The EC2 worked example:

Description, actions, durability, materials

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The EC2 worked example

6-storeys building + 2-storeys underground parking in an urban area (terrain category IV) not close to the at 300 m AMSL (Above Mean Sea Level). The building design working is 50 years.

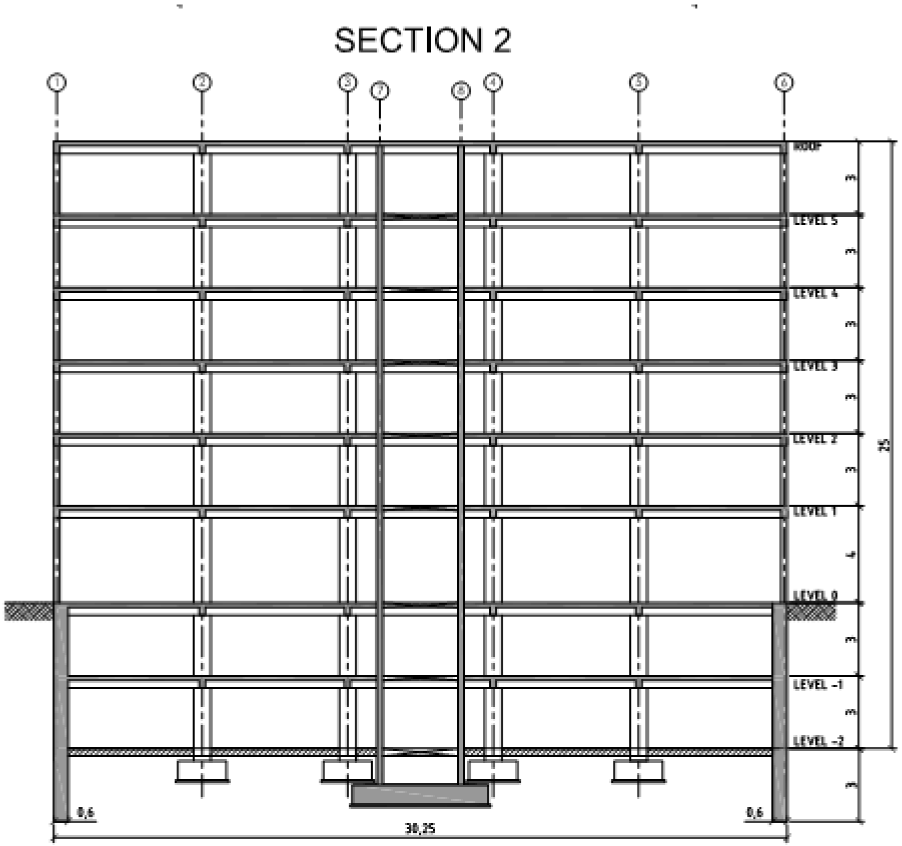
Reinforced cast on site concrete, 3 different floor solutions: slab on beams, flat slab, slab with embedded lighting (clay) elements.

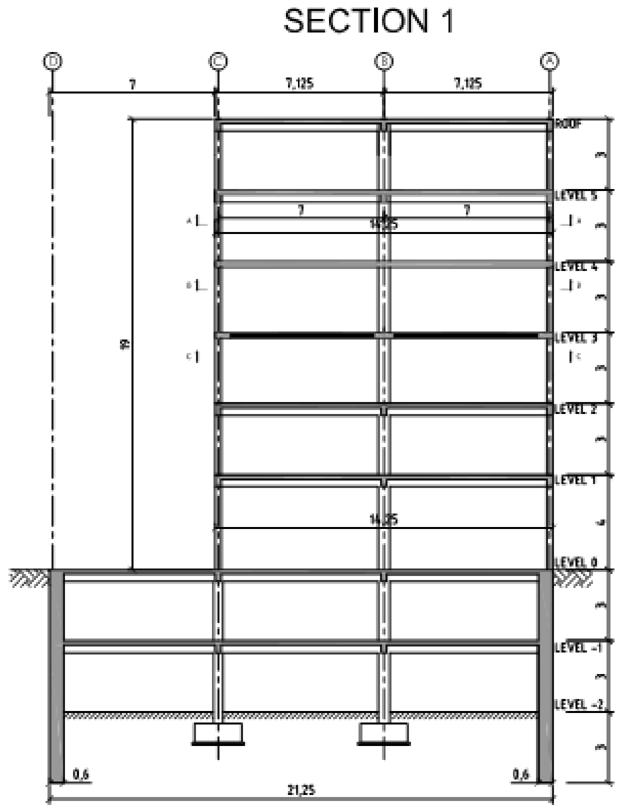
Building similar to the one used for the EC8 example *(documentation available on http://eurocodes.jrc.ec.europa.eu/showpage.php?id=335_2):*

Scope: two “case studies” referring to the same building with the same vertical loads but two different sets of horizontal actions (EC2: vertical loads + high wind; EC8: vertical loads + earthquake).

In comparison with EC8 example, lateral stiffness and strength are still required but less bracing elements (lift core + two walls) are present.

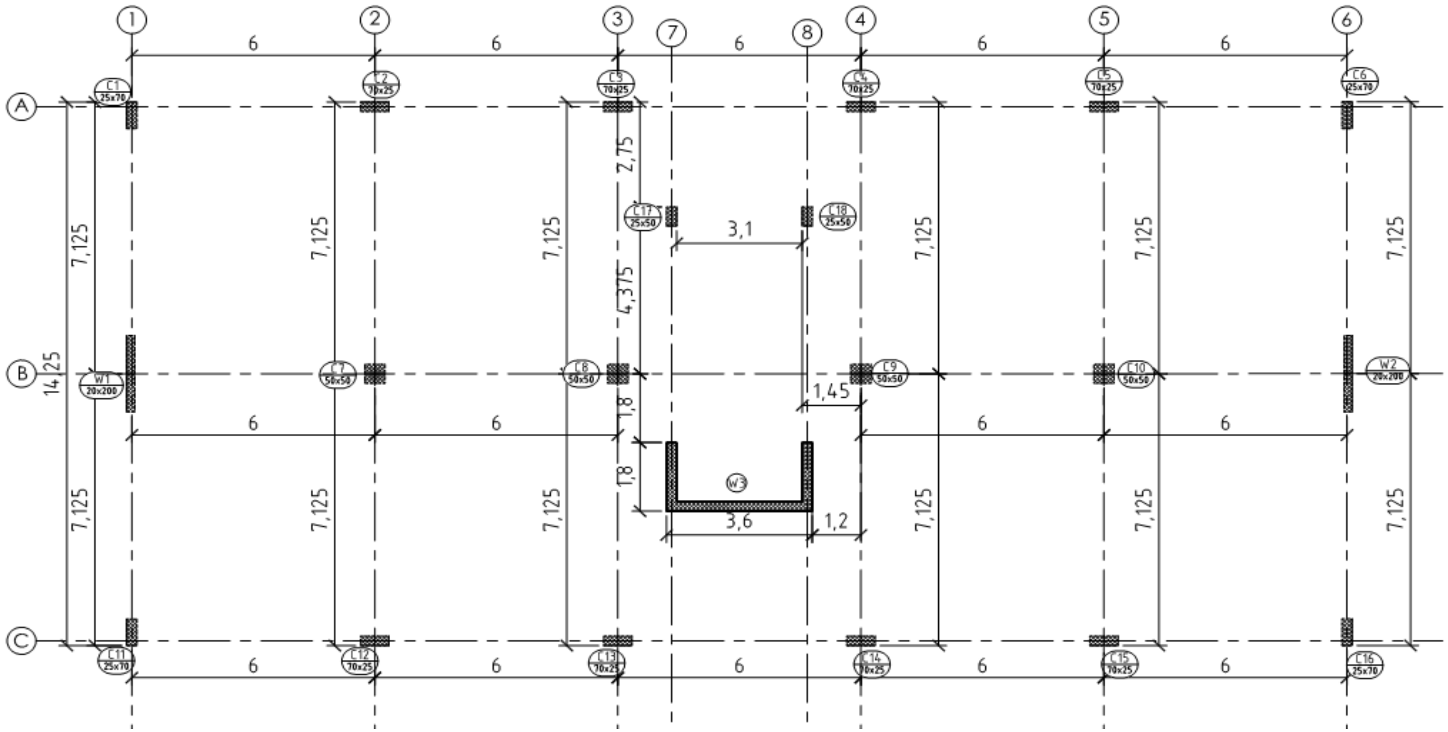
EC2 worked example





- 2-level underground parking
- ground floor: offices open to public, 1st to 5th floor: dwellings
- roof

Columns Layout



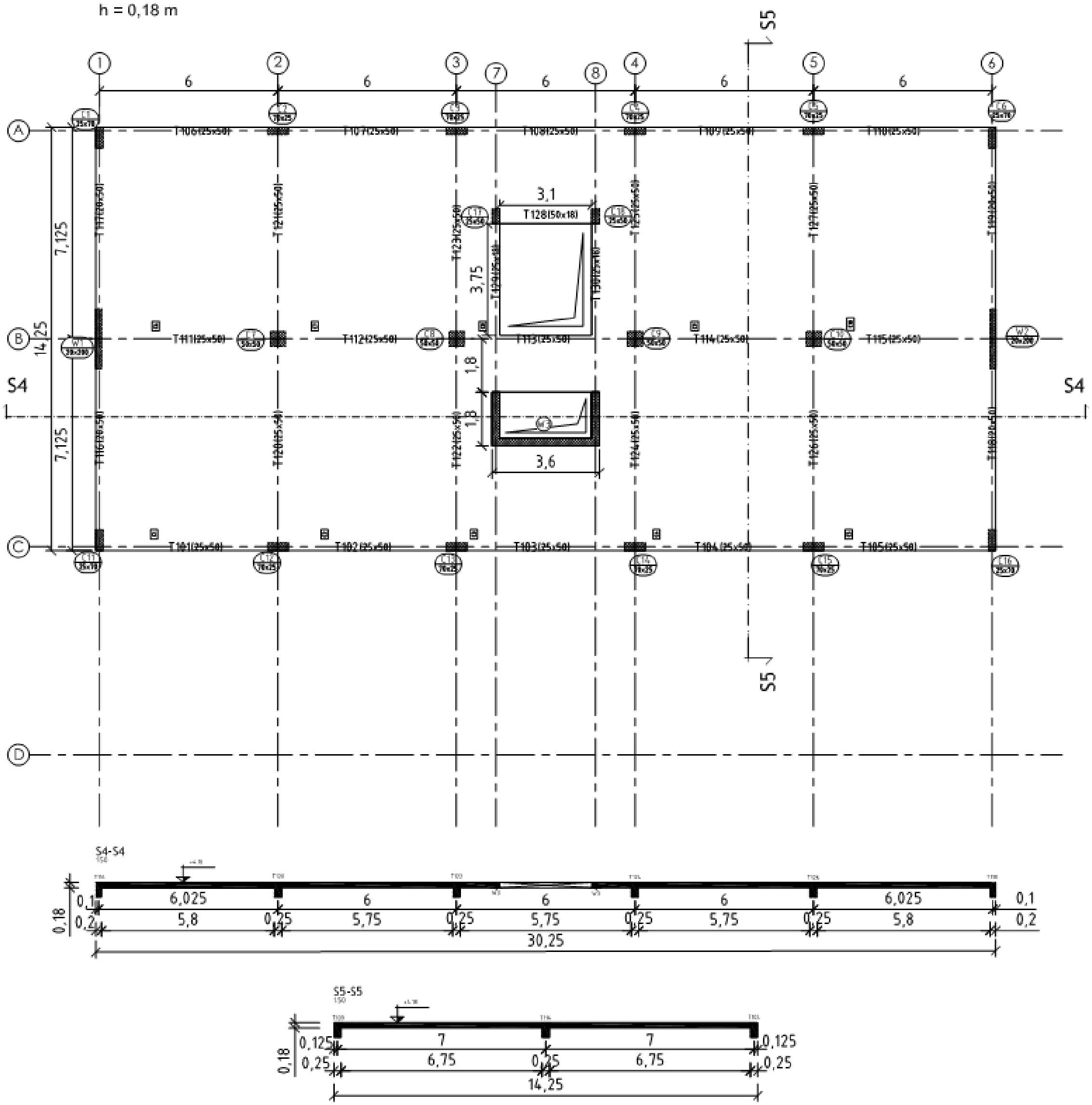
x direction slab/ beams spans: all equal

single central core and stairs

two y-direction walls

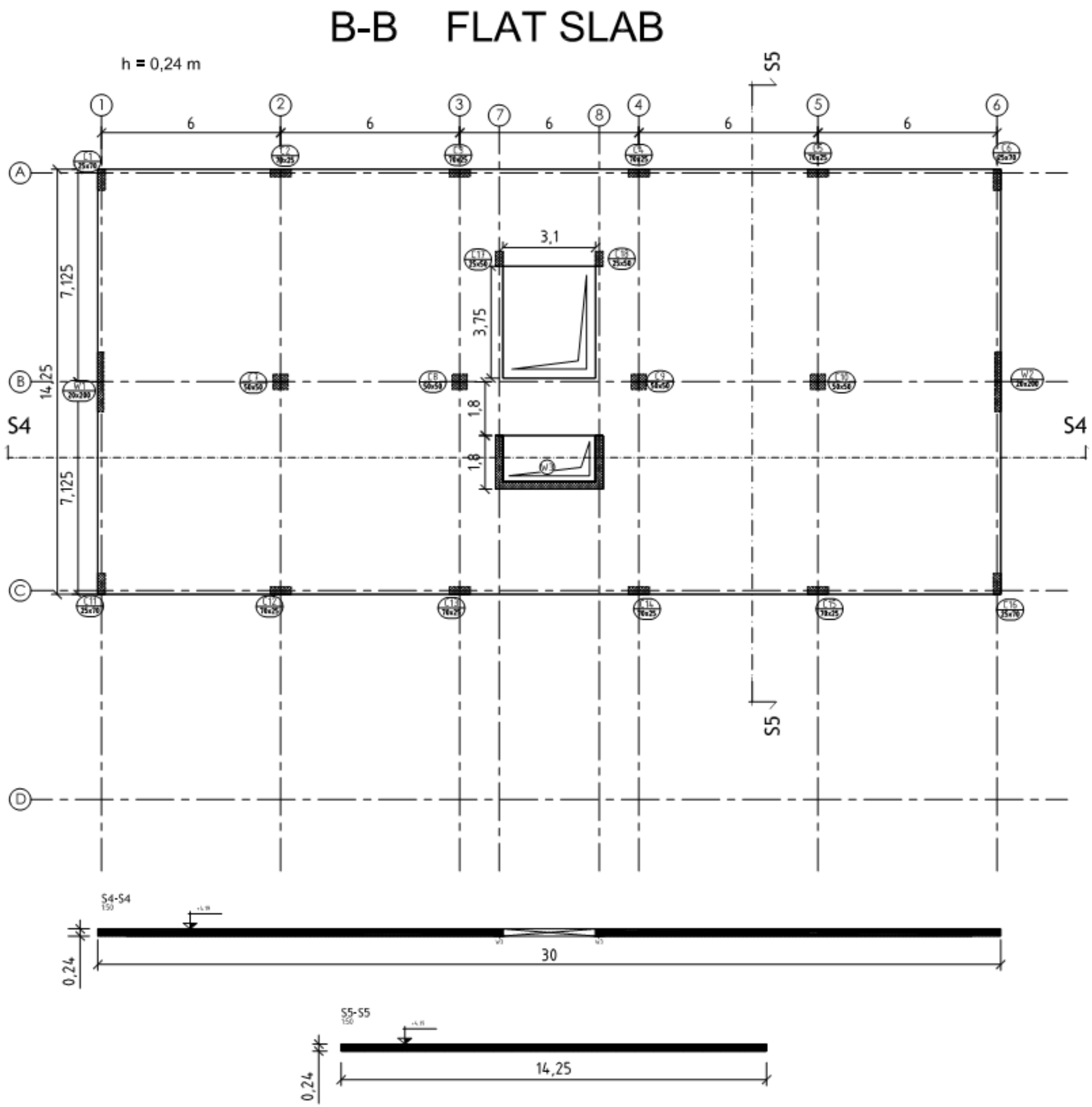
Three solutions: 1) slab on beams

A-A SLAB ON BEAMS



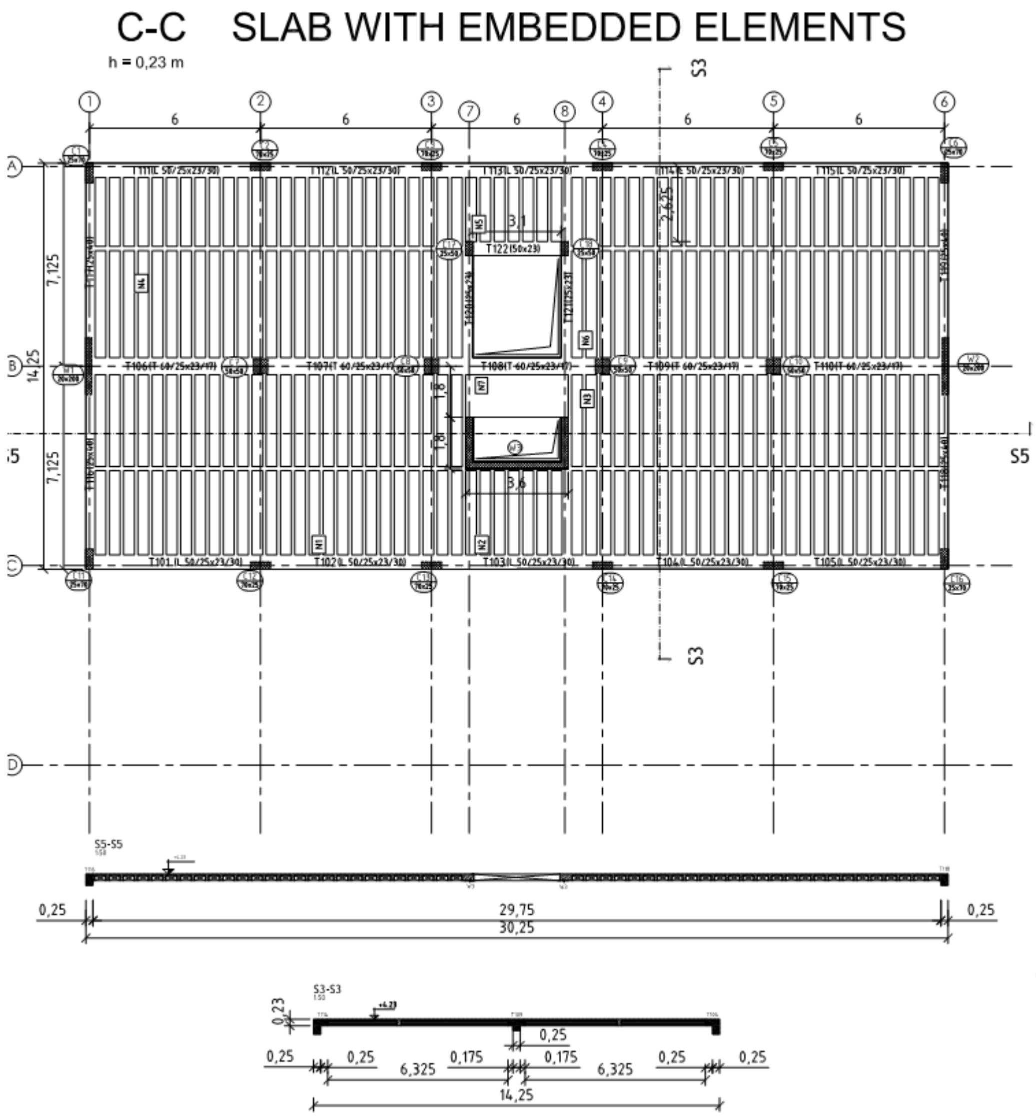
0,18 m slab on 0,40 h beams spanning in both x and y directions

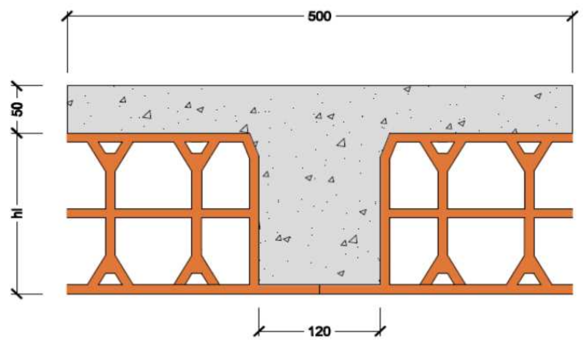
2) flat slab



0,24 m flat slab spanning in x and y directions

3) Monodirectional ribbed slab





Lighting clay elements
b = 500 mm, b_w = 120 mm
$b/b_w = 4,2 > 3$
50 mm flange made of cast on site concrete
h = 0,18 + 0,05 = 0,23 m

T beams h = 0,23+0,17 = 0,40 m

Actions: G loads

Self weight G_1 : based on reinforced concrete unit weight (25kN/m^3) and the geometry of structural elements.

Permanent loads G_2
Finishing, pavement, embedded services, partitions: 3,0 kN/m^2
Walls on external perimeter (windows included): 8,0 kN/m

Variable loads characteristic values and ψ factors

Type	q_k (kN/m^2)	ψ_0	ψ_2
Dwellings	2,00	0,70	0,30
Stairs, office open to public	4,00		
Snow	1,70	0,50	0,00

European wind map

10-minutes median wind velocity at 10-m height above flat, even ground; no gusts

The characteristic value of wind velocity or velocity pressure occurs in the average once every 50 year (p = 0,02, mean return period 50 years)

Wind $\psi_0 = 0{,}60$ $\psi_2 = 0{,}0$

Basic wind velocity $v_b = c_{dir} c_{season} v_{b,0}$ $c_{dir} = c_{season} = 1{,}0$ $v_{b,0} = 30$ m/s $v_b = v_{b,0} = 30$ m/s

Terrain category IV $z_0 = 1$ $z_{min} = 10$ m

Terrain factor $k_r = 0{,}19\left(\frac{z_0}{0{,}05}\right)^{0{,}07} = 0{,}19\left(\frac{10}{0{,}05}\right)^{0{,}07} = 0{,}234$ m/s

Orography factor $c_o = 1{,}0$

Turbulence intensity $k_I = 1{,}0$ $I_v(z) = \cdot\frac{k_I}{c_o(z)\ln z/z_0} = \frac{1}{\ln z/z_0}$

Exposure factor $c_e(z)$ taking into account turbulence

$z \le 10$ m $c_e(z) = c_e(z_{min}) = k_r^2 \cdot c_o \cdot \ln\frac{z}{z_0}\left(7 + c_o \cdot \ln\frac{z}{z_0}\right) = 0{,}23^2 \cdot 1 \cdot \ln\frac{10}{1}\left(7 + 1 \cdot \ln\frac{10}{1}\right) = 1{,}13$ const.

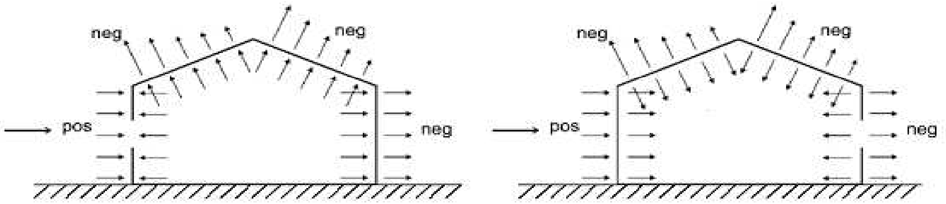
$z > 10$ m $c_e(z) = k_r^2 \cdot c_o \cdot \ln\frac{z}{z_0}\left(7 + c_o \cdot \ln\frac{z}{z_0}\right) = 0{,}053 \cdot \ln z\,(7 + \ln z\,)$.

