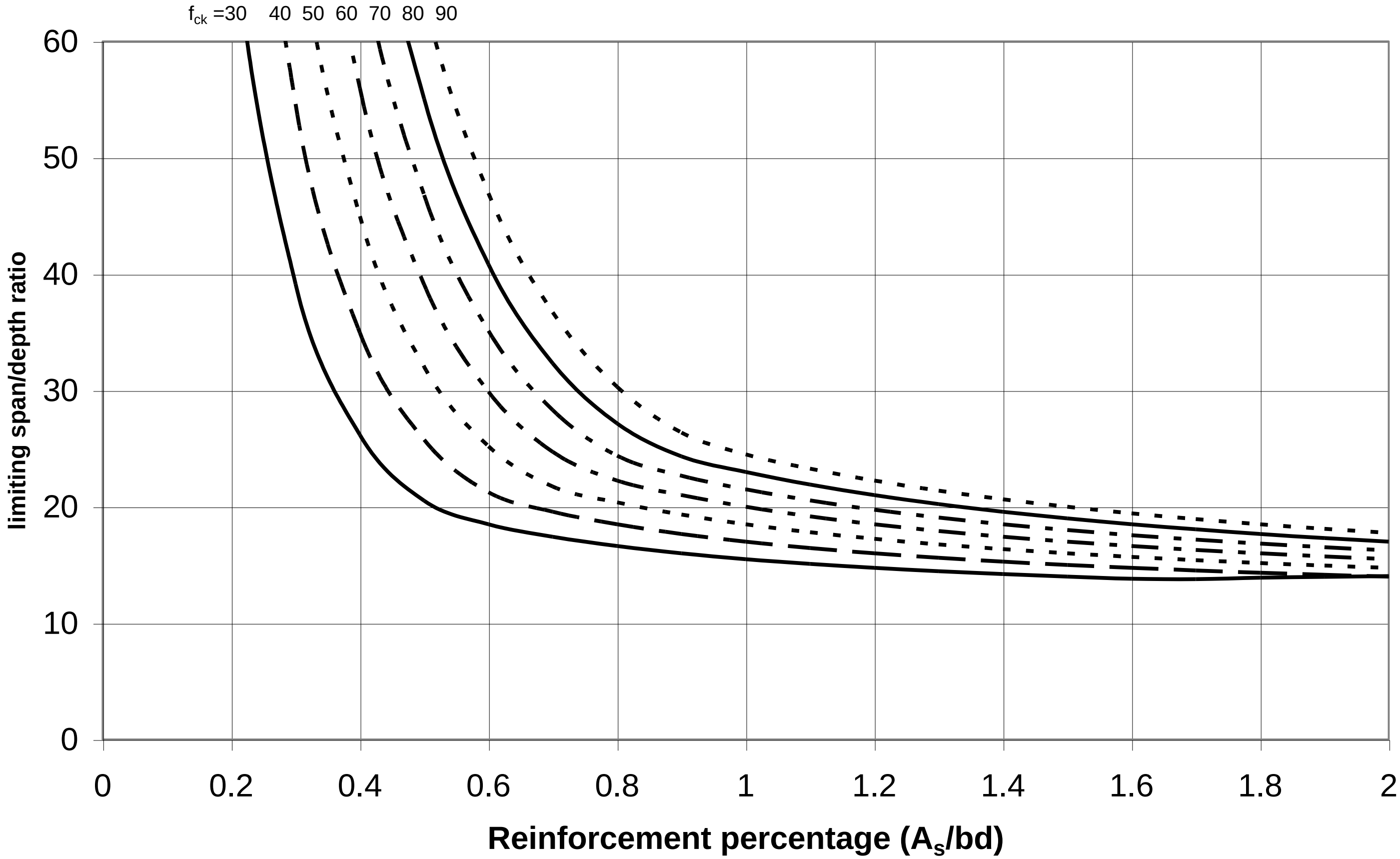
where σ_s is the stress in the reinforcing steel at mid-span

Rules for large spans

For beams and slabs (no flat slabs) with spans larger than 7m, which support partitions liable to damage by excessive deflections, the values l/d given by Eq. (7.16) should be multiplied by $7/l_{eff}$ (l_{eff} in meters).