Basic velocity pressure $q_b = \frac{1}{2}\rho\, v_b^2 = \frac{1}{2} \cdot 1{,}25 \cdot 30^2 10^{-3} = 0{,}563$ kN/m^2

Peak velocity pressure $q_p(z_e) = c_e(z) q_b = c_e(z)\, 0{,}563$ kN/m^2 $z > 10$ m

$q_p(z_e) = c_e(z_{min}) q_b = c_e(10)\, 0{,}563$ kN/m^2 $z \le 10$ m

Wind pressure on external surfaces $c_{pe} = +\,0{,}8$ $c_{pe} = -\,0{,}4$

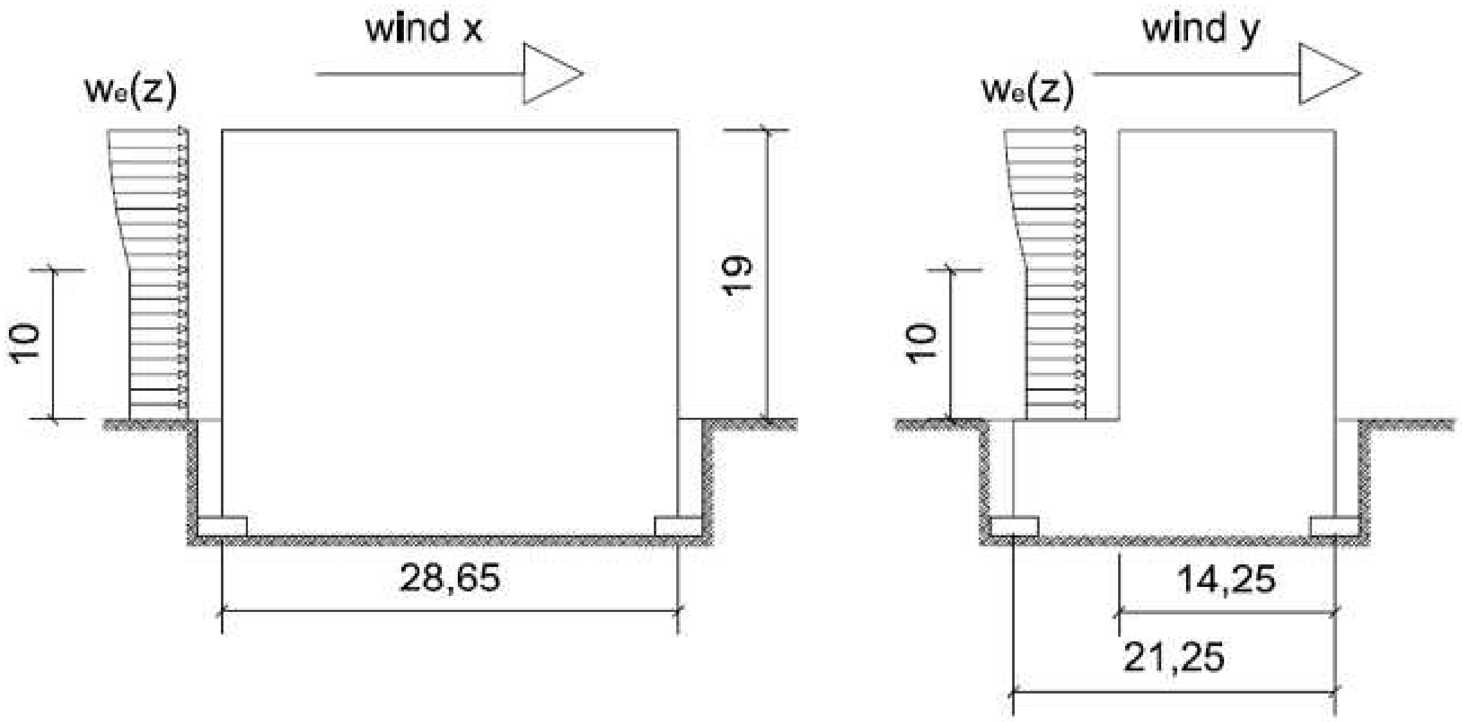


Structural factor $c_s c_d = 1{,}0$ (framed buildings with structural walls less than 100 m high)

Wind pressure on external surfaces

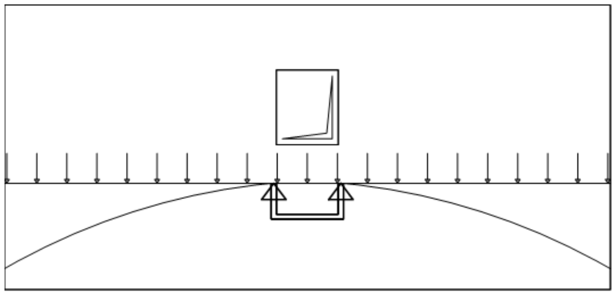
$w_e = q_p(z_e)\, c_{pe} c_s c_d = q_p(z_e)\, (0{,}8 - (-0{,}4)) \cdot 1 = 1{,}2\, q_p(z_e)$ kN/m

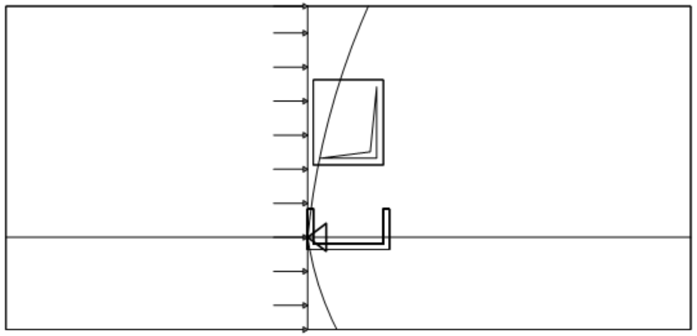
- $z_e > 10$ m $\quad w_e(z_e) = 1{,}2 \cdot c_e(z_e)\, 0{,}56 = 0{,}0357 \cdot \ln(z_e)\, [7 + \ln(z_e)]$ kN/m² $\quad w_e(19) = 1{,}04$ kN/m²
- $z_e \leq 10$ m $\quad w_e(10) = 0{,}0357 \cdot \ln(10)\, [7 + \ln(10)] = 0{,}77$ kN/m²



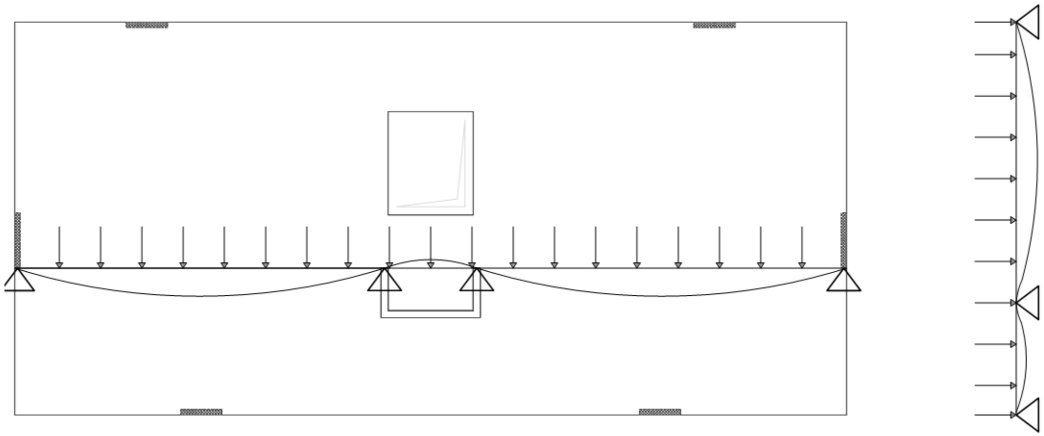
Preliminary evalutation

Horizontal loads: wind y and wind x





To increase torsional rigidity, place bracing elements on all sides (stiffnesses' "centrifugation")



Durability

EC2 2.1.3 Rules for design working life, durability and quality management are in EN1990 Section 2

2.4 Durability

(1)P The structure shall be designed such that deterioration over its design working life does not impair the performance of the structure below that intended, having due regard to its environment and the anticipated level of maintenance.

(3)P The environmental conditions shall be identified at the design stage so that their significance can be assessed in relation to durability and adequate provisions can be made for protection of the materials used in the structure.

EC2 refers to a 50-years design working life and normal maintenance

For concrete structures quality management procedures during execution are described in EN13670.

50-years design working life?

carbonation

α 1959 ω 1971

chlorides

α 1975 ω 2000

R.I.P.

Durability

Traditional “deemed to satisfy” rules related to the exposure conditions of the various structural members, described in:

- EN206-1 Annex F (concrete standard) for material composition
- EN1992-1 for design, based on 1) a required concrete quality and 2) an adequate concrete cover to reinforcement.

Strength is used as a measure for the durability of concrete, with values for maximum w/c ratio and mininum cement concrete

Result: large variation in requirements in different countries (see CEN TR 15868).

BASIC PARAMETERS

- exposure conditions classified using “**exposure classes**”;
- Minimum concrete strength class and concrete cover related to exposure conditions;
- behaviour in use (e.g. cracking) related to exposure conditions.

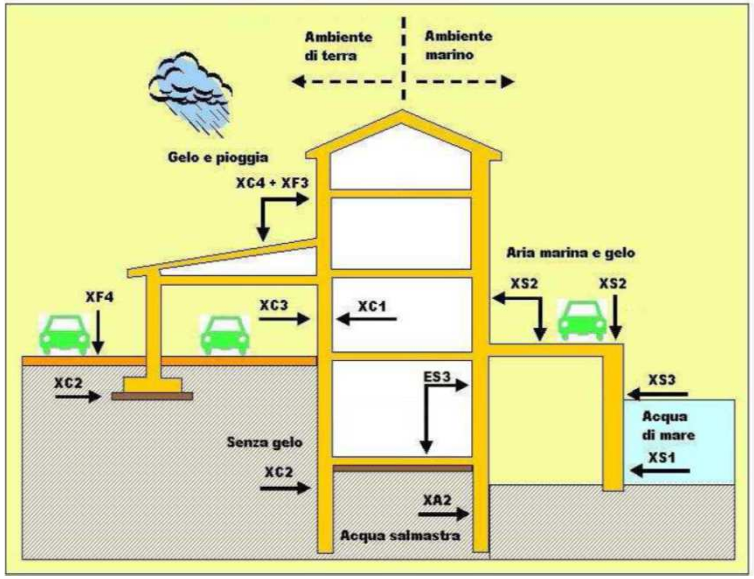
EXPOSURE CLASSES VS. DETERIORATION MECHANISMS

- Corrosion of reinforcement due to **C**arbonation (XC) or chlorides from **D**e-icing agents, industrial wastes, pools (XD) or **S**ea water (XS)
- Deterioration of concrete due to **F**reeze/thaw action (XF) or chemical **A**ttack (XA)

Exposure classes in EN206-1 referred to in EN1992-1

Class designation	Description of the environment	Informative examples where exposure classes may occur
1 No risk of corrosion or attack		
X0	For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack For concrete with reinforcement or embedded metal: very dry	Concrete inside buildings with very low air humidity
2 Corrosion induced by carbonation		
XC1	Dry or permanently wet	Concrete inside buildings with low air humidity Concrete permanently submerged in water
XC2	Wet, rarely dry	Concrete surfaces subject to long-term water contact Many foundations
XC3	Moderate humidity	Concrete inside buildings with moderate or high air humidity External concrete sheltered from rain
XC4	Cyclic wet and dry	Concrete surfaces subject to water contact, not within exposure class XC2
3 Corrosion induced by chlorides		
XD1	Moderate humidity	Concrete surfaces exposed to airborne chlorides
XD2	Wet, rarely dry	Swimming pools Concrete components exposed to industrial waters containing chlorides
XD3	Cyclic wet and dry	Parts of bridges exposed to spray containing chlorides Pavements Car park slabs
4 Corrosion induced by chlorides from sea water		
XS1	Exposed to airborne salt but not in direct contact with sea water	Structures near to or on the coast
XS2	Permanently submerged	Parts of marine structures
XS3	Tidal, splash and spray zones	Parts of marine structures
5. Freeze/Thaw Attack		
XF1	Moderate water saturation, without de-icing agent	Vertical concrete surfaces exposed to rain and freezing
XF2	Moderate water saturation, with de-icing agent	Vertical concrete surfaces of road structures exposed to freezing and airborne de-icing agents
XF3	High water saturation, without de-icing agents	Horizontal concrete surfaces exposed to rain and freezing
XF4	High water saturation with de-icing agents or sea water	Road and bridge decks exposed to de-icing agents Concrete surfaces exposed to direct spray containing de-icing agents and freezing Splash zone of marine structures exposed to freezing
6. Chemical attack		
XA1	Slightly aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water
XA2	Moderately aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water
XA3	Highly aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water

CURRENT SYSTEM : EC2 ch. 4



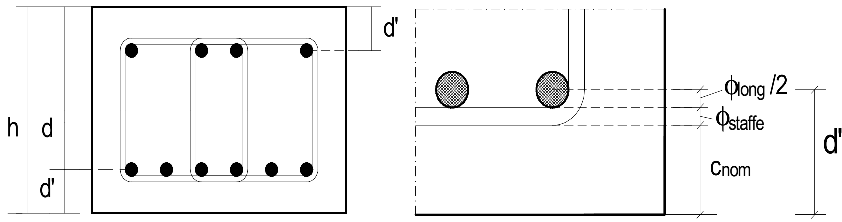
1) Exposure class(es)

2) MINIMUM strength class for the exposure class(es) (EC2 Informative annex E)

3) Nominal concrete cover c_{nom}

$$c_{nom} = \max [(c_{min} + \Delta c); 20 \text{ mm}]$$

$$\Delta c = 0 \text{ -10 mm}$$



$$c_{min} = \max \{c_{min,b}; (c_{min,dur} - \Delta c_{dur,add}); 10 \text{ mm}\}$$

accounts for bond, protection from corrosion and fire resistance

Minimum cover for durability

1) STRUCTURAL CLASS SELECTION - DEFAULT: S4

Table 4.3N: Recommended structural classification

Structural Class							
Criterion	Exposure Class according to Table 4.1						
	X0	XC1	XC2 / XC3	XC4	XD1	XD2 / XS1	XD3 / XS2 / XS3
Design Working Life of 100 years	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2
Strength Class [1) 2)]	≥ C30/37 reduce class by 1	≥ C30/37 reduce class by 1	≥ C35/45 reduce class by 1	≥ C40/50 reduce class by 1	≥ C40/50 reduce class by 1	≥ C40/50 reduce class by 1	≥ C45/55 reduce class by 1
Member with slab geometry (position of reinforcement not affected by construction process)	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1
Special Quality Control of the concrete production ensured	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1

Exp. class XC2/XC3 - 50 years working life, no special QC

Slabs: concrete C25/30 ⇒ S(4 – 1) = S3

Beams and columns: concrete C30/37 ⇒ S4

Minimum cover for durability

2) CONCRETE COVER FOR XC2/3 AND CLASSES S3/S4

Environmental Requirement for $c_{min,dur}$ (mm)							
Structural Class	Exposure Class according to Table 4.1						
	X0	XC1	XC2 / XC3	XC4	XD1 / XS1	XD2 / XS2	XD3 / XS3
S1	10	10	10	15	20	25	30
S2	10	10	15	20	25	30	35
S3	10	10	20	25	30	35	40
S4	10	15	25	30	35	40	45
S5	15	20	30	35	40	45	50
S6	20	25	35	40	45	50	55

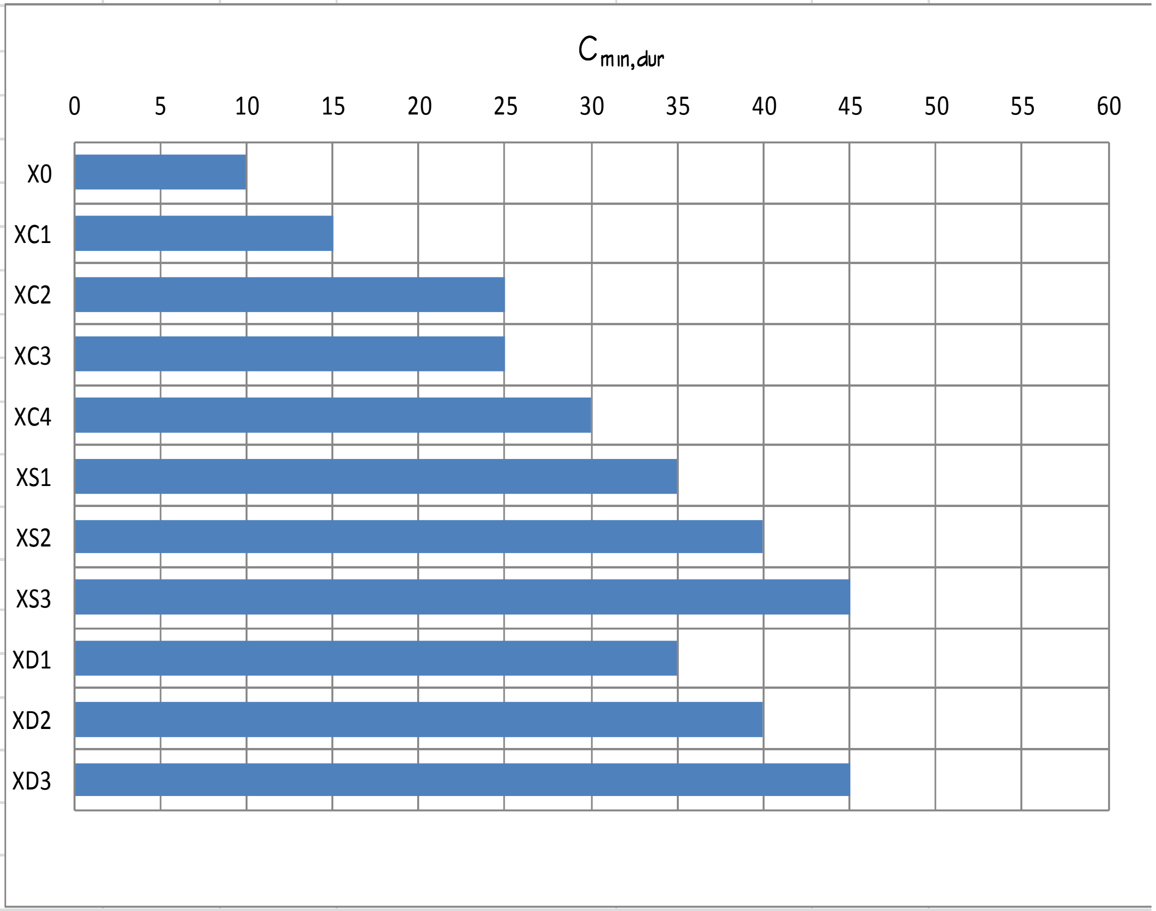
$c_{min,dur}$ slabs = 20 mm

$c_{min,dur}$ columns = 25 mm

Nominal cover evaluation

Excel™ spreadsheet

Concrete cover			
	Parameters	Suggested	User defined
1	Exposure class		XC3
2	Freeze/thaw		-
3	Strenght class	C30/37	C30/37
4	Service life		50
5	Slab or similar?		NO
6	Quality control?		NO
7	Max bar diam. (mm)		16
8	$\Delta c_{dur,st}$	0	0
9	$\Delta c_{dur,\gamma}$	0	0
10	$\Delta c_{dur,add}$	0	0
11	Δc_{toll}		A) Recommended
		10	10
12	Structural class		S4
13	$c_{min,dur}$		25
14	$c_{min,b}$		16
15	c_{min}		25
16	c_{nom}		35



Nominal cover evaluation

Classe di esposizione ambientale	Copriferro $c_{min,dur}$ [mm]: 15	25	30	35	40	45	50	55
XC1	C25/30; 0,60; 300							
XC2	C25/30; 0,60; 300							
XC3	C28/35; 0,55; 320							
XC4	C32/40; 0,50; 340							
XD1	C28/35; 0,55; 320							
XD2	C35/45; 0,45; 360							
XD3	C35/45; 0,45; 360							
XS1	C28/35; 0,55; 320							
XS2	C35/45; 0,45; 360							
XS3	C35/45; 0,45; 360							
XF1	C28/35; 0,50; 320							
XF2 – XF3	C25/30; 0,50; 340							
XF4	C28/35; 0,45; 360							
XA1	C28/35; 0,55; 320							
XA2	C32/40; 0,50; 340							
XA3	C35/45; 0,45; 360							

Tab. 26.02 - Copriferro c_{min} e composizione del calcestruzzo (EN206-1 ed EC2)

National tables

Durability Classes, proposal

Classes	Carbonation resistance class			Chloride resistance class			Frost resistance class	
	Low	Medium	High	Low	Medium	High	Low	High
Definition of class Depth of front after 50 years	XC1 < 40mm	XC1 < 30mm	XC1 < 20mm	XS2 < 60mm	XS2 < 60mm	XS2 < 40mm	XF4 Scaling loss < $nn(g/m^2)$	XF4 Scaling loss < $n(g/m^2)$
Accepted accelerated Test condition	EN XXX	EN XXX	EN XXX	EN YYY	EN YYY	EN YYY	EN ZZZ	EN ZZZ
Deemed to satisfy CEM I	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) = Air = n%	w/c = w/(c +kp) = Air = nn%
Deemed to satisfy CEM II	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) = Air = n%	w/c = w/(c +kp) = Air = n%
Deemed to satisfy CEM III	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) =	w/c = w/(c +kp) = Air = n%	w/c = w/(c +kp) = Air = n%
Deemed to satisfy Binders, additions								

Design for durability, relating exposure to concrete and cover

standard norge

The text on durability in EC2 §4.4.1.2 tables 4.3, 4.4 and 4.5 could then be replaced a table like this;

Here the Durability Class is a national choice (NDP)

Exposure Class	Durability Class (minimum)	Minimum cover $c_{min,dur}$ (mm)	
		50 -years design service life	100 -years design service life
X0	Carbonation Resistance Low	$c_{min,b}$	$c_{min,b}$
XC1	Carbonation Resistance Medium	15	25
XC2, XC3, XC4	Carbonation Resistance High	25	35
XD1, XS1	Chloride Resistance Low	40	50
XD2, XD3, XS2	Chloride Resistance High	40	50
XS3	Chloride Resistance High	50	60

Durability - EC2 Worked example

Due to non uniformity of EU National choices, to avoid country-specific conditions, for the example no exposure classes were selected and nominal cover to reinforcement c_{nom} was fixed:

$$c_{nom} = 20 + 10 = 25 + 5 = 30 \text{ mm}$$

$c_{min,dur}$ = 20/25 mm – exp. class XC2/XC3 for classes S3/S4
$\Delta_{c,dev}$ = 5 - 10 mm for controlled execution

For foundations c_{nom} = 40 mm.

Concrete strength classes have been selected accordingly

Materials: concrete

Foundations, beams and slabs: C25/30

Columns: C30/37 > C25/30 → EC8 “capacity design” rule to avoid soft storey plastic mechanism

Safety factors:

ULS γ_c = 1,50 (persistent and transient design situation) α_{cc} = 1,0

SLS γ_c = 1,0

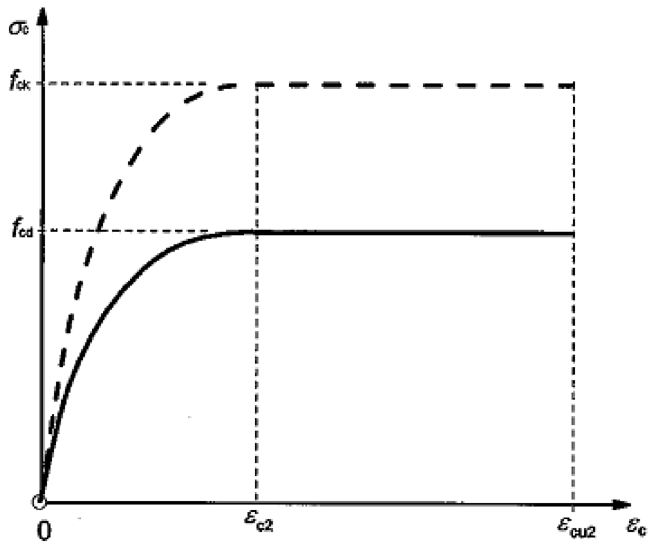


Figure 3.3: Parabola-rectangle diagram for concrete under compression.

Materials: steel

Grade 500 class B

Strength $f_{yk} \geqq 500$ N/mm^2 $f_{y,max} \leqq 1{,}30\ f_{yk}$

Ductility $(f_t/f_y)_k \geqq 1{,}08$ $\varepsilon_{uk} \geqq 5\%$ $\varepsilon_{ud} = 0{,}90\ \varepsilon_{uk} \geqq 4{,}5\%$

Safety factors:

ULS $\gamma_s = 1{,}15$ (persistent and transient design situation

SLS $\gamma_s = 1{,}0$

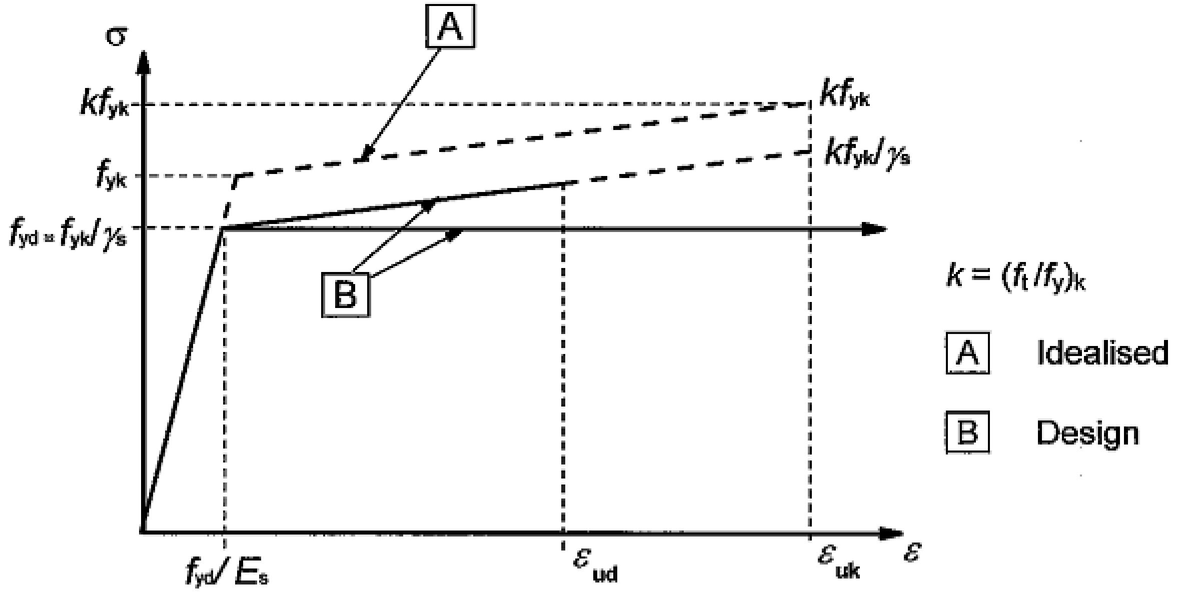


Figure 3.8: Idealised and design stress-strain diagrams for reinforcing steel (for tension and compression)

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EC2 worked example

Conceptual design

Slabs

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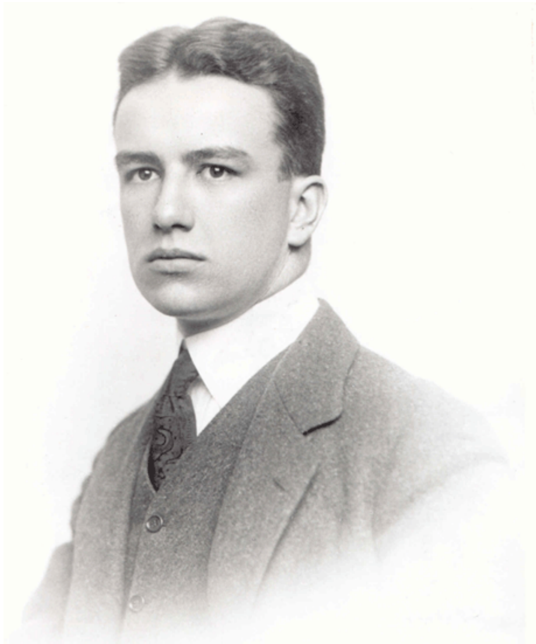
Two (contradictory?) appproaches

"When time is money, it's moral not to waste time. Especially your own."

Theodor W. Adorno



"Keep doing what you've always done and you'll keep getting what you've always got"

Buckminster Fuller

Conceptual Design: definition

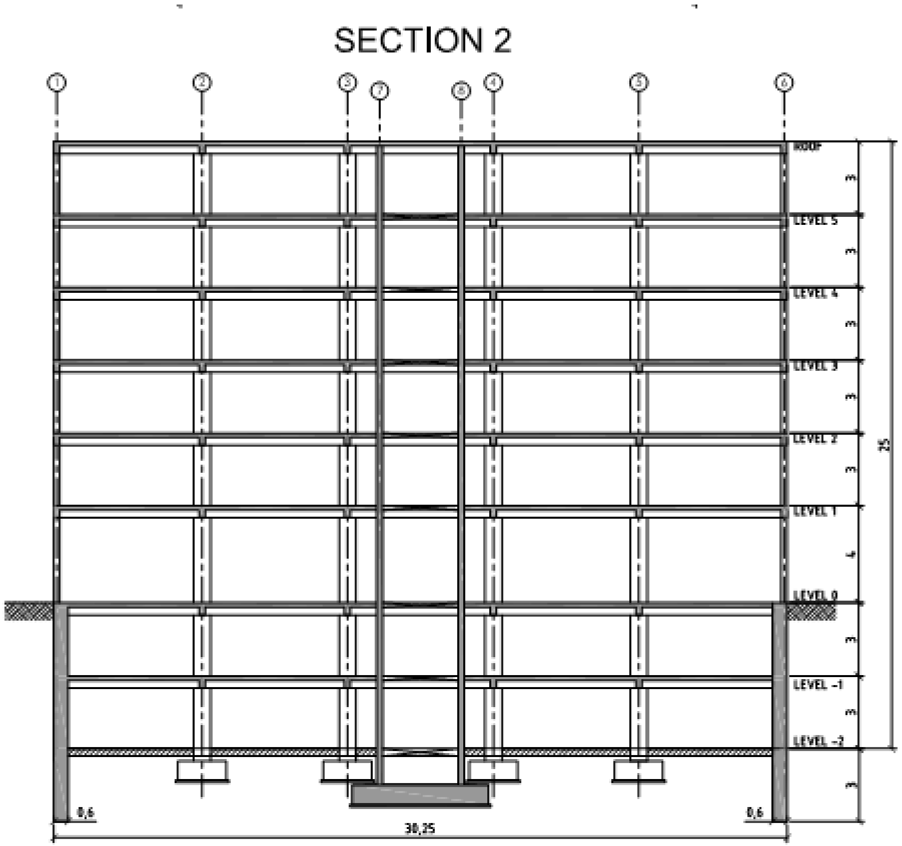
«Choosing an appropriate solution among many possible which must be studied in order to solve a particular problem, taking into account functional, structural, aesthetical and sustainability requirements»

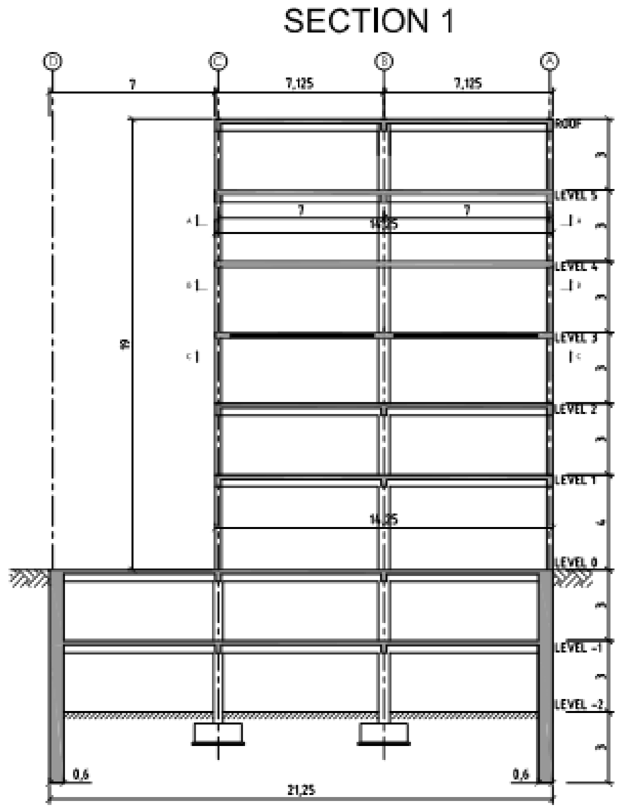
H. Corres Peiretti et al.

(Structural concrete Textbook, fib bulletin 51)

Aesthetical requirements?

EC2 worked example

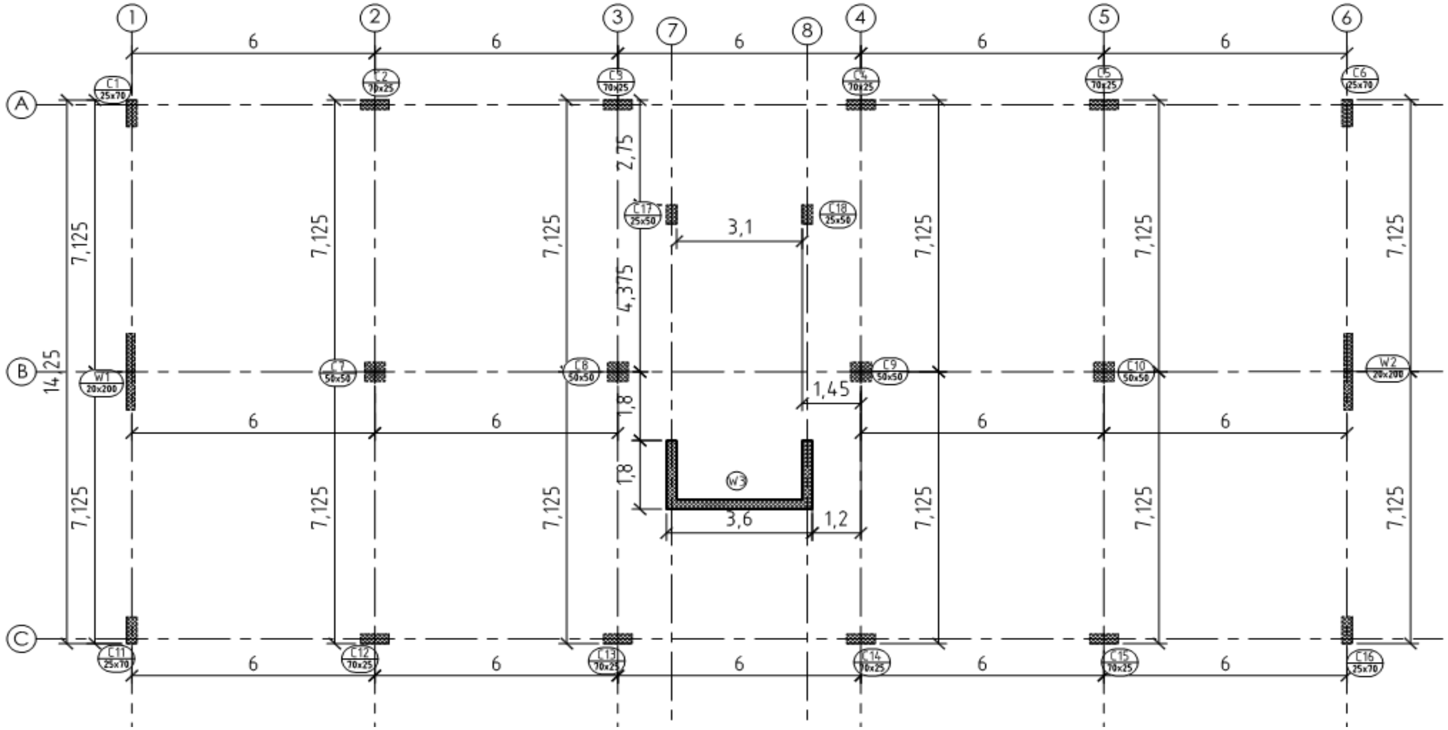




- 2-level underground parking
- ground floor: offices open to public
- 1^{st} to 5^{th} floor: dwellings
- roof

EC2 worked example

Columns Layout



x, y -direction slab/ beams spans all equal
single central core, two y-walls

General assumptions

SLABS AND BEAMS

The design of the geometry of slabs and beams has to fulfill both Ultimate (ULS) and Serviceability Limit States (SLS) requrements.

The depth of all slabs is based on deflection control (EC2 7.4).
For flat slabs, punching may also govern.

The width “b” of the beams is evaluated on the basis of the **span** ULS maximum bending, taking into account SLS of stress limitation and crack control. Maximum bending moments occur generally at the face of supports but redistribution and double reinforcement there can take care of the (M_{sup} – M_{span}) difference.

In the case of T beam, the minimum web width b_w may be governed by ULS shear.

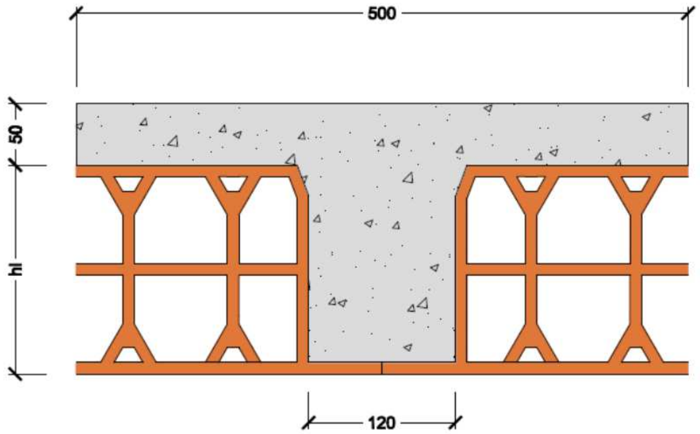
Slab self weight estimation

Self weight based on reinforced concrete unit weight (25 kN/m^3) and the actual or equivalent depth h (m) of the slab.

$$G_1 = 25\ h \qquad (kN/m^2)$$

For lighting embedded clay elements (38+12) cm with 5 cm topping, the equivalent height (= load) is 51 - 55% (average: 53%) of the weight of a flat slab of the same height.

h_{le} [m]	$h = h_{le} + 0{,}05$ [m]	G_1 [kN/m^2]	$h_{eq} = G_1/25$ [m]	h_{eq}/h_{tot}
0,16	0,21	2,89	0,116	**0,55**
0,18	0,23	3,08	0,123	**0,54**
0,20	0,25	3,27	0,131	**0,52**
0,22	0,27	3,46	0,138	**0,51**
0,24	0,29	3,69	0,148	**0,51**



Ex. Total height h = 0,23 m $\qquad$ G = (0,54 x 0,23) x 25 = 3,10 kN/m^2

G and Q loads

Permanent loads G2
Finishing, pavement, embedded services, partitions: 3,0 kN/m^2
Walls on external perimeter (windows included): 8,0 kN/m

Variable loads Q and ψ factors for load combinations

Type	q_k (kN/m^2)	ψ_0	ψ_2
Parking (cars ≦ 30 kN)	2,50	0,70	0,60
Dwellings	2,00	0,70	0,30
Stairs, office open to public	4,00		
Snow	1,70	0,50	0,00

No thermal effects considered as $L_{max} \leq 30$ m - EC2 2.3.3 (3)

Structural model

EC2 5.3.2.2 (1)

l_{eff} = “effective span”

EC2 5.3.2.2 (2)

Slabs analysed on the assumption that supports provide no rotational restraint

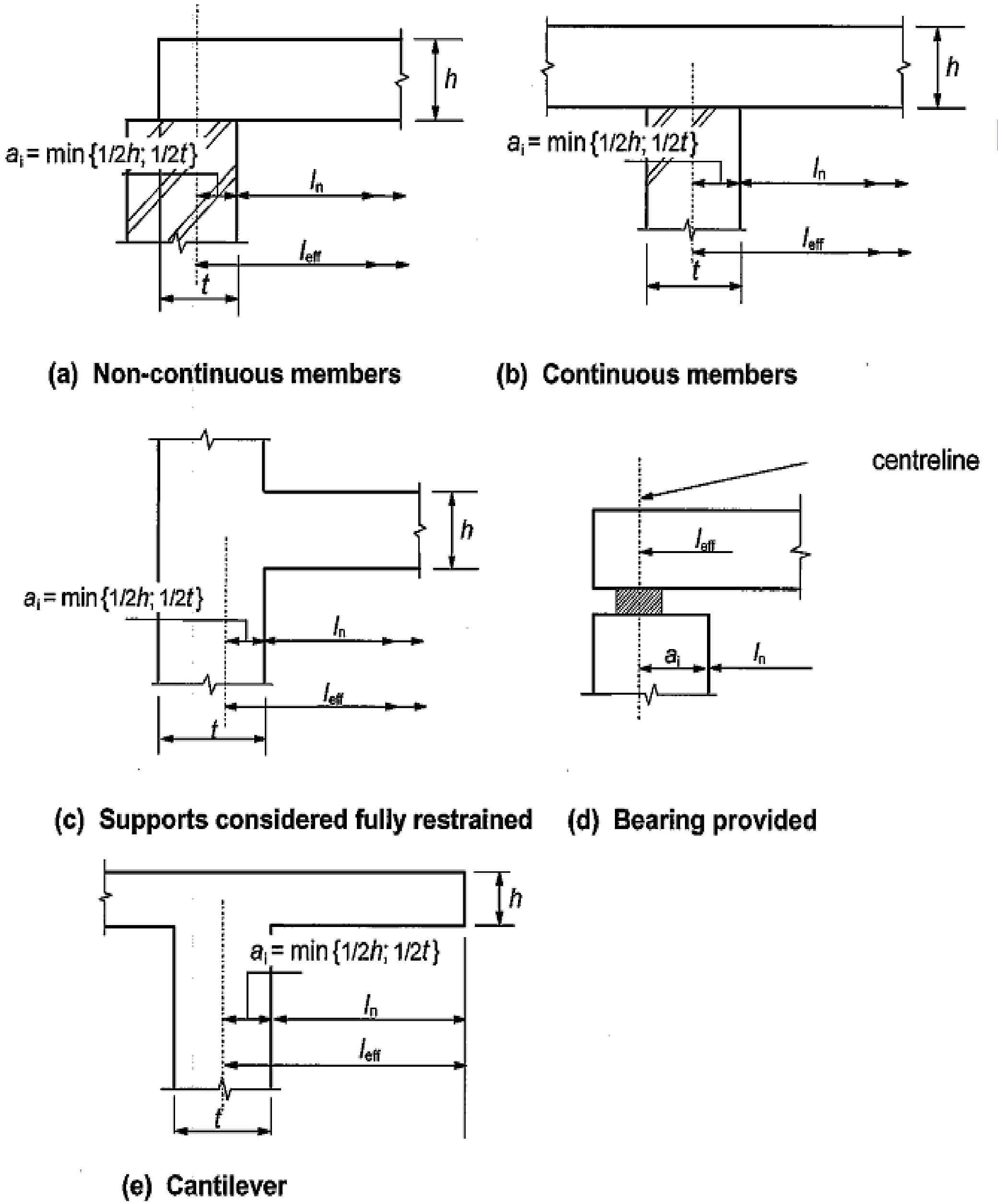
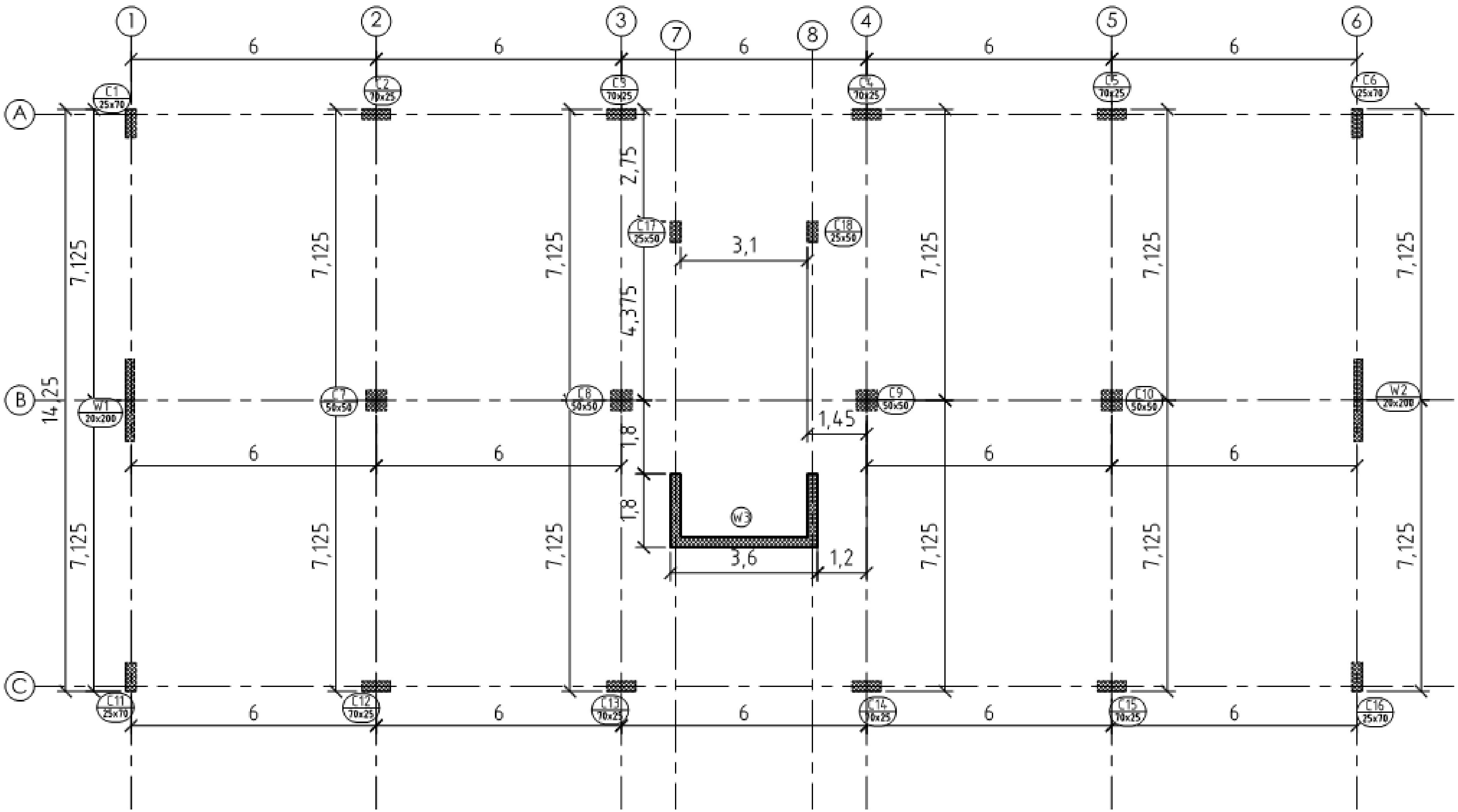


Figure 5.4: Effective span (l_{eff}) for different support conditions

Preliminary evalutation

Columns Layout



Slab depth

EC2 7.4.2 - Deflection control for flat slabs ≦ 8,5m and slab and beams ≦ 7 ml

$$\frac{l_{ef}}{d} = K\, s \frac{310}{\sigma_s}\left(\frac{l}{d}\right)_0 = K\, s \frac{500}{f_{yk}} \frac{A_{s,prov}}{A_{s,req}}\left(\frac{l}{d}\right)_0$$

s(hape) factor

s = 1,0 R section

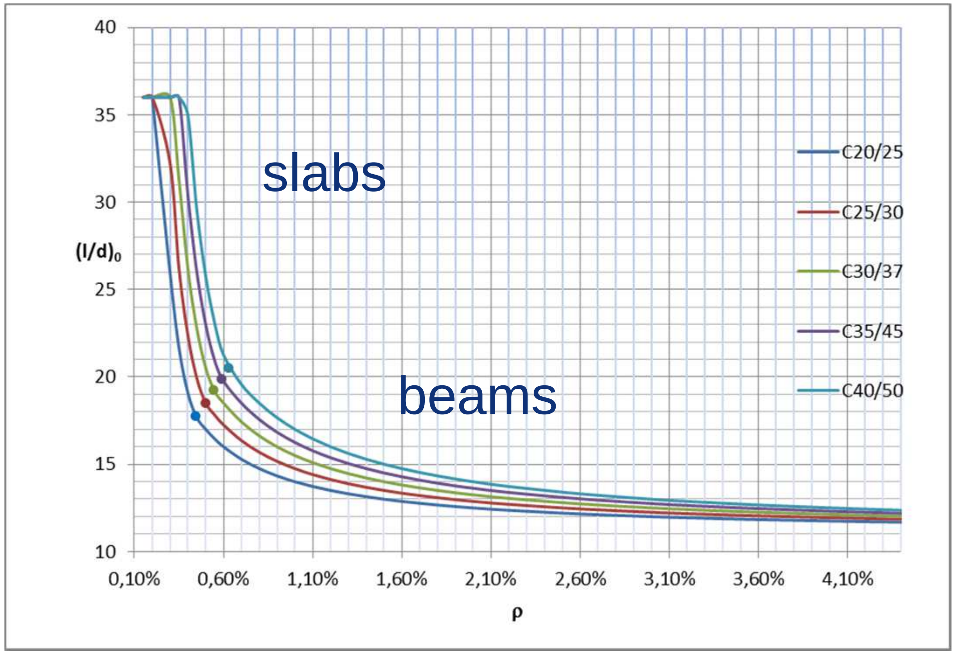
s = 0,8 T sections with $b/b_w > 3$

$$\rho = \frac{A_s}{bd} \qquad \rho' = \frac{A'_s}{bd} \qquad \rho_0 = 10^{-3}\sqrt{f_{ck}}$$

$$\rho \le \rho_0 \qquad \left(\frac{l}{d}\right)_0 = 11 + 1{,}5\sqrt{f_{ck}}\frac{\rho_0}{\rho} + 3{,}2\sqrt{f_{ck}}\sqrt{\left(\frac{\rho_0}{\rho} - 1\right)^3}$$

$$\rho > \rho_0 \qquad \left(\frac{l}{d}\right)_0 = 11 + 1{,}5\sqrt{f_{ck}}\frac{\rho_0}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}}\sqrt{\frac{\rho'}{\rho_0}}$$

	C20/25	C25/30	C30/37	C32/40	C35/45
ρ_0 (%)	0,45	0,50	0,55	0,57	0,59
$(l/d)_0$	19	**20**	20	21	18

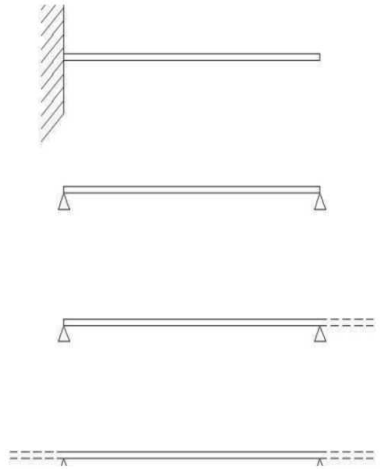


max $(l/d)_o = 36$

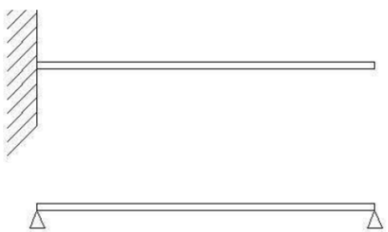
(l/d) values – C30/37, f_{yk} = 500 N/mm²

Structural System	K	Concrete highly stressed ρ = 1,5%	Concrete lightly stressed ρ = 0,5%
Simply supported beam, one- or two-way spanning simply supported slab	1,0	14	20
End span of continuous beam or one-way continuous slab or two-way spanning slab continuous over one long side	1,3	18	26
Interior span of beam or one-way or two-way spanning slab	1,5	20	30
Slab supported on columns without beams (flat slab) (based on longer span)	1,2	17	24
Cantilever	0,4	6	8

Note 1: The values given have been chosen to be generally conservative and calculation may frequently show that thinner members are possible.
Note 2: For 2-way spanning slabs, the check should be carried out on the basis of the shorter span. For flat slabs the longer span should be taken.
Note 3: The limits given for flat slabs correspond to a less severe limitation than a mid-span deflection of span/250 relative to the columns. Experience has shown this to be satisfactory.