For flat slabs where the greater span exceeds 8,5m, and which support partitions to be damaged by excessive deflections, the values l/d given by expression (7.16) should be multiplied by $8{,}5/\ l_{eff}$.

Eq. 7.16 as a graphical representation, assuming K = 1 and σ_s = 310 MPa

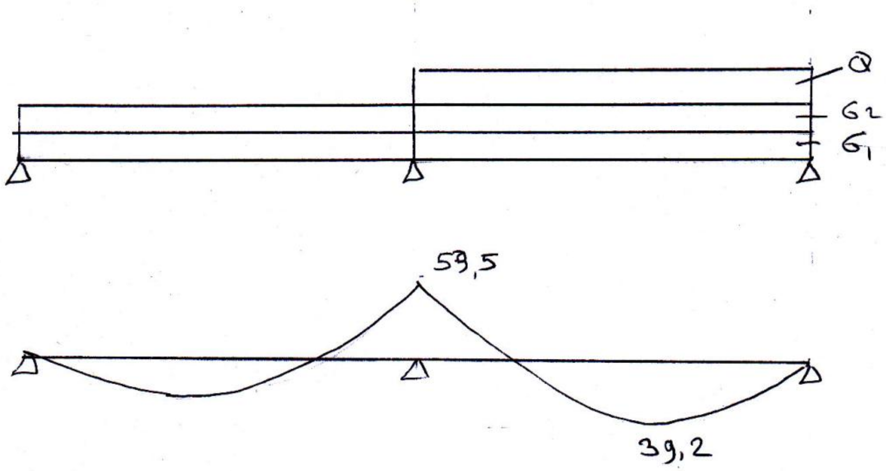


Tabulated values for l/d calculated from Eq. 7.16a/b

The table below gives the values of K (Eq.7.16), corresponding to the structural system. The table furthermore gives limit l/d values for a relatively high (ρ=1,5%) and low (ρ=0,5%) longitudinal reinforcement ratio. These values are calculated for concrete C30 and σ_s = 310 MPa and satisfy the deflection limits given in 7.4.1 (4) and (5).

Structural system	K	ρ = 1,5%	ρ = 0,5%
Simply supported slab/beam	1,0	l/d=14	l/d=20
End span	1,3	l/d=18	l/d=26
Interior span	1,5	l/d=20	l/d=30
Flat slab	1,2	l/d=17	l/d=24
Cantilever	0,4	l/d= 6	l/d=8

Beams with embedded elements: design for bending at midspan



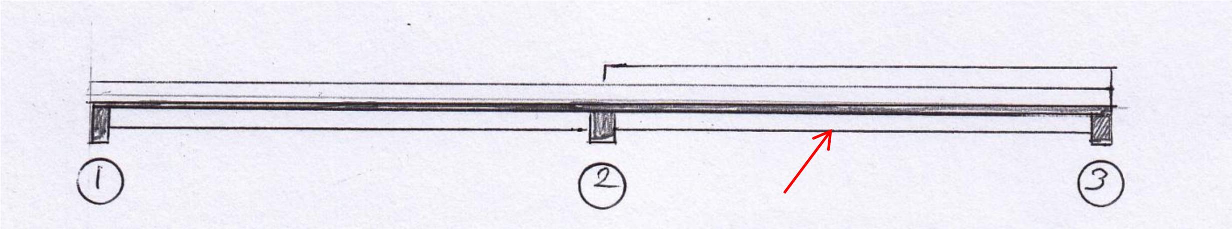
$$K = \frac{M_{Ed}}{bd^2 f_{ck}} = \frac{39{,}2 \cdot 10^6}{1000 \cdot 189^2 \cdot 25} = 0{,}044$$

From diagram z = 0,95d = 0,95·189 = 180 mm

$$A_{sl} = \frac{M_{Ed}}{z \cdot f_{yd}} = \frac{39{,}2 \cdot 10^6}{180 \cdot 435} = 501 mm^2$$

251 mm^2 per rib (e.g. 2∅14 = 308 mm^2)