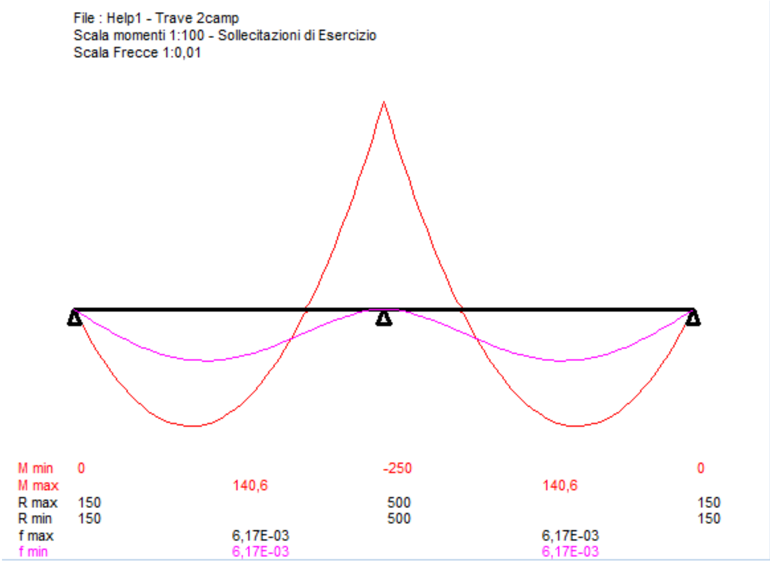
k "normalizes" structural spans to the Simply Supported one

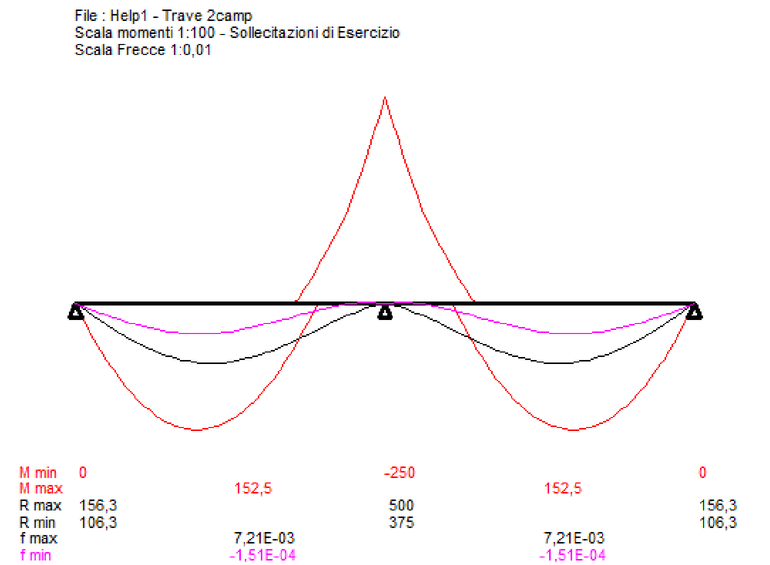


$$f_{CL} = \frac{1}{8}\frac{q}{EJ} l_{CL}^4 \qquad f_{SS} = \frac{5}{384}\frac{q}{EJ} l_{SS}^4 \qquad l_{SS} = \frac{l_{CL}}{k}$$

$$f_{CL} = f_{SS} \Rightarrow l_{CL}^4 = \frac{5}{48} l_{SS}^4 = k^4 l_{SS}^4 \qquad k = \sqrt[4]{\frac{5}{48}} = 0,57$$

EC2 7.4.2 Deflection control by slenderness (l/d)





- the “normalized” span
  $l_n = l/k$

may be used for fast approximate span bending moment evaluation using the “single span beam” formula

$$M = q l_n^2/8$$

$$M_{leff} = \frac{G\, l_{eff}^2}{14{,}2} \qquad l_n = \frac{l_{eff}}{1{,}3}$$

$$M_{ln} = \frac{G\, l_n^2}{8} = \frac{G\, l_{eff}^2}{13{,}5} \qquad + 5\%$$

$$M_{G+Q} \approx \frac{G\,(1+Q/G)\, l_{eff}^2}{13{,}1} \qquad - 3\% \;\; \text{if } Q = 1/3\ G$$

Depth evaluation

	C20/25	C25/30	C30/37	C32/40	C35/45
ρ_0 (%)	0,45	0,50	0,55	0,57	0,59
$(l/d)_0$	19	20	20	21	18

	$l_{ef,x}$	$l_{ef,y}$	l_{ef}	k	l_n	**$(l/d)_0$**	s	d_{min}	A_s
	m	m	m		m			m	cm²/m
Slab on beams	6,0	7,125	6,0	1,3	4,62	20	1,0	0,23	**12,7**
Flat slab	6,0	7,125	7,125	1,2	5,94	20	1,0	0,30	**16,5**
Slab with emb. el.	-	7,125	7,125	1,3	5,48	20	0,8	0,27	**14,9**

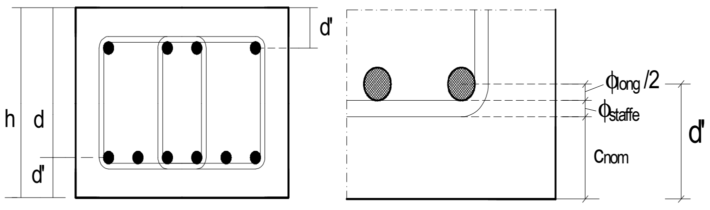
$$l_n = \frac{l_{ef}}{K}$$

$$d_{min} = \frac{l_n}{(l/d)_0\, s}$$

Due to the high reinforcement ratio assumed in the table resulting effective depths d_{min} are too conservative, but may be used for a (safe) preliminary evaluation of slab self weight G_1

$$h_{min} = d_{min} + d'$$

$$d' = c_{nom} + \phi_{st} + ½\, \phi_l = 30 + 0 + 14/2 = 37 \text{ mm}$$



	d_{min}	$h_{min} = d_{min} + d'$	coeff	$h_{c,eq}$	G_1
	m	m		m	kN/m²
Slab on beams	**0,23**	0,27	1,00	0,27	6,69
Flat slab	**0,30**	0,33	1,00	0,33	8,35
Slab with emb. el.	**0,27**	0,31	0,55	0,17	4,28

Iterative refined method

G and Q in kN/m

$$\left(\frac{l_n}{d}\right) = \frac{\lambda_s}{\sqrt[3]{G+\psi_2 Q}}$$

λ_s

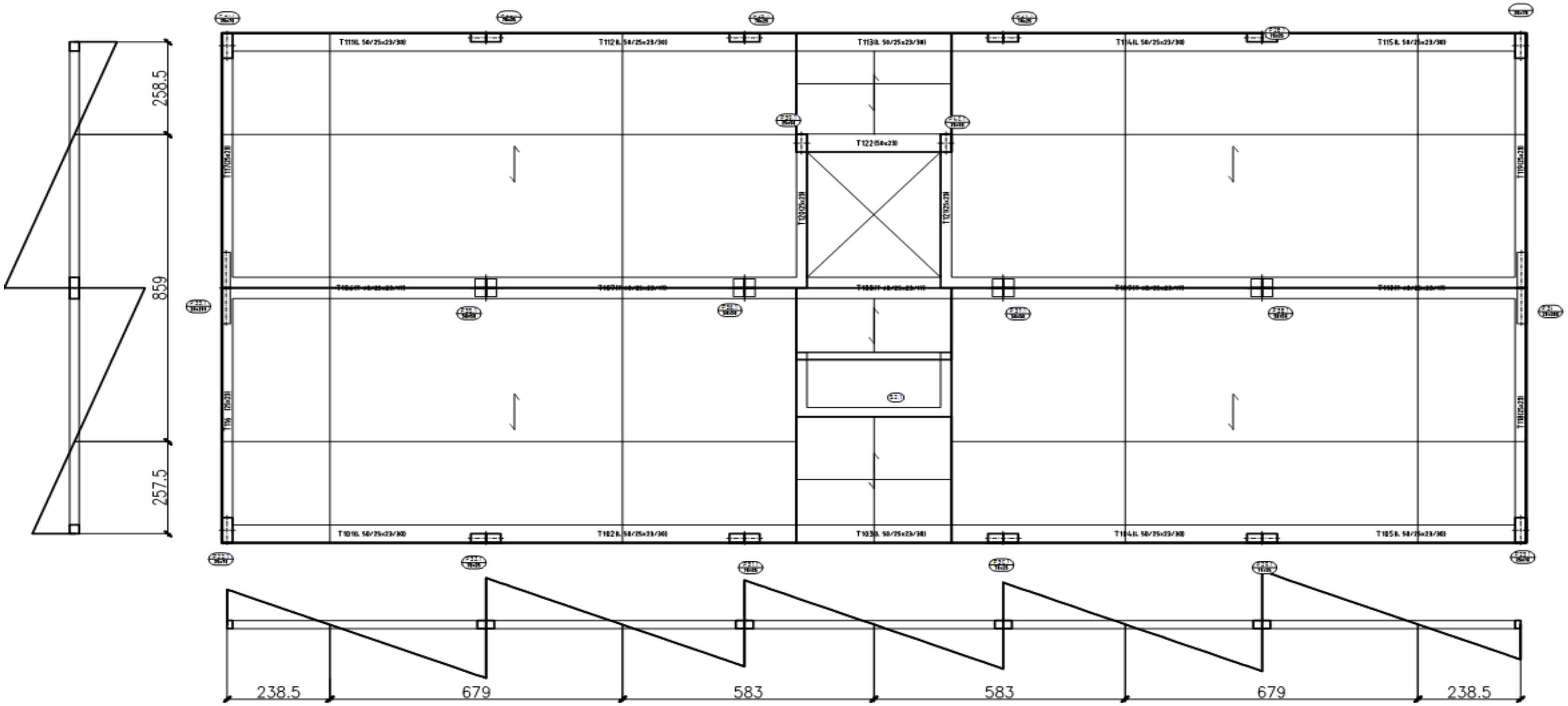
		C20/25	C25/30	C30/37	C35/45	C40/50
S =	**1,0**	53	57	60	63	65
S =	**0,8**	49	53	56	59	61

	d_{min}	$h_{min} = d_{min} + d'$	coeff	h_{eq}	G_1	G_2	Q_k	ψ_2	Tot	λ_s	l_n/d	l_n	d_{min}	
	m	m		m	kN/m	kN/m	kN/m		kN/m			m	m	
Slab on beams	**0,23**	0,27	1,00	0,27	6,69	3,0	2,0	0,30	10,29	60	28	4,62	0,17	-28%
Flat slab	**0,30**	0,33	1,00	0,33	8,35	3,0	2,0	0,30	11,95	60	26	5,94	0,23	-24%
Slab with l. el.	**0,27**	0,31	0,55	0,17	4,28	3,0	2,0	0,30	7,88	56	28	5,48	0,20	-29%
Slab on beams	0,17	0,20	1,00	0,20	5,10	3,0	2,0	0,30	8,70	60	29	4,62	0,16	-6%
Flat slab	0,23	0,26	1,00	0,26	6,56	3,0	2,0	0,30	10,16	60	28	5,94	0,21	-6%
Slab with l. el.	0,20	0,23	0,55	0,13	3,19	3,0	2,0	0,30	6,79	56	30	5,48	0,19	-6%
Slab on beams	0,16	**0,19**	1,00	0,19	4,87	3,0	2,0	0,30	8,47	60	30	4,62	**0,16**	-1%
Flat slab	0,21	**0,25**	1,00	0,25	6,27	3,0	2,0	0,30	9,87	60	28	5,94	**0,21**	-1%
Slab with l. el.	0,19	**0,22**	0,55	0,12	3,06	3,0	2,0	0,30	6,66	56	30	5,48	**0,18**	-1%

h_{min} = 0,19 – 0,25 – 0,22 m

Taking into account $A_{s,req}/A_{s,prov}$ h_{fin} = 0,18 - 0,24 – (0,18+0,05) = 0,23 m

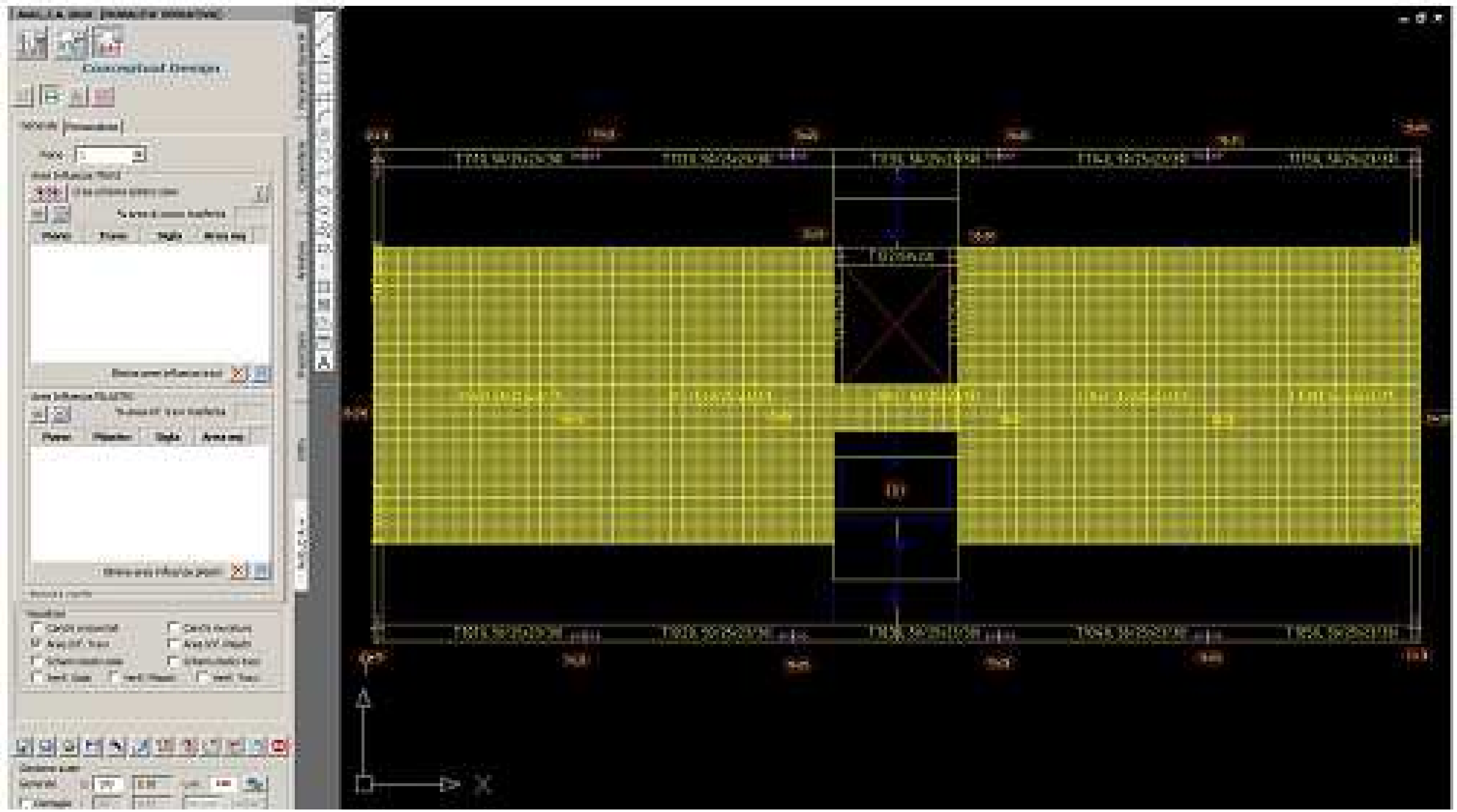
Beams and columns load tributary area



Monodirectional slabs: “zero-shear” lines under uniform q = 1 loading identify **beams** tributary areas; zero shear lines for beams together with the ones for slabs identify **columns** tributary areas

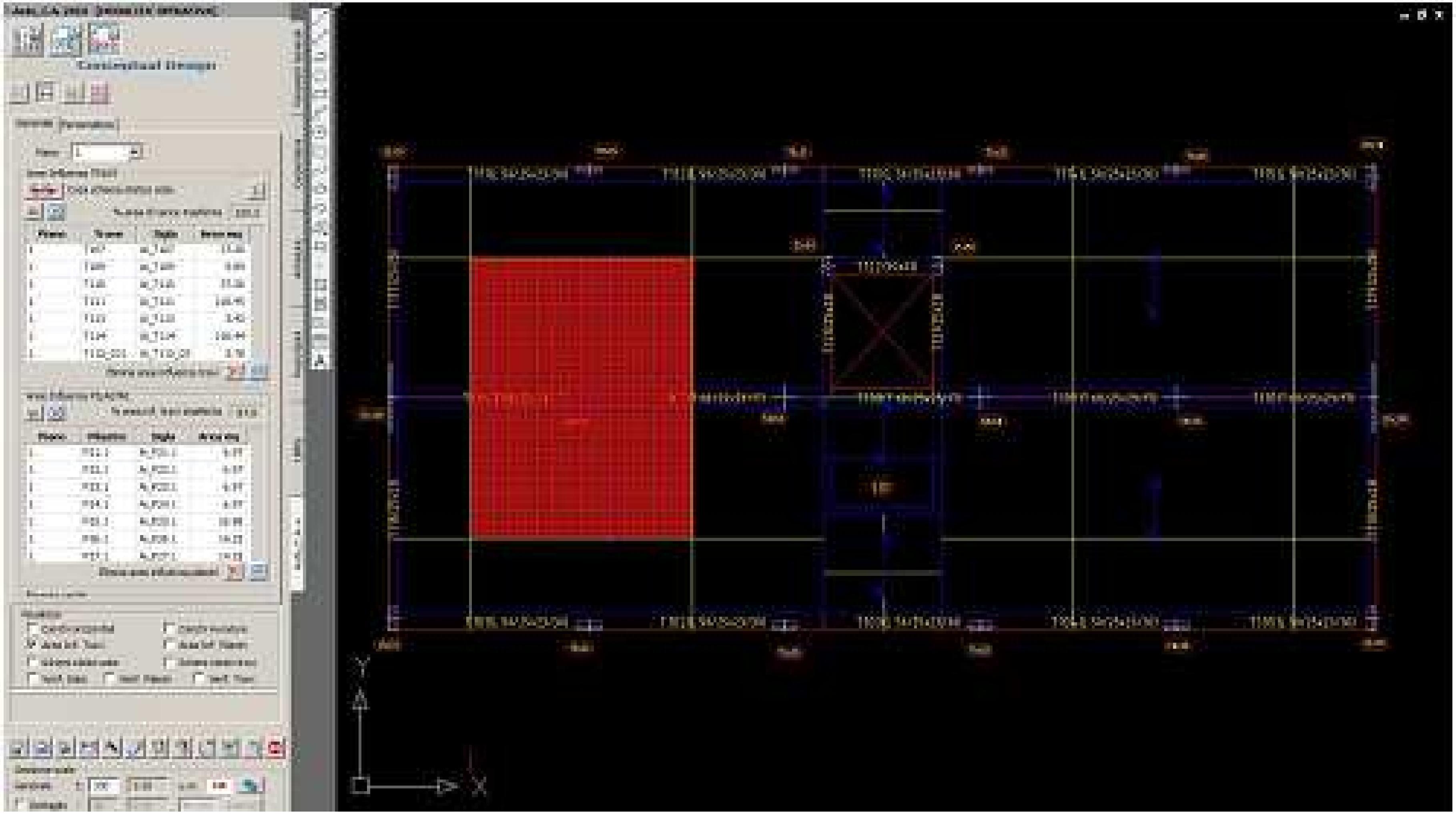
Bi-directional or flat slabs: yield lines approach apply.