Control of deflection slab with embedded elements



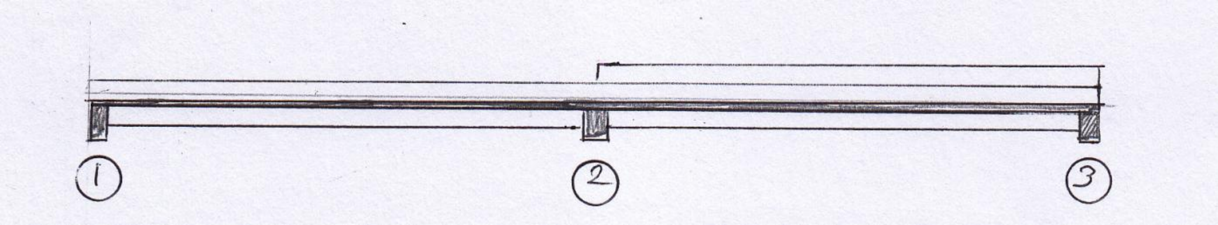
Reinforcement ratio at midspan $\rho = A_{sl}/b_e d = 501/(1000 \cdot 189) = 0{,}265\%$
According to Cl. 7.4.2(2) no detailed calculation is necessary if the l/d ratio of the slab is smaller than the limit value:

$$\frac{l}{d} = K\left[11 + 1{,}5\sqrt{f_{ck}}\,\frac{\rho_0}{\rho} + 3{,}2\sqrt{f_{ck}}\left(\frac{\rho_0}{\rho} - 1\right)^{3/2}\right]$$

So:

$$\frac{l}{d} = 1{,}3[11 + 1{,}5\sqrt{25} \cdot \frac{0{,}5}{0{,}256} + 3{,}2 \cdot 5 \cdot (\frac{0{,}5}{0{,}265} - 1)^{3/2} = 49$$

Control of deflection slab with embedded elements



Moreover correction for real steel stress versus 310 N/mm^2 as default value:

Quasi permanent load: Q_{qp}=2,33 + 3,0 + 0,3·2,0 = 5,93
Ultimate design load: Q_{ed} = 9,93

Steel stress under quasi permanent load σ_2 = (5,93/9,93)·435 = 260 Mpa

Corrected value of l/d is: $$\frac{l}{d} = \frac{310}{\sigma_{s,qp}} \cdot \left(\frac{l}{d}\right) = \frac{310}{260} \cdot 49 = 58{,}4$$

Actual value is l/d = 7,125/189 = 38 so OK

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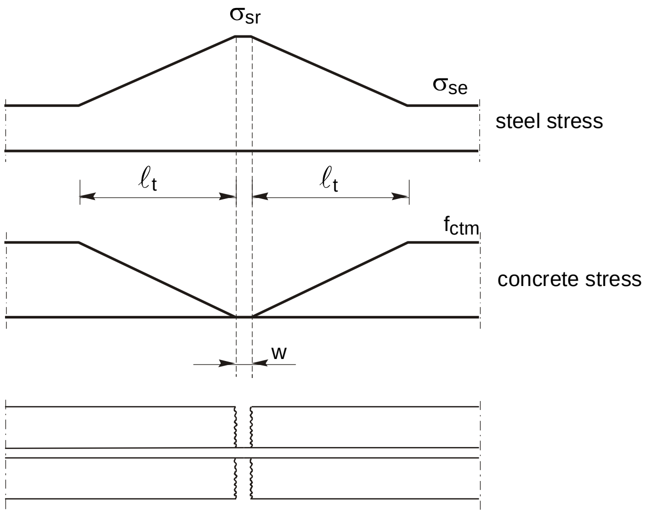
Theory of crack width control

The crack width is the difference between the steel deformation and the concrete deformation over the length $2l_t$, where l_t is the “transmission length”, necessary to build-up the concrete stength from 0 to the tensile strength f_{ctm}. Then the maximum distance between two cracks is $2l_t$ (otherwise a new crack could occur in-between). It can be found that the transmission length is equal to:

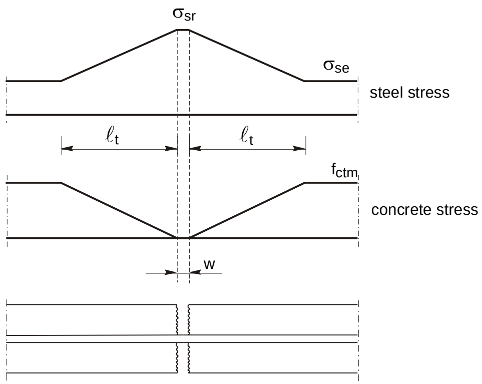


$$l_t = \frac{1}{4}\frac{f_{ctm}}{\tau_{bm}}\frac{\Phi}{\rho}$$

EC-formula's for crack width control

For the calculation of the maximum (or characteristic) crack width, the difference between steel and concrete deformation has to be calculated for the largest crack distance, which is $s_{r,max} = 2l_t$. So

$$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) \qquad \text{Eq. (7.8)}$$



where

$s_{r,max}$ is the maximum crack distance

and

$(\varepsilon_{sm} - \varepsilon_{cm})$ is the difference in deformation between steel and concrete over the maximum crack distance.

Accurate formulations for $s_{r,max}$ and $(\varepsilon_{sm} - \varepsilon_{cm})$ will be given

EC-2 formula's for crack width control

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{p,eff}}(1 + \alpha_e \rho_{p,eff})}{E_s} \geq 0{,}6 \frac{\sigma_s}{E_s} \qquad \text{(Eq. 7.9)}$$

where: σ_s is the stress in the steel assuming a cracked section

α_e is the ratio E_s/E_{cm}

$\rho_{p,eff} = (A_s + \xi A_p)/A_{c,eff}$ (effective reinforcement ratio including eventual prestressing steel A_p

ξ is bond factor for prestressing strands or wires

k_t is a factor depending on the duration of loading (0,6 for short and 0,4 for long term loading)

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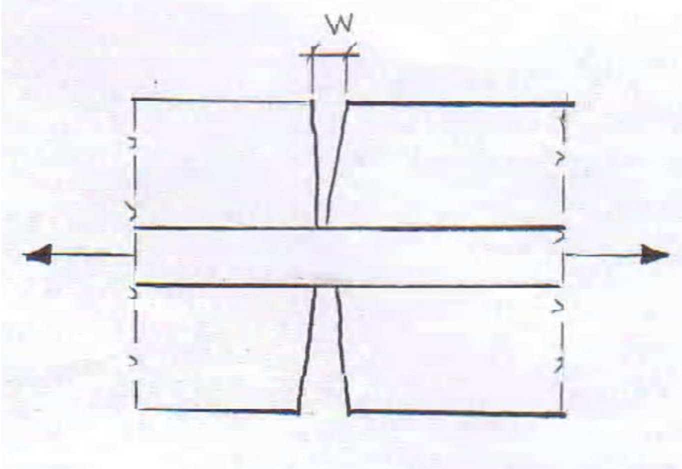
EC-2 formulae for crack width control

For the crack spacing $s_{r,max}$ a modified expression has been derived, including the concrete cover. This is inspired by the experimental observation that the crack at the outer concrete surface is wider than at the reinforcing steel. Moreover, cracks are always measured at the outside of the structure (!)



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EC-3 formula's for crack width control

Maximum final crack spacing $s_{r,max}$

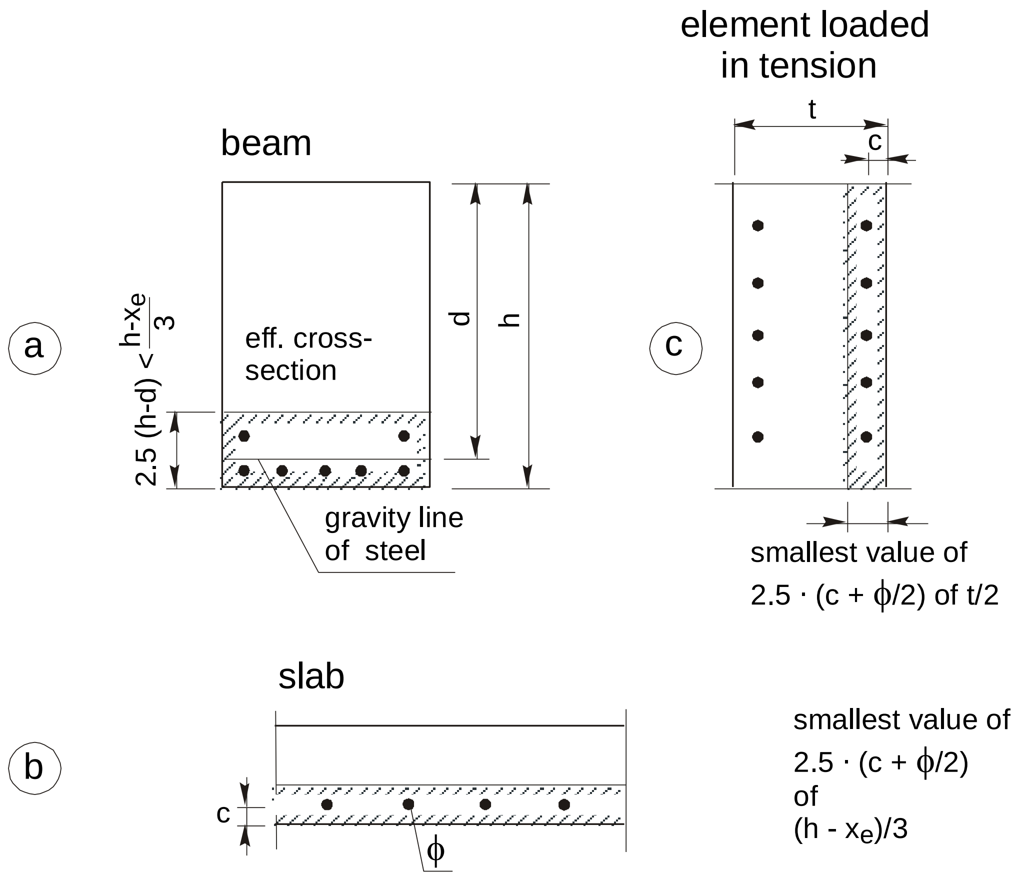
$$s_{r,max} = 3.4c + 0.425\, k_1 k_2 \frac{\phi}{\rho_{p,eff}} \qquad \text{(Eq. 7.11)}$$

where
- c is the concrete cover
- Φ is the bar diameter
- k_1 bond factor (0,8 for high bond bars, 1,6 for bars with an effectively plain surface (e.g. prestressing tendons)
- k_2 strain distribution coefficient (1,0 for tension and 0,5 for bending: intermediate values van be used)

EC-2 formula's for crack width control

In order to be able to apply the crack width formulae, basically valid for a concrete tensile bar, to a structure loaded in bending, a definition of the "effective tensile bar height" is necessary. The effective height $h_{c,ef}$ is the minimum of:

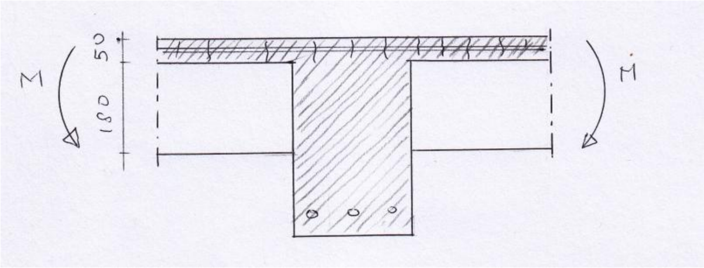
2,5 (h-d)
(h-x)/3
h/2



EC-2 requirements for crack width control (recommended values)

Exposure class	RC or unbonded PSC members	Prestressed members with bonded tendons
	Quasi-permanent load	Frequent load
X0,XC1	0.3	0.2
XC2,XC3,XC4	0.3	
XD1,XD2,XS1,XS2, XS3		Decompression

Crack width control at intermediate support of slabs with embedded elements



Assumption: concentric tension of upper slab of 50 mm.