Beams tributary area – Auto-CA add on for Autocad™



www.auto-ca.it

Columns and cores load tributary area – Auto-Ca



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EC2 worked example

Conceptual design

Slabs

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EC2 worked example

Conceptual design

Beams

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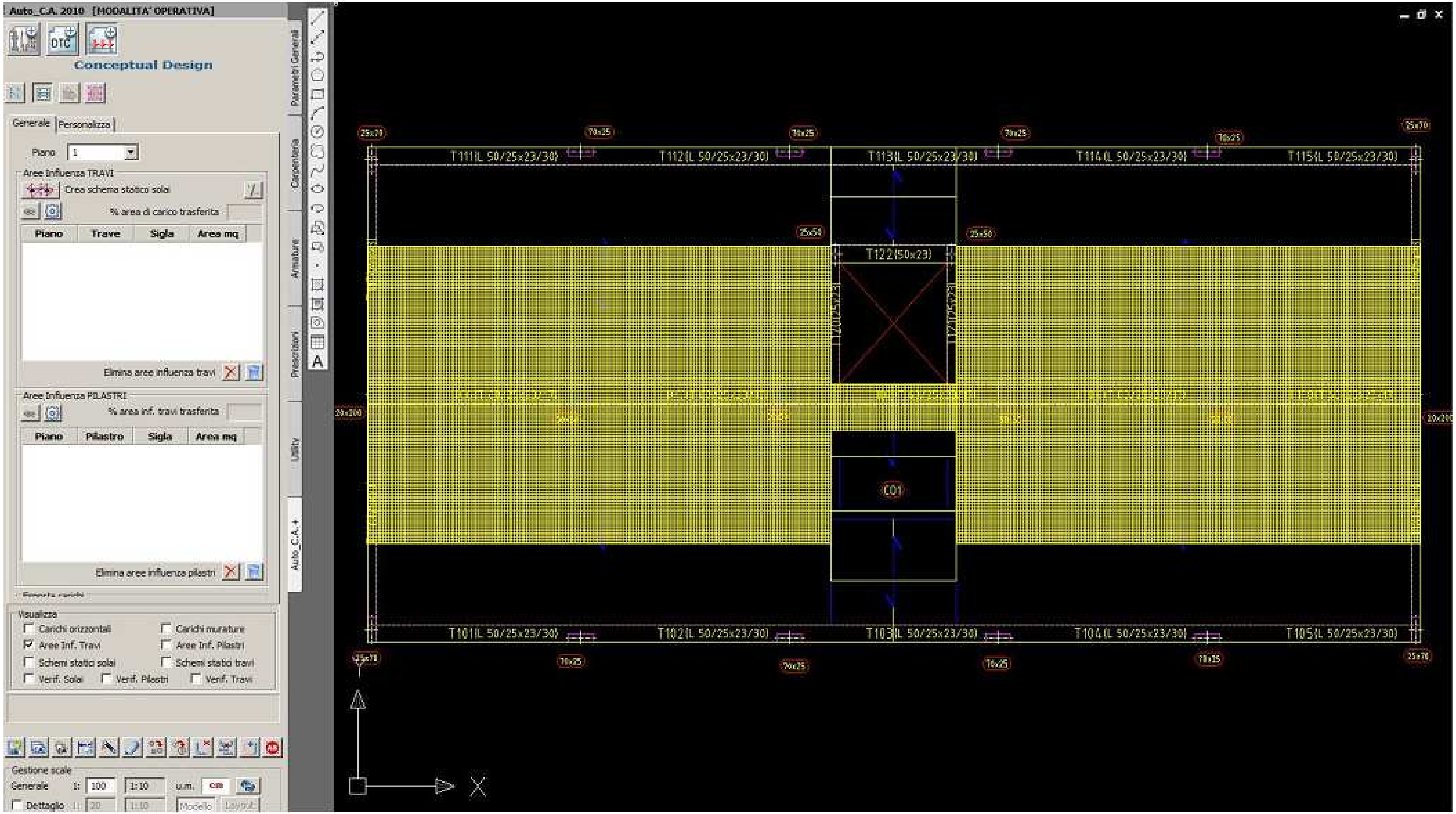
Beams tributary area by Auto-ca, add on for Autocad™



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SLS cracking

Table 7.1N Recommended values of w_{max} (mm)

Exposure Class	Reinforced members and prestressed members with unbonded tendons	Prestressed members with bonded tendons
	Quasi-permanent load combination	Frequent load combination
X0, XC1	0,4[1]	0,2
XC2, XC3, XC4	0,3	0,2[2]
XD1, XD2, XS1, XS2, XS3		Decompression

Note 1: For X0, XC1 exposure classes, crack width has no influence on durability and this limit is set to guarantee acceptable appearance. In the absence of appearance conditions this limit may be relaxed.
Note 2: For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.

w_{max} = 0,3 mm to be evaluated for the Quasi-Permanent (QP) load combination

Maximum diameters - cracked section, QP load combination

Steel 500 B		Concrete class				
		C20/25	**C25/30**	**C30/37**	**C35/45**	**C40/50**
	$f_{ct,eff}$	**2,3**	**2,6**	**2,9**	**3,4**	**3,6**
σ_s	σ_s/f_{yk}	$\phi_{l,max}$ for crack width w_k = 0,30 mm				
160	0,32	24	28	**32**	36	38
170	0,34	22	26	30	34	36
180	0,36	22	24	28	32	34
190	0,38	20	22	26	30	32
200	0,40	18	20	**24**	26	28
210	0,42	16	18	22	24	26
220	0,44	14	16	20	22	24
230	0,46	14	16	18	20	22
240	0,48	12	14	**16**	18	20
260	0,52	10	12	14	16	16
280	0,56	10	10	**12**	14	14

Note: EC2 values up to f_{yk}; 25 mm for σ_s = 200 Mpa

SLS stress limitation

CA combination of loads

1) Longitudinal craks due to excessive concrete compressive stress may affect durability (exposure classes XD,XF, XS only)
2) Excessive steel inelastic strain leads to unacceptable cracking or deformation.

QP combination of loads

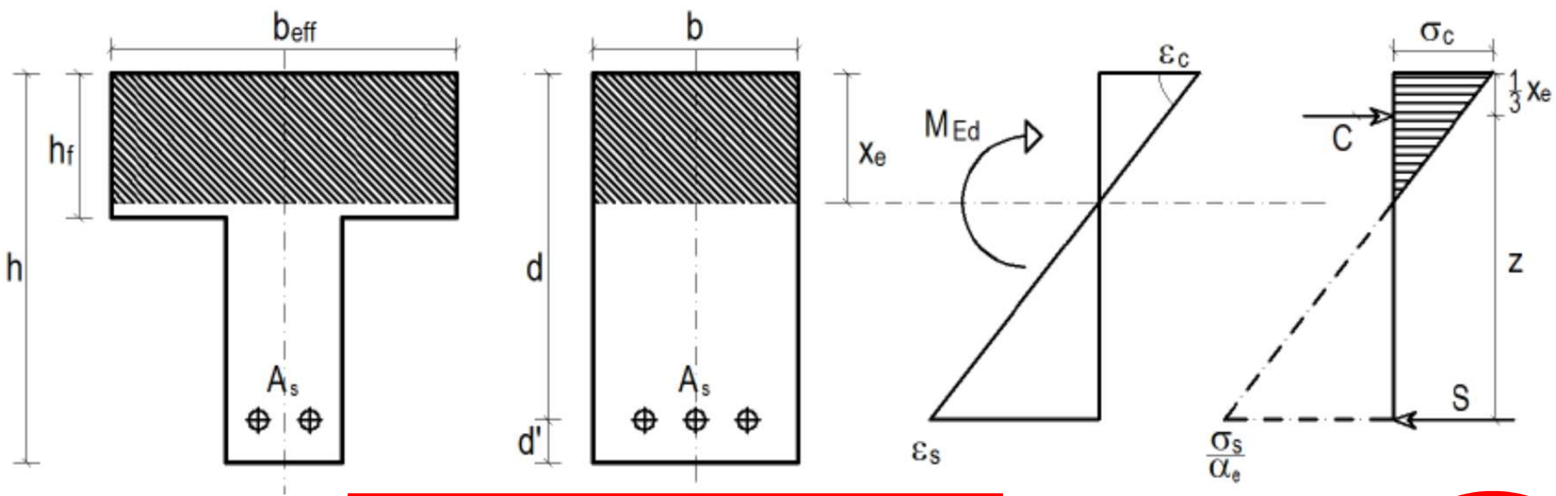
3) Limitation of max concrete compressive stress to confirm linear creep for concrete
4) [Crack width control by maximum bar diameter – see prev. slide]

Concrete σ_c/f_{ck}		Steel σ_s/f_{yk}	
QP	CA	QP	CA
$\leq 0,45$	$\leq 0,60$	$\leq 0,32 – 0,52$	$\leq 0,80$

σ_c/f_{ck} and σ_s/f_{yk} to be evaluated with **an elastic cracked model**

SLS stress evaluation

Single reinforced cracked section - elastic model



$$\xi_e = \frac{x_e}{d} = \frac{1}{1+\frac{\sigma_s}{\alpha_e \sigma_c}}$$

$$\xi_e = \frac{x_e}{d} = \alpha_e \, \rho \left(\sqrt{1+\frac{2}{\alpha_e \rho}} - 1 \right)$$

$$\rho = \frac{A_s}{bd} \qquad \alpha_e = \frac{E_s}{E_c}$$

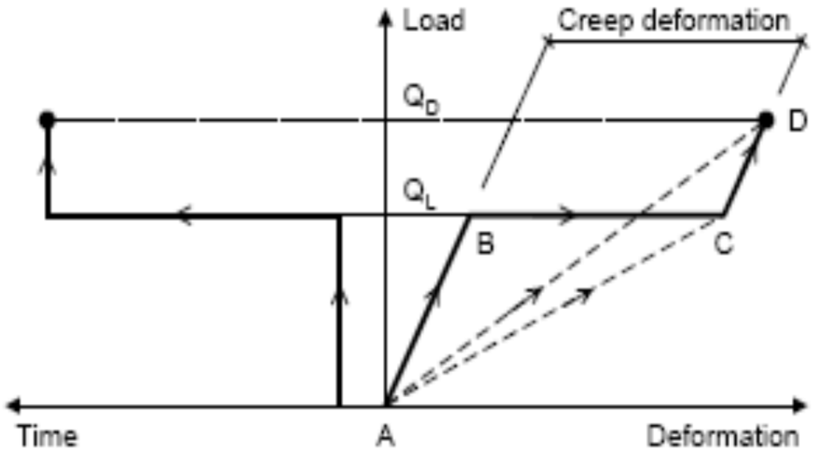
$$\mu_k = \frac{M_{Ek}}{b\,d^2 f_{ck}}$$

$$\frac{\sigma_c}{f_{ck}} = \frac{\mu_k}{\frac{\xi_e}{2}\left(1-\frac{\xi_e}{3}\right)} \qquad \frac{\sigma_s}{f_{yk}} = \frac{\mu_k}{\rho \frac{f_{yk}}{f_{ck}}\left(1-\frac{\xi_e}{3}\right)}$$

$M_{Ek} = M_{EK,QP}$ or $M_{EK,CA}$

$f(\rho)$

α_e coefficient



$$\alpha_e = \frac{E_s}{E_{c,eff}} \qquad E_{c,eff} = \frac{E_{cm}}{(1+\phi_{eff})}$$

$$\phi_{eff} = \phi_{\infty,t_o}$$ QP combination

$$\phi_{eff} = \phi_{\infty,t_o} \frac{M_{Ek,QP}}{M_{Ek,CA}}$$ CA combination

	C16/20	C20/25	C25/30	C30/37	C35/45	C40/50
α_{eQP}	21,0	20,0	19,1	18,3	17,6	17,0
α_{eCA}	16,1	15,4	14,6	14,0	13,5	13,1
α_{eEcm}	7,0	6,7	6,4	6,1	5,9	5,7

ULS – materials' design values

Concrete C25 / 30 f_{ck} = 25/30 N/mm^2

$$f_{cd} = \frac{\alpha_{cc}\, f_{ck}}{\gamma_C} \qquad \alpha_{cc} = 1.0 \qquad \gamma_C = 1.50$$

$$f_{cd} = \frac{1.0 \times 25}{1,50} = 16,7 \text{ N/mm}^2 \qquad f_{cd} = \frac{1.0 \times 30}{1,50} = 20,0 \text{ N/mm}^2$$

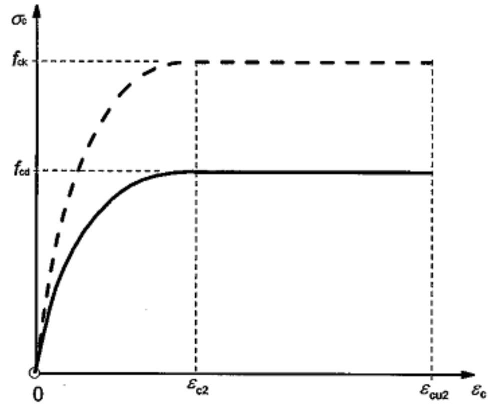


Figure 3.3: Parabola-rectangle diagram for concrete under compression.

Steel 500 B $f_{yk} = 435$ N/mm^2

$$f_{yd} = \frac{f_{yk}}{\gamma_s} \qquad \varepsilon_{syd} = \frac{f_{yd}}{E_s} \qquad \gamma_s = 1,15$$

$$f_{yd} = \frac{500}{1,15} = 435 \text{ N/mm}^2 \qquad \varepsilon_{syd} = \frac{435}{2000} = 0,22\ \%$$

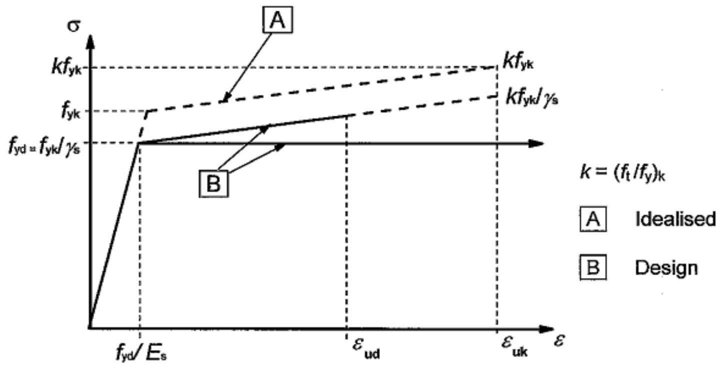
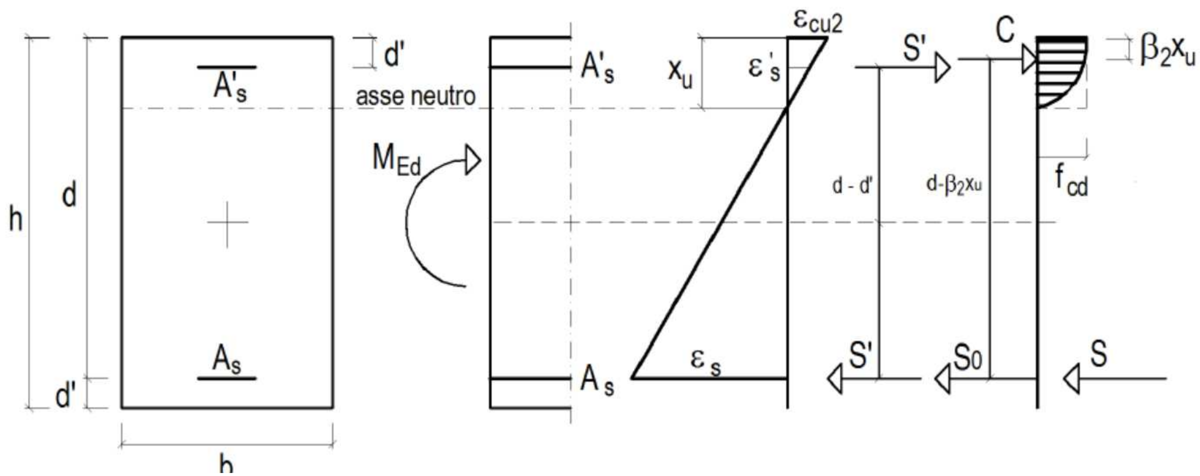


Figure 3.8: Idealised and design stress-strain diagrams for reinforcing steel (for tension and compression)

Single reinforced ($A'_s = 0$) cracked section – plastic model



$$\mu_d = \frac{M_{Ed}}{b d^2 f_{cd}} \qquad \xi_u = \frac{x_u}{d} = \frac{1}{1 + \varepsilon_{cu2}/\varepsilon_s} \qquad \omega = \frac{A_s f_{yd}}{b\, d\, f_{cd}} = \rho \frac{f_{yd}}{f_{cd}} \qquad \rho = \frac{A_s}{b\, d}$$

For single reinforced elements ($A'_s = 0$):

$$\xi_u = 1{,}202 - \sqrt{1{,}445 - 2{,}970\ \mu_d} \qquad \omega = 0{,}973 - \sqrt{0{,}947 - 1{,}946\ \mu_d}$$

EUROCODE 2
Background and Applications

ULS bending "universal" table

$$\mu_d = \frac{M_{Ed}}{b d^2 f_{cd}}$$

$$\omega = \frac{A_s f_{yd}}{b d f_{cd}} = \rho \frac{f_{yd}}{f_{cd}}$$

$$\rho = \frac{A_s}{b d} = \omega \frac{f_{cd}}{f_{yd}}$$

Is any μ_d value ok for design?
What about SLS (deflection, stress limitation)?

							ρ [%]				
ε [‰]	k	ξ_u	ζ_u	μ_u	ω_0	δ'lim %	C20/25	C25/30	C30/37	C35/45	C40/50
67,5	1,00	0,05	0,98	**0,039**	0,040	1,9	**0,13**	0,15	0,18	0,21	0,24
65,2	1,00	0,05	0,98	**0,040**	0,041	1,9	0,13	0,16	0,19	0,22	0,25
51,3	1,00	0,06	0,97	**0,050**	0,052	2,4	0,16	0,20	0,24	0,28	0,32
41,9	1,00	0,08	0,97	**0,060**	0,062	2,9	0,19	0,24	0,29	0,33	0,38
35,3	1,00	0,09	0,96	**0,070**	0,073	3,4	0,22	0,28	0,34	0,39	0,45
30,2	1,00	0,10	0,96	**0,080**	0,084	3,9	0,26	0,32	0,39	0,45	0,52
26,3	1,00	0,12	0,95	**0,090**	0,095	4,4	0,29	0,36	0,44	0,51	0,58
23,2	1,00	0,13	0,95	**0,100**	0,106	5,0	0,33	0,41	0,49	0,57	0,65
22,5	1,00	0,13	0,94	**0,103**	0,109	5,1	0,33	0,42	0,50	0,58	0,67
20,6	1,00	0,15	0,94	**0,110**	0,117	5,5	0,36	0,45	0,54	0,63	0,72
18,5	1,00	0,16	0,93	**0,120**	0,129	6,0	0,40	0,49	0,59	0,69	0,79
16,7	1,00	0,17	0,93	**0,130**	0,140	6,6	0,43	0,54	0,65	0,75	0,86
15,1	1,00	0,19	0,92	**0,140**	0,152	7,1	0,47	0,58	0,70	0,82	0,93
13,8	1,00	0,20	0,92	**0,150**	0,164	7,7	0,50	0,63	0,75	0,88	1,01
12,6	1,00	0,22	0,91	**0,160**	0,176	8,2	0,54	0,68	0,81	0,95	1,08
11,5	1,00	0,23	0,90	**0,170**	0,189	8,8	0,58	0,72	0,87	1,01	1,16
10,6	1,00	0,25	0,90	**0,180**	0,201	9,4	0,62	0,77	0,92	1,08	1,23
10,0	**1,00**	0,26	0,89	**0,187**	**0,210**	9,8	**0,64**	**0,81**	**0,97**	**1,13**	**1,29**
9,8	1,00	0,26	0,89	**0,190**	0,214	10,0	0,66	0,82	0,98	1,15	1,31
9,0	1,00	0,28	0,88	**0,200**	0,227	10,6	0,69	0,87	1,04	1,22	1,39
8,3	1,00	0,30	0,88	**0,210**	0,240	11,2	0,74	0,92	1,10	1,29	1,47
7,7	1,00	0,31	0,87	**0,220**	0,253	11,8	0,78	0,97	1,16	1,36	1,55
7,1	**1,00**	0,33	0,86	**0,230**	**0,267**	12,5	**0,82**	**1,02**	**1,23**	**1,43**	**1,64**
6,6	1,00	0,35	0,86	**0,240**	0,281	13,1	0,86	1,08	1,29	1,51	1,72
6,1	1,00	0,36	0,85	**0,250**	0,295	13,8	0,90	1,13	1,36	1,58	1,81
5,7	1,00	0,38	0,84	**0,260**	0,309	14,5	0,95	1,19	1,42	1,66	1,90
5,2	1,000	0,40	0,83	**0,270**	0,324	15,2	0,99	1,24	1,49	1,74	1,99
4,8	1,000	0,42	0,83	**0,280**	0,339	15,9	1,04	1,30	1,56	1,82	2,08
4,5	1,000	0,44	0,82	**0,290**	0,355	16,6	1,09	1,36	1,63	1,90	2,18
4,28	**1,000**	**0,45**	**0,81**	**0,296**	**0,364**	17,1	**1,12**	**1,40**	**1,68**	**1,96**	**2,23**
4,1	1,000	0,46	0,81	**0,302**	0,374	17,5	1,15	1,44	1,72	2,01	2,30
3,9	1,000	0,48	0,80	**0,309**	0,385	18,0	1,18	1,48	1,77	2,07	2,36
3,6	1,000	0,49	0,80	**0,316**	0,397	18,6	1,22	1,52	1,82	2,13	2,43
3,4	1,000	0,50	0,79	**0,323**	0,409	19,1	1,25	1,57	1,88	2,19	2,51
3,2	1,000	0,52	0,78	**0,330**	0,421	19,7	1,29	1,61	1,94	2,26	2,58
3,0	1,000	0,54	0,78	**0,338**	0,435	20,4	1,33	1,67	2,00	2,33	2,67
2,8	1,000	0,56	0,77	**0,346**	0,449	21,0	1,38	1,72	2,07	2,41	2,76
2,6	1,000	0,57	0,76	**0,354**	0,465	21,8	1,43	1,78	2,14	2,49	2,85
2,4	1,000	0,59	0,75	**0,362**	0,482	22,5	1,48	1,85	2,21	2,58	2,95
2,17	**1,000**	0,62	0,74	**0,371**	**0,499**	23,4	1,53	1,91	2,30	2,68	3,06

EUROCODE 2
Background and Applications

ULS bending "universal" table vs. linear elastic analysis of hyperstatic structures EC2 5.4 – 5.5

$$\partial = \frac{M_{Eel,rid}}{M_{Eel,d}} \geq 0,44 + 1,25\,\frac{x_u}{d}$$

$$0,70 \leq \partial \leq 1,0$$

$$\partial = 1 \quad \text{when } \xi_u = \frac{x_u}{d} = 0,45$$

$$\Downarrow$$

$$\mu_d \leq 0,296$$

ε [‰]	k	ξ_u	ζ_u	μ_u	ω_0	δ'_{lim} %	ρ [%] C20/25	C25/30	C30/37	C35/45	C40/50
67,5	1,00	0,05	0,98	0,039	0,040	1,9	0,13	0,15	0,18	0,21	0,24
65,2	1,00	0,05	0,98	0,040	0,041	1,9	0,13	0,16	0,19	0,22	0,25
51,3	1,00	0,06	0,97	0,050	0,052	2,4	0,16	0,20	0,24	0,28	0,32
41,9	1,00	0,08	0,97	0,060	0,062	2,9	0,19	0,24	0,29	0,33	0,38
35,3	1,00	0,09	0,96	0,070	0,073	3,4	0,22	0,28	0,34	0,39	0,45
30,2	1,00	0,10	0,96	0,080	0,084	3,9	0,26	0,32	0,39	0,45	0,52
26,3	1,00	0,12	0,95	0,090	0,095	4,4	0,29	0,36	0,44	0,51	0,58
23,2	1,00	0,13	0,95	0,100	0,106	5,0	0,33	0,41	0,49	0,57	0,65
22,5	1,00	0,13	0,94	0,103	0,109	5,1	0,33	0,42	0,50	0,58	0,67
20,6	1,00	0,15	0,94	0,110	0,117	5,5	0,36	0,45	0,54	0,63	0,72
18,5	1,00	0,16	0,93	0,120	0,129	6,0	0,40	0,49	0,59	0,69	0,79
16,7	1,00	0,17	0,93	0,130	0,140	6,6	0,43	0,54	0,65	0,75	0,86
15,1	1,00	0,19	0,92	0,140	0,152	7,1	0,47	0,58	0,70	0,82	0,93
13,8	1,00	0,20	0,92	0,150	0,164	7,7	0,50	0,63	0,75	0,88	1,01
12,6	1,00	0,22	0,91	0,160	0,176	8,2	0,54	0,68	0,81	0,95	1,08
11,5	1,00	0,23	0,90	0,170	0,189	8,8	0,58	0,72	0,87	1,01	1,16
10,6	1,00	0,25	0,90	0,180	0,201	9,4	0,62	0,77	0,92	1,08	1,23
10,0	1,00	0,26	0,89	0,187	0,210	9,8	0,64	0,81	0,97	1,13	1,29
9,8	1,00	0,26	0,89	0,190	0,214	10,0	0,66	0,82	0,98	1,15	1,31
9,0	1,00	0,28	0,88	0,200	0,227	10,6	0,69	0,87	1,04	1,22	1,39
8,3	1,00	0,30	0,88	0,210	0,240	11,2	0,74	0,92	1,10	1,29	1,47
7,7	1,00	0,31	0,87	0,220	0,253	11,8	0,78	0,97	1,16	1,36	1,55
7,1	1,00	0,33	0,86	0,230	0,267	12,5	0,82	1,02	1,23	1,43	1,64
6,6	1,00	0,35	0,86	0,240	0,281	13,1	0,86	1,08	1,29	1,51	1,72
6,1	1,00	0,36	0,85	0,250	0,295	13,8	0,90	1,13	1,36	1,58	1,81
5,7	1,00	0,38	0,84	0,260	0,309	14,5	0,95	1,19	1,42	1,66	1,90
5,2	1,000	0,40	0,83	0,270	0,324	15,2	0,99	1,24	1,49	1,74	1,99
4,8	1,000	0,42	0,83	0,280	0,339	15,9	1,04	1,30	1,56	1,82	2,08
4,5	1,000	0,44	0,82	0,290	0,355	16,6	1,09	1,36	1,63	1,90	2,18
4,28	1,000	0,45	0,81	0,296	0,364	17,1	1,12	1,40	1,68	1,96	2,23
4,1	1,000	0,46	0,81	0,302	0,374	17,5	1,15	1,44	1,72	2,01	2,30
3,9	1,000	0,48	0,80	0,309	0,385	18,0	1,18	1,48	1,77	2,07	2,36
3,6	1,000	0,49	0,80	0,316	0,397	18,6	1,22	1,52	1,82	2,13	2,43
3,4	1,000	0,50	0,79	0,323	0,409	19,1	1,25	1,57	1,88	2,19	2,51
3,2	1,000	0,52	0,78	0,330	0,421	19,7	1,29	1,61	1,94	2,26	2,58
3,0	1,000	0,54	0,78	0,338	0,435	20,4	1,33	1,67	2,00	2,33	2,67
2,8	1,000	0,56	0,77	0,346	0,449	21,0	1,38	1,72	2,07	2,41	2,76
2,6	1,000	0,57	0,76	0,354	0,465	21,8	1,43	1,78	2,14	2,49	2,85
2,4	1,000	0,59	0,75	0,362	0,482	22,5	1,48	1,85	2,21	2,58	2,95
2,17	1,000	0,62	0,74	0,371	0,499	23,4	1,53	1,91	2,30	2,68	3,06