Steel stress $\sigma_{s,qp}$ under quasi permanent load:

$$\sigma_{s,qp} = \frac{Q_{qp}}{Q_{Ed}} \cdot \frac{A_{s,req}}{A_{s,prov}} \cdot f_{yd} = 0{,}597 \cdot 0{,}85 \cdot 435 = 220 MPa$$

Reinforcement ratio: $\rho_{s,eff} = A_{sl}/bd = 959/(1000 \cdot 50) = 1{,}92\%$

Crack distance: $$\sigma_{s,\max} = k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{s,eff}} = 3{,}4 \cdot 19 + 0{,}8 \cdot 1{,}0 \cdot 0{,}425 \cdot \frac{12}{0{,}0192} = 277 mm$$

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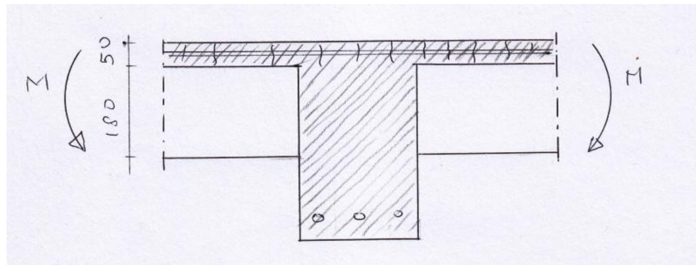
Crack width control at intermediate support of slabs with embedded elements



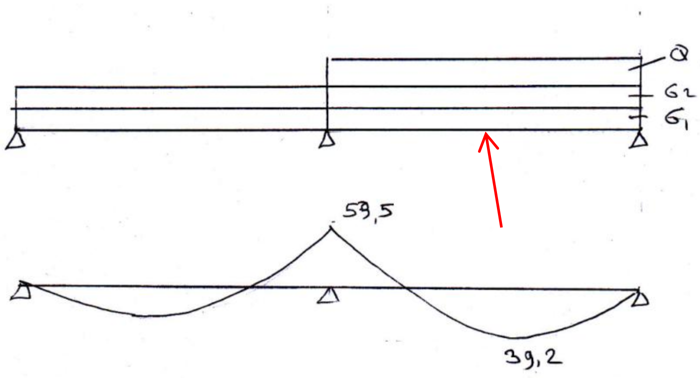
Average strain: $$\varepsilon_{sm}-\varepsilon_{cm}=\frac{\sigma_s-k_t\dfrac{f_{ct,eff}}{\rho_{p,eff}}(1+\alpha_e\rho_{p,eff})}{E_s}\geq 0{,}6\frac{\sigma_s}{E_s}$$

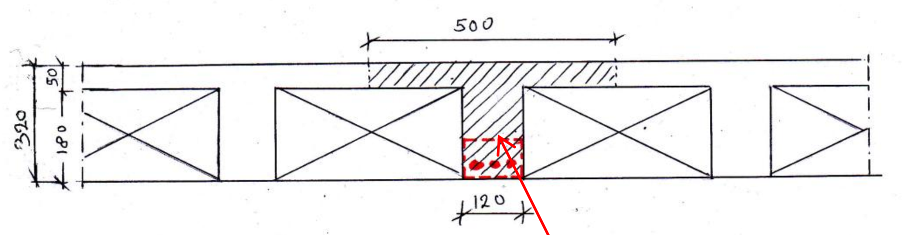
$$\varepsilon_{sm}-\varepsilon_{cm}=\frac{220-0{,}4\dfrac{2{,}6}{0{,}0192}(1+7\cdot 0{,}0192)}{200.000}=0{,}79\cdot 10^{-3}$$

Characteristic crack width: $$w_k=s_{r,\max}\{\varepsilon_{sm}-\varepsilon_{cm}\}=227\cdot 0{,}79\cdot 10^{-3}=0{,}18mm\leq 0{,}30mm$$

so, OK

Crack width at mid-span beams with embedded elements





Cross-section of tensile bar

Height of tensile bar: smallest value of 2,5(h-d), (h-x)/3 or h/2.

Critical value 2,5(h-d) = 2,5·29 = 72 mm.

$\rho_{s,eff} = A_{sl}/bh_{eff} = 308/(120 \cdot 72) = 3,56\%$

$$\sigma_{s,qp} = \frac{Q_{qp}}{Q_{Ed}} \cdot \frac{A_{s,req}}{A_{s,prov}} \cdot f_{yd} = 0,597 \cdot 0,81 \cdot 435 = 210 MPa$$

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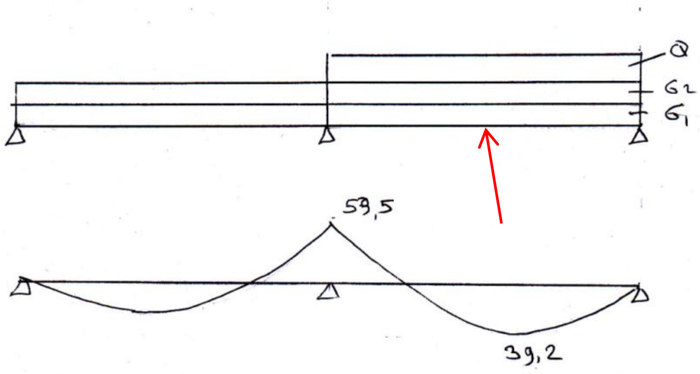
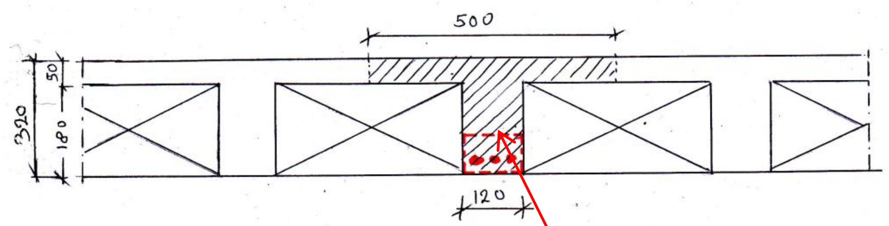
Crack width at mid-span beams with embedded elements





Cross-section of tensile bar

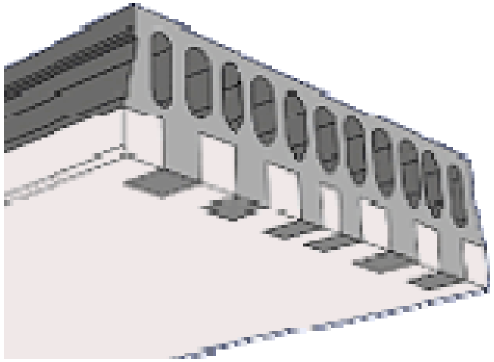
$$\sigma_{s,\max} = k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{s,eff}} = 3{,}4 \cdot 29 + 0{,}8 \cdot 0{,}5 \cdot 0{,}425 \cdot \frac{12}{0{,}0356} = 156mm$$

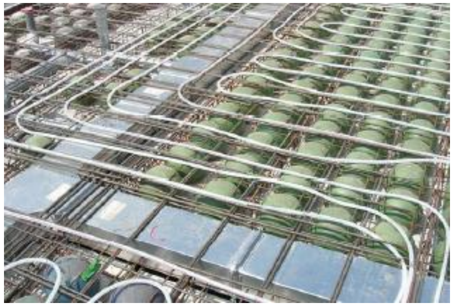
$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{p,eff}}(1 + \alpha_e \rho_{p,eff})}{E_s} = \frac{210 - 0{,}4 \dfrac{2{,}6}{0{,}0356}(1 + 7 \cdot 0{,}0356)}{200.000} = 0{,}87 \cdot 10^{-3}$$

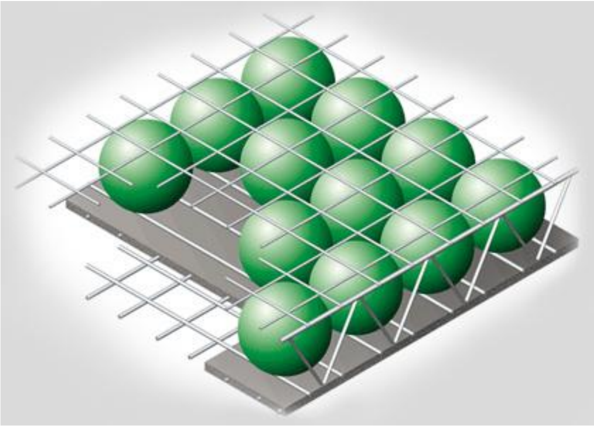
$$w_k = s_{r,\max}(\varepsilon_{sm} - \varepsilon_{cm}) = 156 \cdot 0{,}87 \cdot 10^{-3} = 0{,}14mm$$

OK

Different cultures: different floors







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