EUROCODE 2
Background and Applications

Universal table vs. SLS deflection

$$\rho \leq \rho_0 \qquad \left(\frac{l}{d}\right)_0 = 11 + 1{,}5\sqrt{f_{ck}}\,\frac{\rho_0}{\rho} + 3{,}2\sqrt{f_{ck}}\sqrt{\left(\frac{\rho_0}{\rho} - 1\right)^3}$$

$$\rho > \rho_0 \qquad \left(\frac{l}{d}\right)_0 = 11 + 1{,}5\sqrt{f_{ck}}\,\frac{\rho_0}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}}\sqrt{\frac{\rho'}{\rho_0}}$$

Increasing μ_d the maximum allowed "slenderness" $(l/d)_0$ (so l_n /d as k, are given) <u>decreases:</u>

high bending M → high curvature → high deflection, → less slenderness-

				$(l/d)_0$			
μ_u	ω_0	C16/20	C20/25	C25/30	C30/37	C35/45	C40/50
0,039	0,040	67,8	88,6	90,2	84,5	79,8	75,9
0,040	0,041	67,8	92,5	85,6	80,1	75,7	71,9
0,050	0,052	67,8	65,0	59,9	55,9	52,7	49,9
0,060	0,062	52,9	48,9	44,9	41,9	39,4	37,4
0,070	0,073	41,9	38,6	35,5	33,1	31,2	29,7
0,080	0,084	34,3	31,7	29,2	27,4	26,0	24,9
0,090	0,095	29,1	26,9	25,0	23,5	22,5	21,8
0,100	0,106	25,2	23,5	22,0	21,0	20,4	20,2
0,103	0,109	24,5	22,8	21,4	20,5	20,0	20,0
0,110	0,117	22,4	21,0	19,9	19,4	19,3	19,3
0,120	0,129	20,3	19,3	18,6	18,6	18,6	18,6
0,130	0,140	18,8	18,1	18,0	18,0	18,0	18,0
0,140	0,152	17,7	17,4	17,4	17,4	17,4	17,4
0,150	0,164	17,0	17,0	17,0	17,0	17,0	17,0
0,160	0,176	16,6	16,6	16,6	16,6	16,6	16,6
0,170	0,189	16,2	16,2	16,2	16,2	16,2	16,2
0,180	0,201	15,9	15,9	15,9	15,9	15,9	15,9
0,187	**0,210**	15,7	15,7	15,7	15,7	15,7	15,7
0,190	0,214	15,6	15,6	15,6	15,6	15,6	15,6
0,200	0,227	15,3	15,3	15,3	15,3	15,3	15,3
0,210	0,240	15,1	15,1	15,1	15,1	15,1	15,1
0,220	0,253	14,9	14,9	14,9	14,9	14,9	14,9
0,230	**0,267**	14,7	14,7	14,7	14,7	14,7	14,7
0,240	0,281	14,5	14,5	14,5	14,5	14,5	14,5
0,250	0,295	14,3	14,3	14,3	14,3	14,3	14,3
0,260	0,309	14,2	14,2	14,2	14,2	14,2	14,2
0,270	0,324	14,0	14,0	14,0	14,0	14,0	14,0
0,280	0,339	13,9	13,9	13,9	13,9	13,9	13,9
0,290	0,355	13,8	13,8	13,8	13,8	13,8	13,8
0,296	**0,364**	**13,7**	**13,7**	**13,7**	**13,7**	**13,7**	**13,7**
0,302	0,374	13,6	13,6	13,6	13,6	13,6	13,6
0,309	0,385	13,5	13,5	13,5	13,5	13,5	13,5
0,316	0,397	13,5	13,5	13,5	13,5	13,5	13,5
0,323	0,409	13,4	13,4	13,4	13,4	13,4	13,4
0,330	0,421	13,3	13,3	13,3	13,3	13,3	13,3
0,338	0,435	13,2	13,2	13,2	13,2	13,2	13,2
0,346	0,449	13,2	13,2	13,2	13,2	13,2	13,2
0,354	0,465	13,1	13,1	13,1	13,1	13,1	13,1
0,362	0,482	13,0	13,0	13,0	13,0	13,0	13,0
0,371	**0,499**	13,0	13,0	13,0	13,0	13,0	13,0

EUROCODE 2

Background and Applications

Universal table vs. SLS stress lim.

$$\xi_e = \frac{x_e}{d} = \alpha_e \rho \left(\sqrt{1 + \frac{2}{\alpha_e \rho}} - 1 \right)$$

For each μ_d one ξ_u, many ξ_e (one for each concrete class) increasing with μ_d

$$\frac{\sigma_c}{f_{ck}} = \frac{\mu_k}{\frac{\xi_e}{2}\left(1 - \frac{\xi_e}{3}\right)} \qquad \frac{\sigma_s}{f_{yk}} = \frac{\mu_k}{\rho \frac{f_{yk}}{f_{ck}}\left(1 - \frac{\xi_e}{3}\right)}$$

$$\mu_k = \mu_d / k \qquad k > 1$$

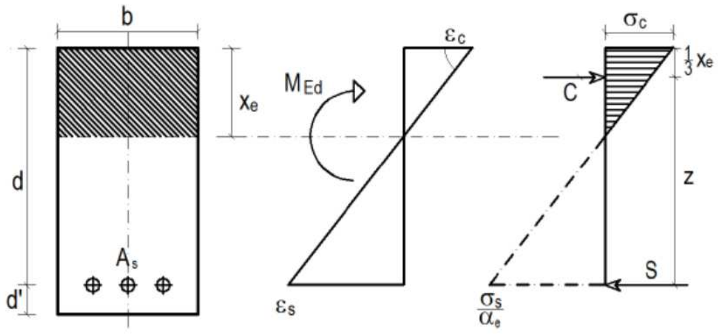
Increasing μ_d

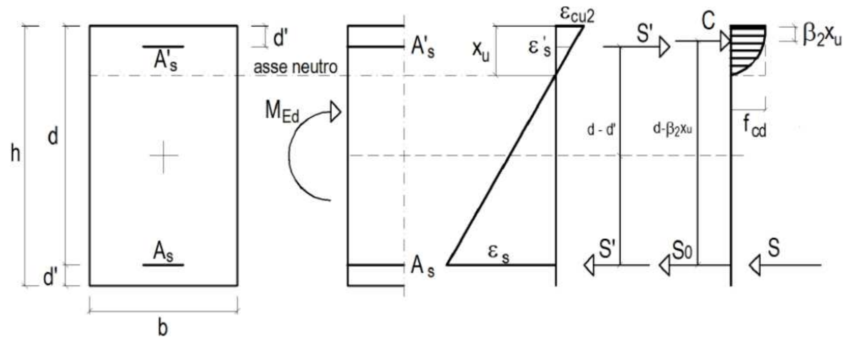
σ_c increases

σ_s decreases.

						$\xi_{e,QP}$				
ξ_u	μ_u	ω_0	C16/20	C20/25	C25/30	C30/37	C35/45	C40/50	σc/fck	$\sigma s/f_{yk}$
0,05	0,039	0,040	0,21	0,20	0,21	0,23	0,24	0,25	100%	100%
0,05	0,040	0,041	0,21	0,20	0,22	0,23	0,24	0,25	102%	100%
0,06	0,050	0,052	0,21	0,22	0,24	0,25	0,27	0,28	116%	100%
0,08	0,060	0,062	0,22	0,24	0,26	0,28	0,29	0,30	129%	100%
0,09	0,070	0,073	0,24	0,26	0,28	0,29	0,31	0,32	142%	101%
0,10	0,080	0,084	0,25	0,27	0,29	0,31	0,33	0,34	154%	101%
0,12	0,090	0,095	0,27	0,29	0,31	0,33	0,34	0,36	166%	101%
0,13	0,100	0,106	0,28	0,30	0,32	0,34	0,36	0,37	177%	100%
0,13	0,103	0,109	0,28	0,30	0,33	0,35	0,36	0,38	179%	100%
0,15	0,110	0,117	0,29	0,31	0,34	0,36	0,37	0,39	187%	100%
0,16	0,120	0,129	0,30	0,33	0,35	0,37	0,39	0,40	198%	100%
0,17	0,130	0,140	0,31	0,34	0,36	0,38	0,40	0,41	208%	100%
0,19	0,140	0,152	0,33	0,35	0,37	0,39	0,41	0,43	218%	100%
0,20	0,150	0,164	0,34	0,36	0,38	0,41	0,42	0,44	228%	100%
0,22	0,160	0,176	0,34	0,37	0,39	0,42	0,43	0,45	238%	99%
0,23	0,170	0,189	0,35	0,38	0,40	0,43	0,44	0,46	247%	99%
0,25	0,180	0,201	0,36	0,39	0,41	0,44	0,46	0,47	257%	99%
0,26	0,187	0,210	0,37	0,40	0,42	0,44	0,46	0,48	263%	98%
0,26	0,190	0,214	0,37	0,40	0,42	0,45	0,46	0,48	266%	98%
0,28	0,200	0,227	0,38	0,41	0,43	0,46	0,47	0,49	275%	98%
0,30	0,210	0,240	0,39	0,42	0,44	0,46	0,48	0,50	284%	97%
0,31	0,220	0,253	0,40	0,42	0,45	0,47	0,49	0,51	292%	97%
0,33	0,230	0,267	0,40	0,43	0,46	0,48	0,50	0,52	301%	97%
0,35	0,240	0,281	0,41	0,44	0,47	0,49	0,51	0,53	310%	96%
0,36	0,250	0,295	0,42	0,45	0,48	0,50	0,52	0,54	318%	96%
0,38	0,260	0,309	0,43	0,46	0,48	0,51	0,53	0,54	326%	95%
0,40	0,270	0,324	0,43	0,46	0,49	0,51	0,53	0,55	335%	94%
0,42	0,280	0,339	0,44	0,47	0,50	0,52	0,54	0,56	343%	94%
0,44	0,290	0,355	0,45	0,48	0,51	0,53	0,55	0,57	351%	93%
0,45	**0,296**	**0,364**	**0,45**	**0,48**	**0,51**	**0,53**	**0,55**	**0,57**	356%	93%
0,46	0,302	0,374	0,46	0,49	0,52	0,54	0,56	0,58	360%	92%
0,48	0,309	0,385	0,46	0,49	0,52	0,54	0,56	0,58	366%	92%
0,49	0,316	0,397	0,47	0,50	0,52	0,55	0,57	0,59	371%	91%
0,50	0,323	0,409	0,47	0,50	0,53	0,55	0,57	0,59	376%	91%
0,52	0,330	0,421	0,48	0,51	0,54	0,56	0,58	0,60	382%	90%
0,54	0,338	0,435	0,48	0,51	0,54	0,56	0,58	0,60	388%	90%
0,56	0,346	0,449	0,49	0,52	0,55	0,57	0,59	0,61	394%	89%
0,57	0,354	0,465	0,49	0,52	0,55	0,58	0,60	0,61	400%	88%
0,59	0,362	0,482	0,50	0,53	0,56	0,58	0,60	0,62	406%	88%
0,62	0,371	0,499	0,50	0,53	0,56	0,59	0,61	0,63	412%	87%
			C15/20	C20/25	C25/30	C28/35	C32/40	C35/45	σc/fck	$\sigma s/f_{yk}$

Stress increase/decrease





$$M_{Ek} = \frac{\sigma_c \, x_e}{2} b \left(d - \frac{x_e}{3} \right) \qquad M_{Ek} = \sigma_s \, A_s \left(d - \frac{x_e}{3} \right)$$

$$M_{Ed} = 0,810 \, b \, x_u f_{cd} \left(d - 0,416 x_u \right) \qquad M_{Ed} = A_s \, f_{yd} \left(d - 0,416 x_u \right)$$

$$\frac{\sigma_c}{f_{ck}} = 1,08 \cdot \frac{x_u}{x_e} \cdot \left(\frac{d - 0,416\, x_u}{d - 0,333 x_e} \right) \cdot \frac{M_{Ek}}{M_{Ed}} = 1,08 \frac{\xi_U}{\xi_E} \cdot \left(\frac{1 - 0,416\, \xi_U}{1 - 0,333\, \xi_E} \right) \cdot \frac{M_{Ek}}{M_{Ed}} = k_{\sigma c} \cdot \frac{M_{Ek}}{M_{Ed}}$$

$$\frac{\sigma_s}{f_{yk}} = \frac{1}{1,15} \left(\frac{d - 0,416 x_u}{d - 0,333 x_e} \right) \cdot \frac{M_{Ek}}{M_{Ed}} = 0,87 \left(\frac{1 - 0,416\, \xi_u}{1 - 0,333 \xi_e} \right) \cdot \frac{M_{Ek}}{M_{Ed}} = k_{\sigma s} \cdot \frac{M_{Ek}}{M_{Ed}}$$

EUROCODE 2

Background andApplications

$$\frac{\sigma_c}{f_{ck}} = 1{,}08\frac{\xi_U}{\xi_E}\cdot\left(\frac{1-0{,}416\,\xi_U}{1-0{,}333\,\xi_E}\right)\cdot\frac{M_{Ek}}{M_{Ed}} = k_{\sigma c}\cdot\frac{M_{Ek}}{M_{Ed}}$$

$$\frac{\sigma_s}{f_{yk}} = 0{,}87\left(\frac{1-0{,}416\,\xi_u}{1-0{,}333\,\xi_e}\right)\cdot\frac{M_{Ek}}{M_{Ed}} = k_{\sigma s}\cdot\frac{M_{Ek}}{M_{Ed}}$$

For each μ_d a single ξ_u but one ξ_e for each concrete class

one $k_{\sigma c}$ increasing with μ_d
one $k_{\sigma s}$ decreasing with μ_d
for each concrete class

	C20/25		C25/30		C30/37		C35/45		C40/50	
μ_u	$k_{\sigma c}$	$k_{\sigma s}$	$k_{\sigma c}$	$k_{\sigma s}$	$k_{\sigma c}$	$k_{\sigma s}$	$k_{\sigma c}$	$k_{\sigma s}$	$k_{\sigma c}$	$k_{\sigma s}$
0,006	0,04	0,93	0,04	0,93	0,04	0,93	0,04	0,93	0,04	0,94
0,010	0,07	0,93	0,07	0,93	0,07	0,93	0,07	0,93	0,07	0,93
0,020	0,14	0,92	0,14	0,92	0,14	0,92	0,14	0,93	0,13	0,93
0,030	0,21	0,92	0,21	0,92	0,20	0,92	0,20	0,92	0,19	0,92
0,039	0,27	0,91	0,26	0,92	0,25	0,92	0,24	0,93	0,23	0,93
0,040	0,29	0,91	0,27	0,92	0,25	0,92	0,24	0,93	0,23	0,93
0,050	0,33	0,91	0,30	0,92	0,29	0,92	0,28	0,93	0,27	0,93
0,060	0,36	0,92	0,34	0,92	0,32	0,93	0,31	0,93	0,30	0,94
0,070	0,40	0,92	0,37	0,92	0,35	0,93	0,34	0,93	0,33	0,94
0,080	0,43	0,92	0,40	0,92	0,38	0,93	0,37	0,93	0,36	0,94
0,090	0,46	0,92	0,43	0,92	0,41	0,93	0,40	0,93	0,38	0,94
0,100	0,49	0,91	0,46	0,92	0,44	0,93	0,42	0,93	0,41	0,94
0,103	0,50	0,91	0,47	0,92	0,45	0,93	0,43	0,93	0,42	0,94
0,110	0,52	0,91	0,49	0,92	0,47	0,93	0,45	0,93	0,44	0,94
0,120	0,55	0,91	0,52	0,92	0,50	0,93	0,48	0,93	0,46	0,94
0,130	0,58	0,91	0,55	0,92	0,52	0,92	0,50	0,93	0,49	0,94
0,140	0,61	0,91	0,57	0,92	0,55	0,92	0,53	0,93	0,51	0,93
0,150	0,63	0,90	0,60	0,91	0,57	0,92	0,55	0,93	0,54	0,93
0,160	0,66	0,90	0,62	0,91	0,60	0,92	0,58	0,92	0,56	0,93
0,170	0,69	0,90	0,65	0,91	0,62	0,92	0,60	0,92	0,58	0,93
0,180	0,71	0,90	0,67	0,90	0,64	0,91	0,62	0,92	0,61	0,93
0,187	0,73	0,89	0,69	0,90	0,66	0,91	0,64	0,92	0,62	0,92
0,190	0,74	0,89	0,70	0,90	0,67	0,91	0,65	0,92	0,63	0,92
0,200	0,76	0,89	0,72	0,90	0,69	0,91	0,67	0,91	0,65	0,92
0,210	0,78	0,88	0,74	0,89	0,71	0,90	0,69	0,91	0,67	0,92
0,220	0,81	0,88	0,77	0,89	0,74	0,90	0,71	0,91	0,69	0,91
0,230	0,83	0,88	0,79	0,89	0,76	0,89	0,74	0,90	0,72	0,91
0,240	0,85	0,87	0,81	0,88	0,78	0,89	0,76	0,90	0,74	0,90
0,250	0,88	0,87	0,83	0,88	0,80	0,88	0,78	0,89	0,76	0,90
0,260	0,90	0,86	0,86	0,87	0,82	0,88	0,80	0,89	0,78	0,89
0,270	0,92	0,86	0,88	0,87	0,85	0,87	0,82	0,88	0,80	0,89
0,280	0,94	0,85	0,90	0,86	0,87	0,87	0,84	0,88	0,82	0,88
0,290	0,96	0,85	0,92	0,86	0,89	0,86	0,86	0,87	0,84	0,88
0,296	**0,98**	**0,84**	**0,93**	**0,85**	**0,90**	**0,86**	**0,87**	**0,87**	**0,85**	**0,87**
0,302	0,99	0,84	0,95	0,85	0,91	0,86	0,89	0,86	0,87	0,87
0,309	1,00	0,83	0,96	0,84	0,93	0,85	0,90	0,86	0,88	0,86
0,316	1,02	0,83	0,97	0,84	0,94	0,85	0,91	0,85	0,89	0,86
0,323	1,03	0,82	0,99	0,83	0,95	0,84	0,93	0,85	0,91	0,86
0,330	1,05	0,82	1,00	0,83	0,97	0,84	0,94	0,84	0,92	0,85
0,338	1,06	0,81	1,02	0,82	0,98	0,83	0,96	0,84	0,94	0,84
0,346	1,08	0,81	1,03	0,82	1,00	0,83	0,97	0,83	0,95	0,84
0,354	1,09	0,80	1,05	0,81	1,01	0,82	0,99	0,83	0,97	0,83
0,362	1,11	0,79	1,06	0,80	1,03	0,81	1,00	0,82	0,98	0,82
0,371	1,13	0,79	1,08	0,80	1,05	0,80	1,02	0,81	1,00	0,82

M_{Ek}/M_{Ed} range of values

In case of linear elastic analysis

$$\frac{M_{Ek}}{M_{Ed}} = \frac{(G+\psi\, Q_k)}{(1{,}35\, G + 1{,}50\, Q_k)}\frac{(l^2_{eff}\,/\,k)}{(l^2_{eff}\,/\,k)} = \frac{1+\psi\dfrac{Q_k}{G}}{1{,}35 + 1{,}50\,\dfrac{Q_k}{G}}$$

$\psi = \psi_2$ for QP $\qquad \psi = 1$ for CA

		γ_g =	1,35		γ_Q =	1,50							
						M_{Ek}/MEd for Qk/Gk							
LC	ψ	0,05	0,1	0,2	0,33	0,4	0,5	0,75	1	1,5	2	4	10
QP	0	0,70	0,67	0,61	0,54	0,51	0,48	0,40	0,35	0,28	0,23	0,14	0,06
	0,2	0,71	0,68	0,63	0,58	0,55	0,52	0,46	0,42	0,36	0,32	0,24	0,18
	0,3	0,71	0,69	0,64	0,60	0,57	0,55	0,49	0,46	0,40	0,37	0,30	0,24
	0,6	0,72	0,71	0,68	0,65	0,64	0,62	0,59	0,56	0,53	0,51	0,46	0,43
	0,8	0,73	0,72	0,70	0,69	0,68	0,67	0,65	0,63	0,61	0,60	0,57	0,55
CA	1	0,74	0,73	0,73	0,72	0,72	0,71	0,71	0,70	0,69	0,69	0,68	0,67
ULS		**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**

$M_{Ek,QP}/M_{Ed}$ large variation $f(Q_k/G)$, max 0,73

$M_{Ek,CA}/M_{Ed}$ limited variation around 0,70

SLS - $M_{ek,QP}/M_{ed}$ vs. μ_d

QP comb.

$$\frac{\sigma_c}{f_{ck}} = k_{\sigma c} \cdot \frac{M_{Ek,QP}}{M_{Ed}} = 0{,}45$$

$$\Downarrow$$

$$\frac{M_{Ek,QP}}{M_{Ed}} = \frac{0{,}45}{k_{\sigma c}}$$

Only

$M_{ek,QP}/M_{ed} \leqq 0{,}73$

are possible!

Use the table for the choice of a suitable μ_d!

	M_{EkQP}/M_{Ed} and $(l/d)_0$ for concrete class									
	C20/25		C25/30		C30/37		C35/45		C40/50	
μ_u	M_{EkQP}/M_{Ed}	$(l/d)_0$	M_{EkQP}/M_{Ed}	$(l/d)_0$	M_{EkQP}/M_{Ed}	$(l/d)_0$	M_{EkQP}/M_{Ed}	$(l/d)_0$	M_{EkQP}/M_{Ed}	$(l/d)_0$
0,150	0,71	17,0								
0,160	0,68	16,6	0,72	16,6						
0,170	0,66	16,2	0,69	16,2	0,72	16,2				
0,180	0,63	15,9	0,67	15,9	0,70	15,9	0,72			
0,187	0,62	15,7	0,65	15,7	0,68	15,7	0,70	15,7	0,72	15,7
0,190	0,61	15,6	0,65	15,6	0,67	15,6	0,70	15,6	0,72	15,6
0,200	0,59	15,3	0,62	15,3	0,65	15,3	0,67	15,3	0,69	15,3
0,210	0,57	15,1	0,60	15,1	0,63	15,1	0,65	15,1	0,67	15,1
0,220	0,56	14,9	0,59	14,9	0,61	14,9	0,63	14,9	0,65	14,9
0,230	0,54	14,7	0,57	14,7	0,59	14,7	0,61	14,7	0,63	14,7
0,240	0,53	14,5	0,55	14,5	0,58	14,5	0,59	14,5	0,61	14,5
0,250	0,51	14,3	0,54	14,3	0,56	14,3	0,58	14,3	0,59	14,3
0,260	0,50	14,2	0,53	14,2	0,55	14,2	0,56	14,2	0,58	14,2
0,270	0,49	14,0	0,51	14,0	0,53	14,0	0,55	14,0	0,56	14,0
0,280	0,48	13,9	0,50	13,9	0,52	13,9	0,53	13,9	0,55	13,9
0,290	0,47	13,8	0,49	13,8	0,51	13,8	0,52	13,8	0,53	13,8
0,296	0,46	13,7	0,48	13,7	0,50	13,7	0,51	13,7	0,53	13,7
0,302	0,45	13,6	0,48	13,6	0,49	13,6	0,51	13,6	0,52	13,6
0,309	0,45	13,5	0,47	13,5	0,49	13,5	0,50	13,5	0,51	13,5
0,316	0,44	13,5	0,46	13,5	0,48	13,5	0,49	13,5	0,50	13,5
0,323	0,44	13,4	0,46	13,4	0,47	13,4	0,48	13,4	0,50	13,4
0,330	0,43	13,3	0,45	13,3	0,46	13,3	0,48	13,3	0,49	13,3
0,338	0,42	13,2	0,44	13,2	0,46	13,2	0,47	13,2	0,48	13,2
0,346	0,42	13,2	0,44	13,2	0,45	13,2	0,46	13,2	0,47	13,2
0,354	0,41	13,1	0,43	13,1	0,44	13,1	0,46	13,1	0,46	13,1
0,362	0,41	13,0	0,42	13,0	0,44	13,0	0,45	13,0	0,46	13,0
0,371	0,40	13,0	0,42	13,0	0,43	13,0	0,44	13,0	0,45	13,0

Expanded ULS universal table

						$M_{EkQP}/M_{ed,}$ ρ and $(l/d)_0$ for concrete class														
ε [‰]	ξ_u	ζ_u	μ_u	ω_0	δ'_{lim} %	C20/25			C25/30			C30/37			C35/45			C40/50		
						M_{EkQP}/M_{Ed}	ρ [%]	$(l/d)_0$	M_{EkQP}/M_{Ed}	ρ [%]	$(l/d)_0$	M_{EkQP}/M_{Ed}	ρ [%]	$(l/d)_0$	M_{EkQP}/M_{Ed}	ρ [%]	$(l/d)_0$	M_{EkQP}/M_{Ed}	ρ [%]	$(l/d)_0$
13,8	0,20	0,92	0,150	0,164	7,7	0,71	0,50	17,0												
12,6	0,22	0,91	0,160	0,176	8,2	0,68	0,54	16,6	0,72	0,68	16,6									
11,5	0,23	0,90	0,170	0,189	8,8	0,66	0,58	16,2	0,69	0,72	16,2	0,72	0,87	16,2						
10,6	0,25	0,90	0,180	0,201	9,4	0,63	0,62	15,9	0,67	0,77	15,9	0,70	0,92	15,9	0,72	1,08				
10,0	0,26	0,89	0,187	0,210	9,8	0,62	0,64	15,7	0,65	0,81	15,7	0,68	0,97	15,7	0,70	1,13	15,7	0,72	1,29	15,7
9,8	0,26	0,89	0,190	0,214	10,0	0,61	0,66	15,6	0,65	0,82	15,6	0,67	0,98	15,6	0,70	1,15	15,6	0,72	1,31	15,6
9,0	0,28	0,88	0,200	0,227	10,6	0,59	0,69	15,3	0,62	0,87	15,3	0,65	1,04	15,3	0,67	1,22	15,3	0,69	1,39	15,3
8,3	0,30	0,88	0,210	0,240	11,2	0,57	0,74	15,1	0,60	0,92	15,1	0,63	1,10	15,1	0,65	1,29	15,1	0,67	1,47	15,1
7,7	0,31	0,87	0,220	0,253	11,8	0,56	0,78	14,9	0,59	0,97	14,9	0,61	1,16	14,9	0,63	1,36	14,9	0,65	1,55	14,9
7,1	0,33	0,86	0,230	0,267	12,5	0,54	0,82	14,7	0,57	1,02	14,7	0,59	1,23	14,7	0,61	1,43	14,7	0,63	1,64	14,7
6,6	0,35	0,86	0,240	0,281	13,1	0,53	0,86	14,5	0,55	1,08	14,5	0,58	1,29	14,5	0,59	1,51	14,5	0,61	1,72	14,5
6,1	0,36	0,85	0,250	0,295	13,8	0,51	0,90	14,3	0,54	1,13	14,3	0,56	1,36	14,3	0,58	1,58	14,3	0,59	1,81	14,3
5,7	0,38	0,84	0,260	0,309	14,5	0,50	0,95	14,2	0,53	1,19	14,2	0,55	1,42	14,2	0,56	1,66	14,2	0,58	1,90	14,2
5,2	0,40	0,83	0,270	0,324	15,2	0,49	0,99	14,0	0,51	1,24	14,0	0,53	1,49	14,0	0,55	1,74	14,0	0,56	1,99	14,0
4,8	0,42	0,83	0,280	0,339	15,9	0,48	1,04	13,9	0,50	1,30	13,9	0,52	1,56	13,9	0,53	1,82	13,9	0,55	2,08	13,9
4,5	0,44	0,82	0,290	0,355	16,6	0,47	1,09	13,8	0,49	1,36	13,8	0,51	1,63	13,8	0,52	1,90	13,8	0,53	2,18	13,8
4,3	0,45	0,81	0,296	0,364	17,1	0,46	1,12	13,7	0,48	1,40	13,7	0,50	1,68	13,7	0,51	1,96	13,7	0,53	2,23	13,7
4,1	0,46	0,81	0,302	0,374	17,5	0,45	1,15	13,6	0,48	1,44	13,6	0,49	1,72	13,6	0,51	2,01	13,6	0,52	2,30	13,6
3,9	0,48	0,80	0,309	0,385	18,0	0,45	1,18	13,5	0,47	1,48	13,5	0,49	1,77	13,5	0,50	2,07	13,5	0,51	2,36	13,5
3,6	0,49	0,80	0,316	0,397	18,6	0,44	1,22	13,5	0,46	1,52	13,5	0,48	1,82	13,5	0,49	2,13	13,5	0,50	2,43	13,5
3,4	0,50	0,79	0,323	0,409	19,1	0,44	1,25	13,4	0,46	1,57	13,4	0,47	1,88	13,4	0,48	2,19	13,4	0,50	2,51	13,4
3,2	0,52	0,78	0,330	0,421	19,7	0,43	1,29	13,3	0,45	1,61	13,3	0,46	1,94	13,3	0,48	2,26	13,3	0,49	2,58	13,3
3,0	0,54	0,78	0,338	0,435	20,4	0,42	1,33	13,2	0,44	1,67	13,2	0,46	2,00	13,2	0,47	2,33	13,2	0,48	2,67	13,2
2,8	0,56	0,77	0,346	0,449	21,0	0,42	1,38	13,2	0,44	1,72	13,2	0,45	2,07	13,2	0,46	2,41	13,2	0,47	2,76	13,2
2,6	0,57	0,76	0,354	0,465	21,8	0,41	1,43	13,1	0,43	1,78	13,1	0,44	2,14	13,1	0,46	2,49	13,1	0,46	2,85	13,1
2,4	0,59	0,75	0,362	0,482	22,5	0,41	1,48	13,0	0,42	1,85	13,0	0,44	2,21	13,0	0,45	2,58	13,0	0,46	2,95	13,0
2,2	0,62	0,74	0,371	0,499	23,4	0,40	1,53	13,0	0,42	1,91	13,0	0,43	2,30	13,0	0,44	2,68	13,0	0,45	3,06	13,0

Fast design – verification of single and double reinforced beams

Cont. beam - ULS section design

For each beam of a continuous beam

1) Calculate G and Q_k on the basis of tributary area
2) Estimate $M_{Ek,QP}$ and M_{Ed} on the basis of l_n
3) Enter design table with $M_{Ek,QP}/M_{Ed}$ for the selected concrete class
4) Identify $\mu_{d,i}$ and $(l/d)_0$: adopt $\mu_{d,i}$ =min ($\mu_{d,i}$; 0,296)
5) Identify the " geometry leading" beam by calculating

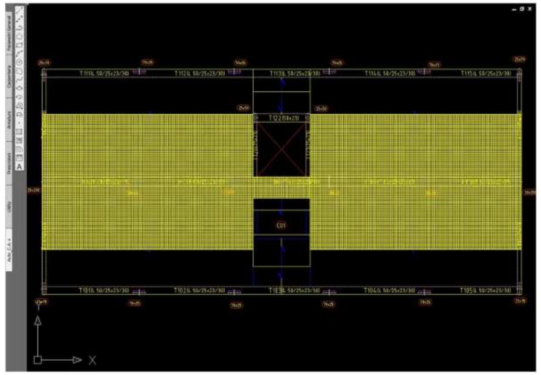
$$\mu_d = \frac{M_{Ed}}{b\,d^2 f_{cd}}$$

$$\Downarrow$$

$$\max\,(b\,d^2) = \frac{1}{f_{cd}} \max\left(\frac{M_{Ed,i}}{\mu_{d,i}}\right)$$

Axis A and B beams

f_{ck}	γ_c	f_{cd}	f_{yk}	γ_c	f_{yd}	c_{nom}	ϕ_{st}	ϕ	d'	ψ_2
N/mm²		N/mm²	N/mm²		N/mm²	mm	mm	mm	mm	-
25	1,5	16,7	500	1,15	435	30	8	16	46	0,30



Beam	l_{eff}	K	l_{ta}	l_n	l_0	b_w	b_{eff}	g_1	g_2	q_k	$G=(g_1+g_2)\cdot l_{ta}$	$Q=q_k\cdot l_c$	$G+\psi_2Q$	1,3G+1,5Q	$M_{EK,QP}$	M_{Ed}	$M_{EK,QP}/M_{Ed}$
	[m]	-	[m]	[m]	[m]	[m]	[m]	[kN/m²]	[kN/m²]	[kN/m²]	[kN/m]	[kN/m]	[kN/m]	[kN/m]	[kNm]	[kNm]	-
B1-B2	6	1,3	8,91	4,62	5,1	0,25	1,02	3,08	3	2	54,2	17,8	59,5	97,2	**158,5**	**258,7**	0,61
B2-B3	6	1,5	8,91	4,00	4,2	0,25	0,84	3,08	3	2	54,2	17,8	59,5	97,2	**119,0**	**194,3**	0,61
B3-B4	6	1,5	3,42	4,00	4,2	0,25	0,84	3,42	3	4	22,0	13,7	26,1	49,1	**52,1**	**98,1**	0,53
B4-B5	6	1,5	8,91	4,00	4,2	0,25	0,84	3,08	3	2	54,2	17,8	59,5	97,2	**119,0**	**194,3**	0,61
B5-B6	6	1,3	8,91	4,62	5,1	0,25	1,02	3,08	3	2	54,2	17,8	59,5	97,2	**158,5**	**258,7**	0,61

Beam	l_{eff}	K	l_{ta}	l_n	l_0	b_w	b_{eff}	g_1	g_2	q_k	$G=(g_1+g_2)\cdot l_{ta}$	$Q=q_k\cdot l_c$	$G+\psi_2Q$	1,3G+1,5Q	$M_{EK,QP}$	M_{Ed}	$M_{EK,QP}/M_{Ed}$
	[m]	-	[m]	[m]	[m]	[m]	[m]	[kN/m²]	[kN/m²]	[kN/m²	[kN/m]	[kN/m]	[kN/m]	[kN/m]	[kNm]	[kNm]	-
A1-A2	6	1,3	2,75	4,62	5,1	0,25	0,76	3,08	5,91	2	24,7	5,5	26,4	40,4	**70,2**	**107,5**	0,65
A2-A3	6	1,5	2,75	4,00	4,2	0,25	0,67	3,08	5,91	2	24,7	5,5	26,4	40,4	**52,7**	**80,8**	0,65
A3-A4	6	1,5	1,89	4,00	4,2	0,25	0,67	3,08	7,23	2	19,5	3,8	20,6	31,0	**41,3**	**62,0**	0,67
A4-A5	6	1,5	2,75	4,00	4,2	0,25	0,67	3,08	5,91	2	24,7	5,5	26,4	40,4	**52,7**	**80,8**	0,65
A5-A6	6	1,3	2,75	4,62	5,1	0,25	0,76	3,08	5,91	2	24,7	5,5	26,4	40,4	**70,2**	**107,5**	0,65

Axis B and A beams - cont.d

from design table

Beam	$M_{EK,QP}/M_{Ed}$	$(l/d)_0$	s	d_{min}	μ_d	$M_{Ed}/(\mu_d f_{cd})$	b_{min}	b_{fin}	d	h	h_{fin}	d_{fin}	μ_d
	-			[m]			[m]	[m]	[m]	[m]	[m]	[m]	
B1-B2	0,61	14,9	1,0	0,31	0,220	71	0,74	0,60	0,34	0,39	0,40	0,35	0,206
B2-B3	0,61	14,9	1,0	0,27	0,220	53	0,74	0,60	0,30	0,34	0,40	0,35	0,155
B3-B4	0,53	14,0	1,0	0,29	0,270	22	0,27	0,60	0,19	0,24	0,40	0,35	0,078
B4-B5	0,61	14,9	1,0	0,27	0,220	53	0,74	0,60	0,30	0,34	0,40	0,35	0,155
B5-B6	0,61	14,9	1,0	0,31	0,220	71	0,74	0,60	0,34	0,39	0,40	0,35	0,206

Beam	$M_{EK,QP}/M_{Ed}$	$(l/d)_0$	s	d_{min}	μ_d	$M_{Ed}/(\mu_d f_{cd})$	b_{min}	b_{fin}	d	h	h_{fin}	d_{fin}	μ_d
	-			[m]			[m]	[m]	[m]	[m]	[m]	[m]	
A1-A2	0,65	15,3	1,0	0,30	0,200	32	0,35	0,50	0,25	0,30	0,40	0,35	0,103
A2-A3	0,65	15,3	1,0	0,26	0,200	24	0,35	0,50	0,22	0,27	0,40	0,35	0,077
A3-A4	0,67	15,6	1,0	0,26	0,190	20	0,30	0,50	0,20	0,24	0,40	0,35	0,059
A4-A5	0,65	15,3	1,0	0,26	0,200	24	0,35	0,50	0,22	0,27	0,40	0,35	0,077
A5-A6	0,65	15,3	1,0	0,30	0,200	32	0,35	0,50	0,25	0,30	0,40	0,35	0,103

$$d_{min} = \frac{l_n}{s\ (l/d)_o} \qquad b_{min} = \left(\frac{M_{Ed,i}}{\mu_{d,i} f_{cd}}\right)\frac{1}{d^2_{min}} \qquad \mu_{d,i} = \text{const} \Rightarrow b_{min}\, d^2_{min} = b_{fin}\, d^2$$

Axis B and A beams – cont.d

Beam	M_{Ed}	$M_{EK,QP}/M_{Ed}$	$(l/d)_0$	s	μ_d	$M_{Ed}/(\mu_d f_{cd})$	b_{fin}	h_{fin}	d_{fin}	μ_d	ω	ρ	A_s	$(l/d)_0$	$k_{\sigma c}$	σ_c/f_{ck}
	[kNm]	-					[m]	[m]	[m]				mm^2			
B1-B2	**258,7**	0,61	14,9	1,0	0,220	71	0,60	0,40	0,35	**0,206**	0,235	0,90%	1910	15,2	0,73	0,45
B2-B3	**194,3**	0,61	14,9	1,0	0,220	53	0,60	0,40	0,35	**0,155**	0,170	0,65%	1382	16,8	0,61	0,37
B3-B4	**98,1**	0,53	14,0	1,0	0,270	22	0,60	0,40	0,35	**0,078**	0,082	0,31%	664	23,0	0,40	0,21
B4-B5	**194,3**	0,61	14,9	1,0	0,220	53	0,60	0,40	0,35	**0,155**	0,170	0,65%	1382	16,8	0,61	0,37
B5-B6	**258,7**	0,61	14,9	1,0	0,220	71	0,60	0,40	0,35	**0,206**	0,235	0,90%	1910	15,2	0,73	0,45

Beam	M_{Ed}	$M_{EK,QP}/M_{Ed}$	$(l/d)_0$	s	μ_d	$M_{Ed}/(\mu_d f_{cd}$	b_{fin}	h_{fin}	d_{fin}	μ_d	ω	ρ	A_s	$(l/d)_0$	$k_{\sigma c}$	σ_c/f_{ck}
	[kNm]	-					[m]	[m]	[m]				mm^2			
A1-A2	**107,5**	0,65	15,3	1,0	0,200	32	0,50	0,40	0,35	**0,103**	0,109	0,42%	739	20,0	0,47	0,31
A2-A3	**80,8**	0,65	15,3	1,0	0,200	24	0,50	0,40	0,35	**0,077**	0,081	0,31%	546	23,1	0,41	0,27
A3-A4	**62,0**	0,67	15,6	1,0	0,190	20	0,50	0,40	0,35	**0,059**	0,061	0,23%	415	27,0	0,34	0,23
A4-A5	**80,8**	0,65	15,3	1,0	0,200	24	0,50	0,40	0,35	**0,077**	0,081	0,31%	546	23,1	0,41	0,27
A5-A6	**107,5**	0,65	15,3	1,0	0,200	32	0,50	0,40	0,35	**0,103**	0,109	0,42%	739	20,0	0,47	0,31

"Green light" everywhere

If $(l/d)_0$ is not verified: take account of steel in compression

Conclusions

Simple method (one table), consistent and coherent driving engineers to comprehensive evaluation of section geometry by proper choice of SLU design parameters while taking into account relevant SLS.

No wasted time, no “trial and error” approach.

Easy to be implemented in spreadsheets and computer programs.

Conceptual design – Beams

Thanks for your attention!

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EC2 worked example

Conceptual design

Columns

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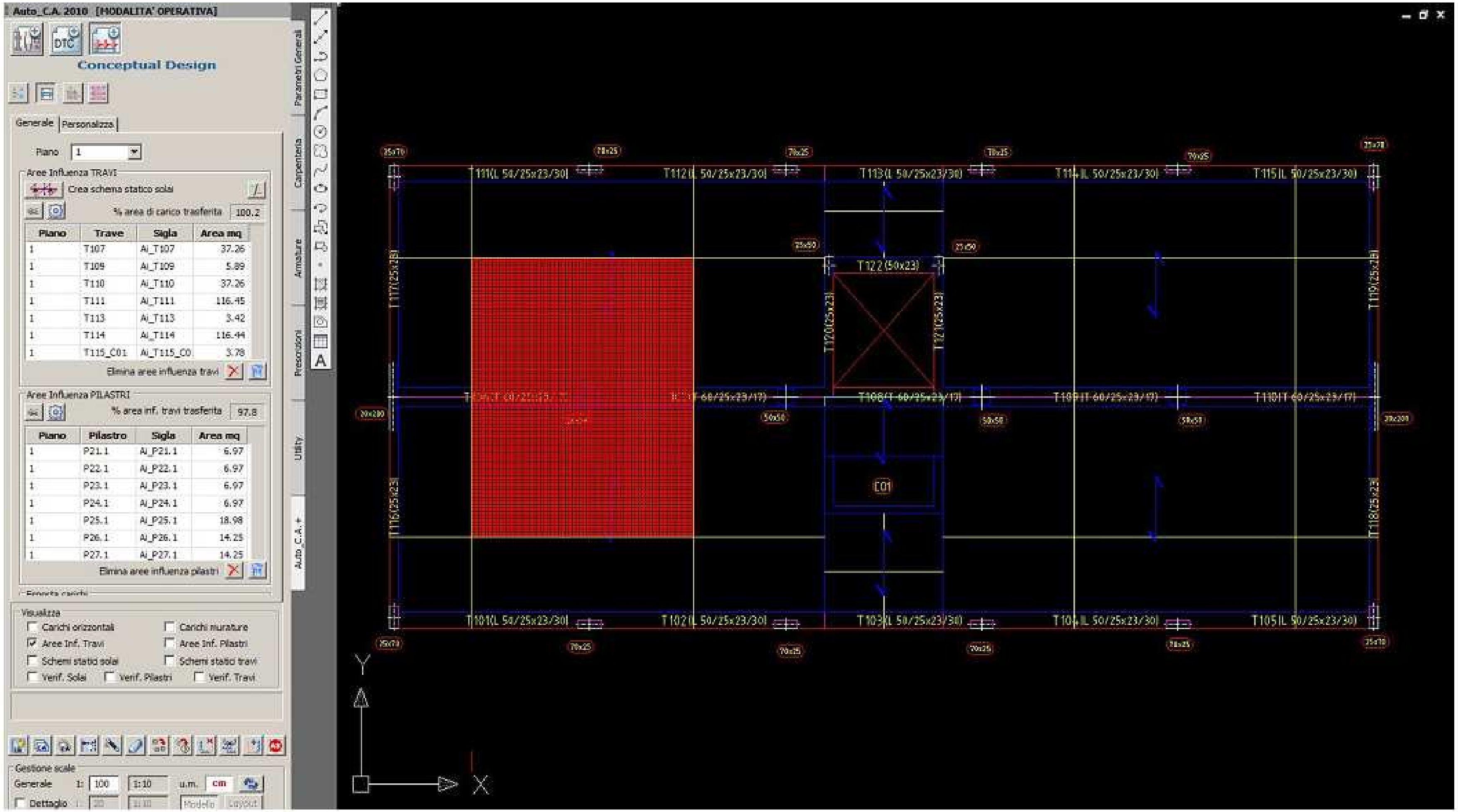
Column B2 tributary area by Auto-ca, for Autocad™

www.auto-ca.it



Objective: define column area and (minimum) size

Global 2nd order effects

EC2.5.8.3.3 In buildings with sufficiente torsional stiffness (defined later) 2nd order global effects may be ignored if :

$$F_{V,Ed} \leq k_1 \frac{n_s}{n_s + 1{,}6} \frac{\sum E_{cd} I_c}{L^2}$$

n_s = number of (real of "equivalent") storeys free of moving

$F_{V,ed}$ = total weight of these storeys, increasing of the same amount per storey : $F_{V,ed} \approx n_s A_s (1{,}3G+1{,}5Q_k)$

K_1 = 0,31(cracked) 0,62 (uncracked) sections at ULS

I_c = inertia of bracing members (uncracked concrete section)

$E_{cd} = E_{cm}/1{,}20$ elasticity modulus of (vertical) bracing elements

Global second effects design formula

$$\Sigma I_c \geq \frac{3{,}87}{E_{cm}}(1{,}3\,G + 1{,}5\,Q_k)A_s(n_s + 1{,}6)L^2$$

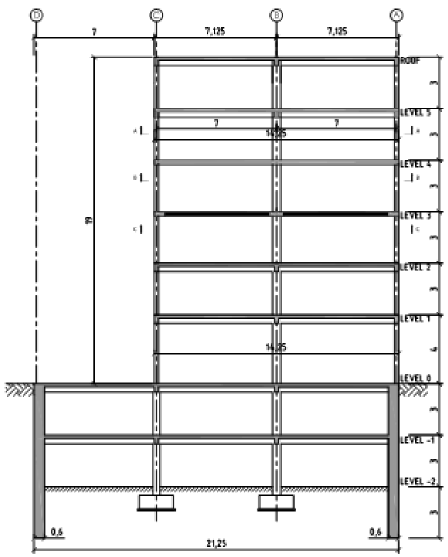
3,87 = 1,20/0,31; if uncracked, use 3,87/2 = 1,94

Units: L [m] As [m^2] I_c [m^4]

G, Q_k [kN/m^2] E_{cm} [kN/m^2] = 10^3 E_{cm} [N/mm^2]

Example : flat slab h = 0,24 cm

SECTION 1



$n_s = 6$ L = 19 m A_s = 30x 14,25 = 427,5 m^2

G = 0,24x25+3,0+ 8x2x(30+14,25)/427,5=10,66 kN/m^2

Q = (5x2+0x1,7)/6 = 1,66 kN/m^2 snow $\psi_2 = 0$

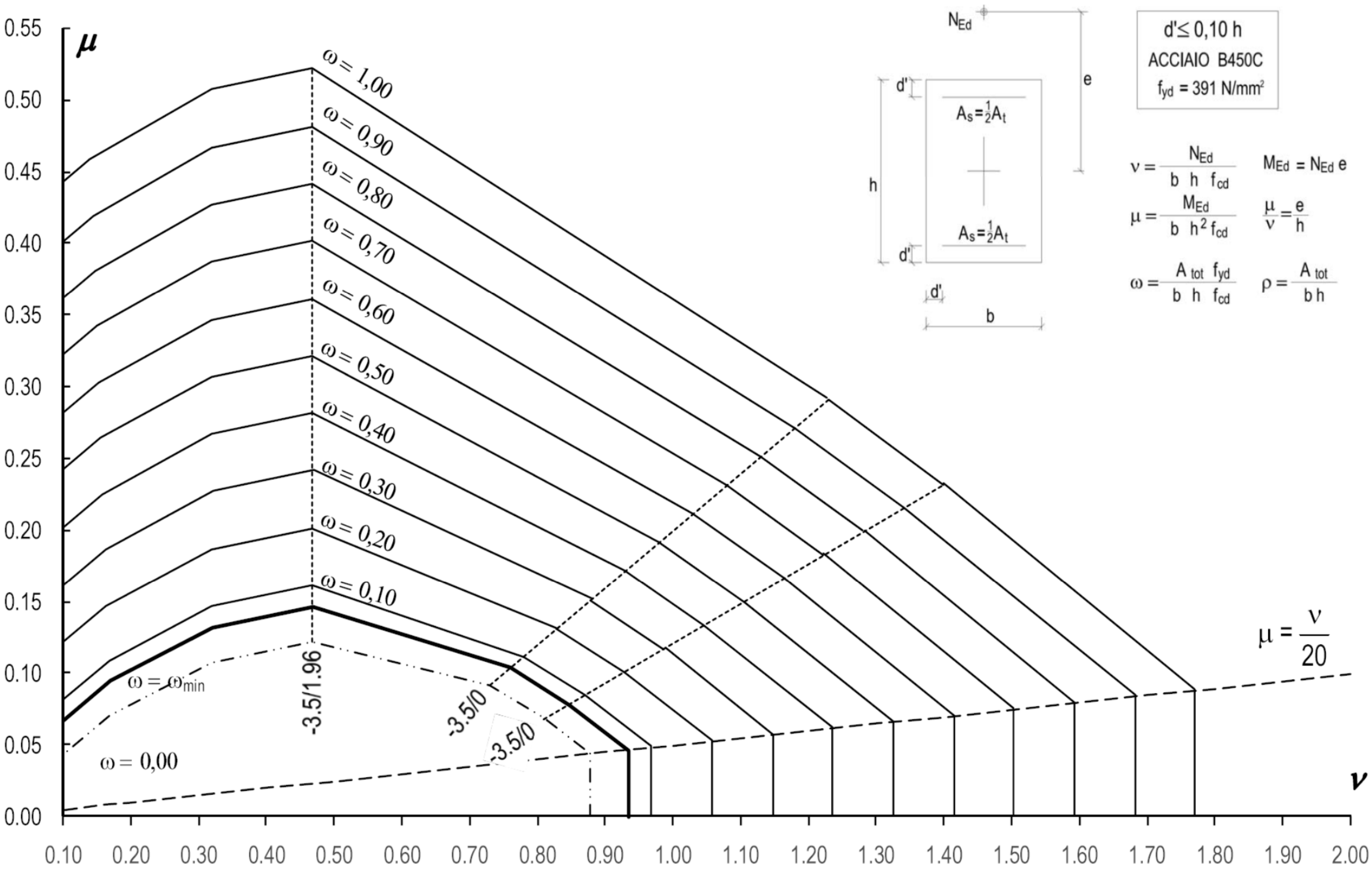
E_{cm} (C30/37) = 33 x 10^6 kN/m^2

$$\Sigma I_c \geq \frac{3{,}87}{33\text{x}10^6}(1{,}3\text{x}10{,}66 + 1{,}5\ \text{x}1.66)427{,}5(6+1{,}6)19^2 = 2{,}25\ m^4$$

$$I_x = \frac{1}{12}(1{,}8\text{x}3{,}6^3 - 1{,}6\text{x}3{,}2^3) = 2{,}62 > 2{,}25\ m^4 \quad \text{OK}$$

$$I_y = \frac{2}{12}(0{,}2\text{x}2^3) + 0{,}413 = 0{,}68 < 2{,}25\ m^4 \quad \text{NO}$$

(ν,μ) interaction diagram



Single B2 column design

G_1 = 0,24x25= 6 kN/m^2 G_2 = 3 kN/m^2 (dwellings + office)
Q_k = 3,0 (dwel); 4,0 (offi); 2,5 (park);1,7 (snow) kN/m^2

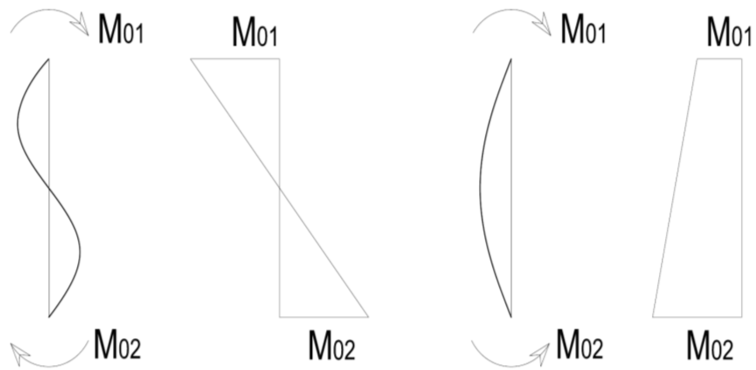
N_{Ed} = 58,3x[1,35x(6x(6+3)+1x6))+1,50x(5x3+1x4+1x0,70x2,5+0x1,70)]=
= 58,3x[81,0+31,13] =6537 kN + self weight

Geometric imperfections and 2nd order have to be taken into account; bending moments mainly due to horizontal actions (wind) resisted by the bracing system → N_{max} related to min M: $\nu = 1 + \omega = 1{,}10$ assuming $\omega = 0{,}10$ (ν = n in EC2)

$$\nu = \frac{N_{Ed}}{A_c f_{cd}} = 1{,}10 \qquad f_{cd} = 20\,\mathrm{N/mm^2} \qquad A_c = 6537 \times 10^3 / (1{,}10 \times 20) \times 10^{-6} = 0{,}30\ \mathrm{m^2}$$

Column B2 - foundation level $A_t = 58{,}3\ m^2$

$$\lambda = \frac{l_0}{i_{min}} \geq \lambda_{lim} \qquad \lambda_{lim} = 20\frac{ABC}{\sqrt{\nu}}$$



$$\nu = \frac{N_{Ed}}{A_c f_{cd}}$$

$$A = \frac{1}{1+0{,}2\varphi_{EF}} \qquad B = \sqrt{1+2\omega} \qquad C = 1{,}7 - \frac{M_{01}}{M_{02}} \qquad |M_{02}| \geq |M_{01}|$$

EC2 Default values: $A = 0{,}7$ ($\varphi_{EF} = 2$) $B = 1{,}1$ ($\omega = 0{,}1$)
$C = 0{,}7$ for buildings with insufficient bracing elements

Column B2 - foundation level $A_t = 58,3\ m^2$

$$\lambda_{lim} = 20\frac{0,7\cdot 1,1\cdot 0,7}{\sqrt{\nu}} = \frac{10,8}{\sqrt{\nu}} \qquad \nu = 1,10 \qquad \lambda_{lim} = 10,3$$

$$i_{min} = \frac{l_0}{\lambda_{lim}} = \frac{l_0}{10,3} = 0,097\ l_0 \qquad l_0 = (0,7 \div 1,0)\ h \text{ for frames}$$

$$\text{sez R: } (b,h)_{min} = i_{min}\sqrt{12} = 0,34\ l_0$$

$$h = 3m \quad l_0 = 0,85h = 2,55m \qquad (b,h)_{min} = 0,34x2,55 = 0,87m$$

Column (0,50x0,50) m $\quad A_c = (0,50x0,50) = 0,25 < 0,30\ m^2$

Torsional rigidity

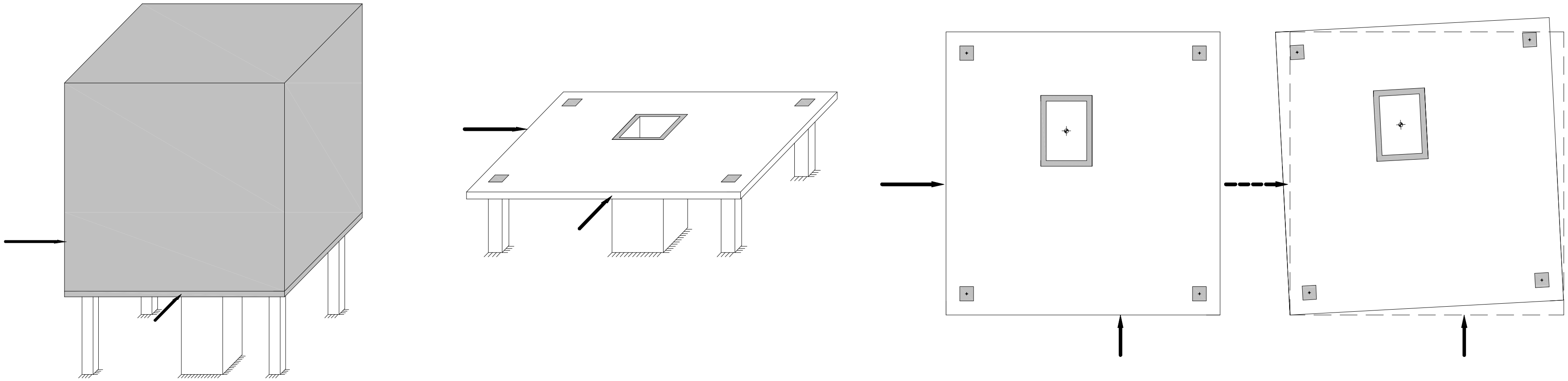
Asymmetry of wind loading causes dangerous torsional effects: torsional rigidity should always to be looked at.

EC8 rules to verify if the plane distribution of bracing elements is correct («regularity in plan»)

Horizonatal forces (wind, earthquake) resultant is applied at a given point in (x,y) direction

The intersection of (x,y) directions identify the conventional «center of masses» CM. In case of an earthquake, CM is the centroid of masses.

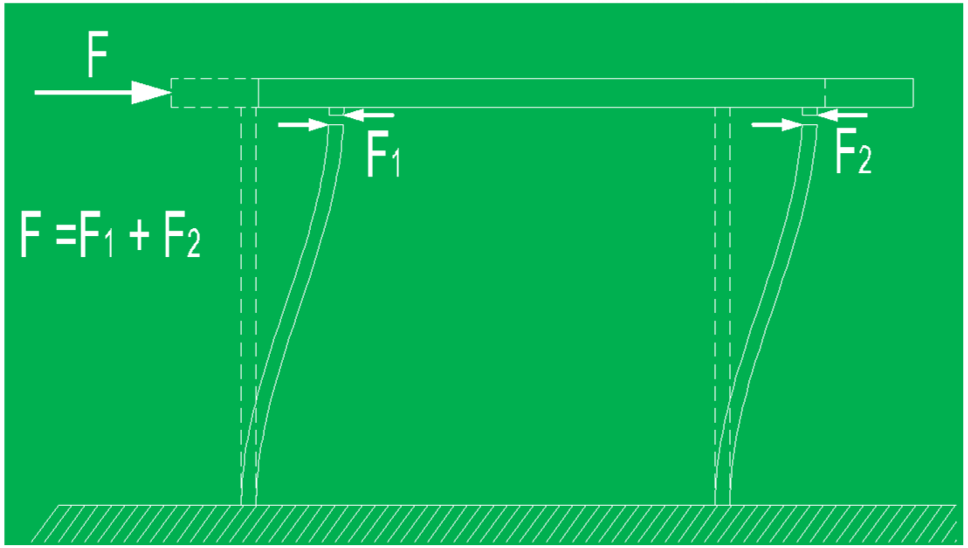
EC2 Appendix I : shear walls simplified action distribution.



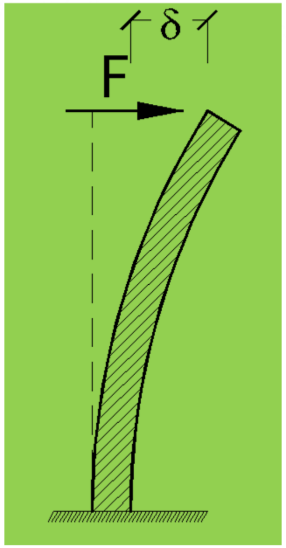
Horizontal forces transferred to cores by rigid plane behaviour.

Lateral stiffness

Shear type



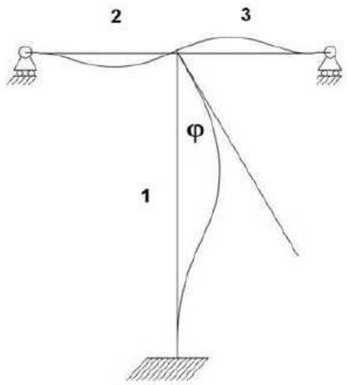
Bending type.



Interaction beetween frames, cores and walls

Columns in frames are retained by walls at lower levesl and retain walls at upper levels

Lateral stiffness



MMM - Modified Muto Model
(including shear flexibility)

Columns + beams subframe

$$k_{x,y} = \frac{\beta_{in} E_{cm} A}{l} \frac{1}{\left[\alpha \left(\frac{l}{\rho_{y,x}} \right)^2 + 2 \beta_{in} t (1+\nu) \right]}$$

$$\text{columns: } \alpha = \frac{1}{12} \frac{1}{\left(1 - \frac{3K_1}{4K_1 + 3K_2 + 3K_3} \right)}; \quad \text{cores, walls: } \alpha = \frac{n_s}{3} \quad n_s = \text{n. storeys}$$

LATERAL GLOBALSTIFFNESSES

$$K_X = \Sigma k_{xi} \qquad K_Y = \Sigma k_{yi}$$

STIFNESSES CENTER

$$x_{CR} = \frac{\Sigma k_{yi} x_i}{K_y} \qquad y_{CR} = \frac{\Sigma k_{xi} y_i}{K_x}$$

TORSIONAL STIFFNESS

$$K_T = \Sigma\, k_{yi}\, (x_i - x_{CR})^2 + \Sigma\, k_{xi}\, (y_i - y_{CR})^2$$

Lateral stiffness

$$e_{0x} = \left| x_{CR} - x_{CM} \right| \qquad e_{0y} = \left| y_{CR} - y_{CM} \right|$$

“Torsional” radius

$$r_X = \sqrt{\frac{K_T}{K_x}} \qquad r_Y = \sqrt{\frac{K_T}{K_Y}}$$

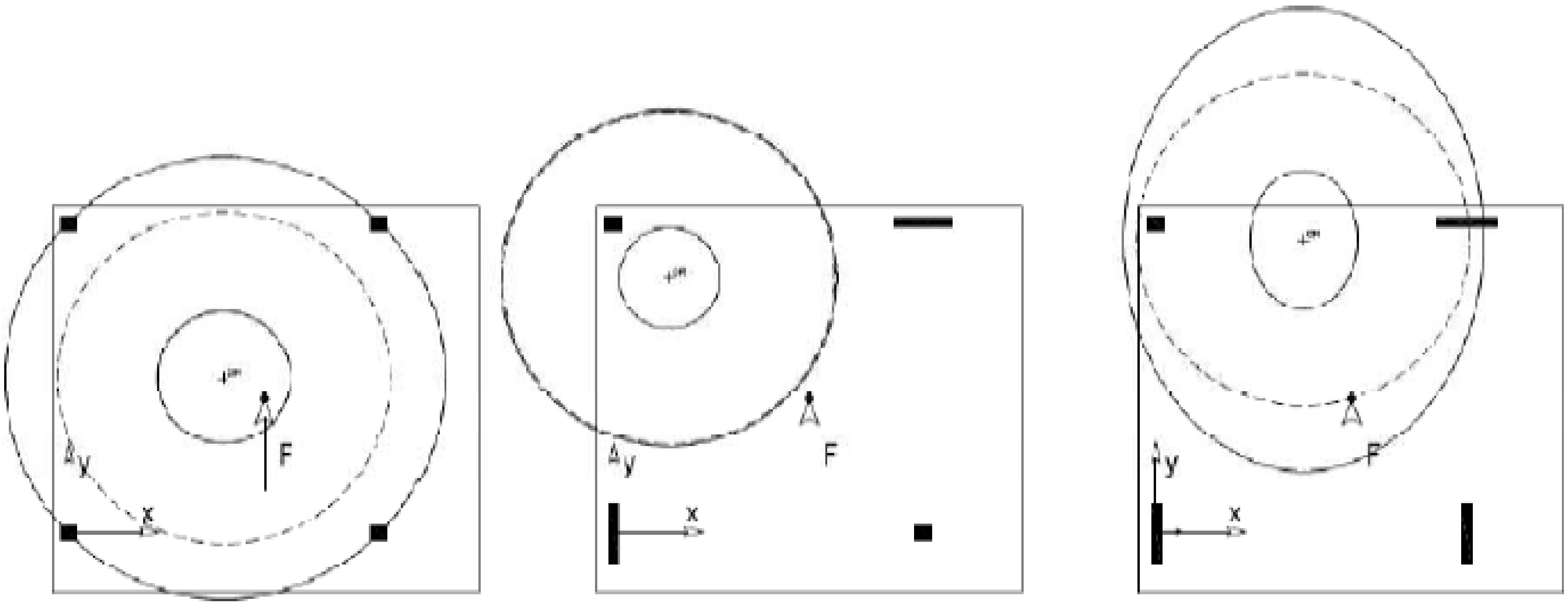
EC8: the bracing system is «torsionally rigid» if:

$$e_{0X} / r_x \leq 0{,}30 \qquad e_{0y} / r_y \leq 0{,}30$$

Ellypsis of stiffnesses

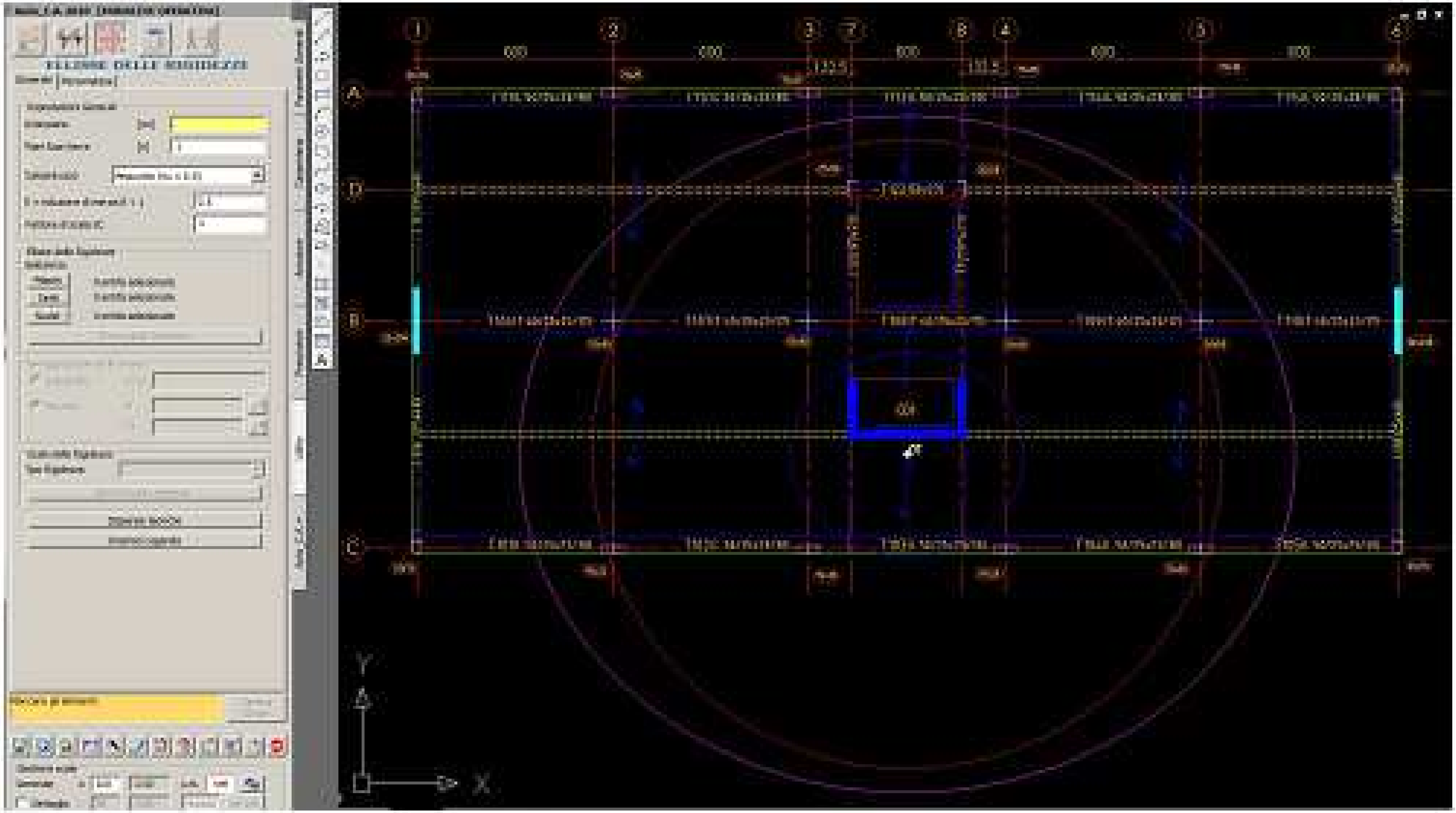
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$

STIFFNESSES' VARIATION AROUND CR



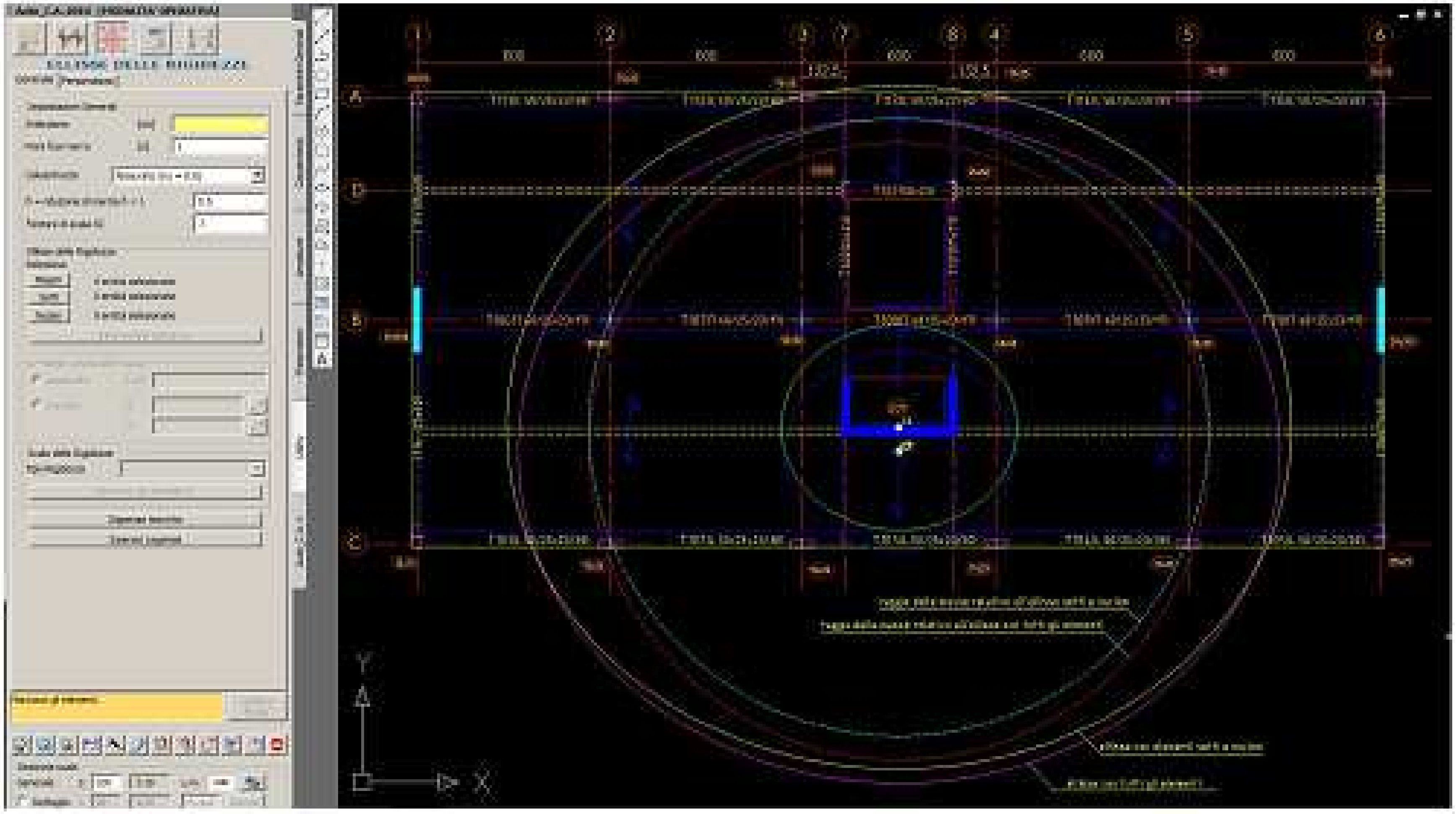
Ellypsis of stiffnesses

CORES AND WALLS ONLY («PRIMARY ELEMENTS»)



Ellypsis of stiffnesses

CORES, WALLS AND COLUMNS («SECONDARY ELEMENTS»)



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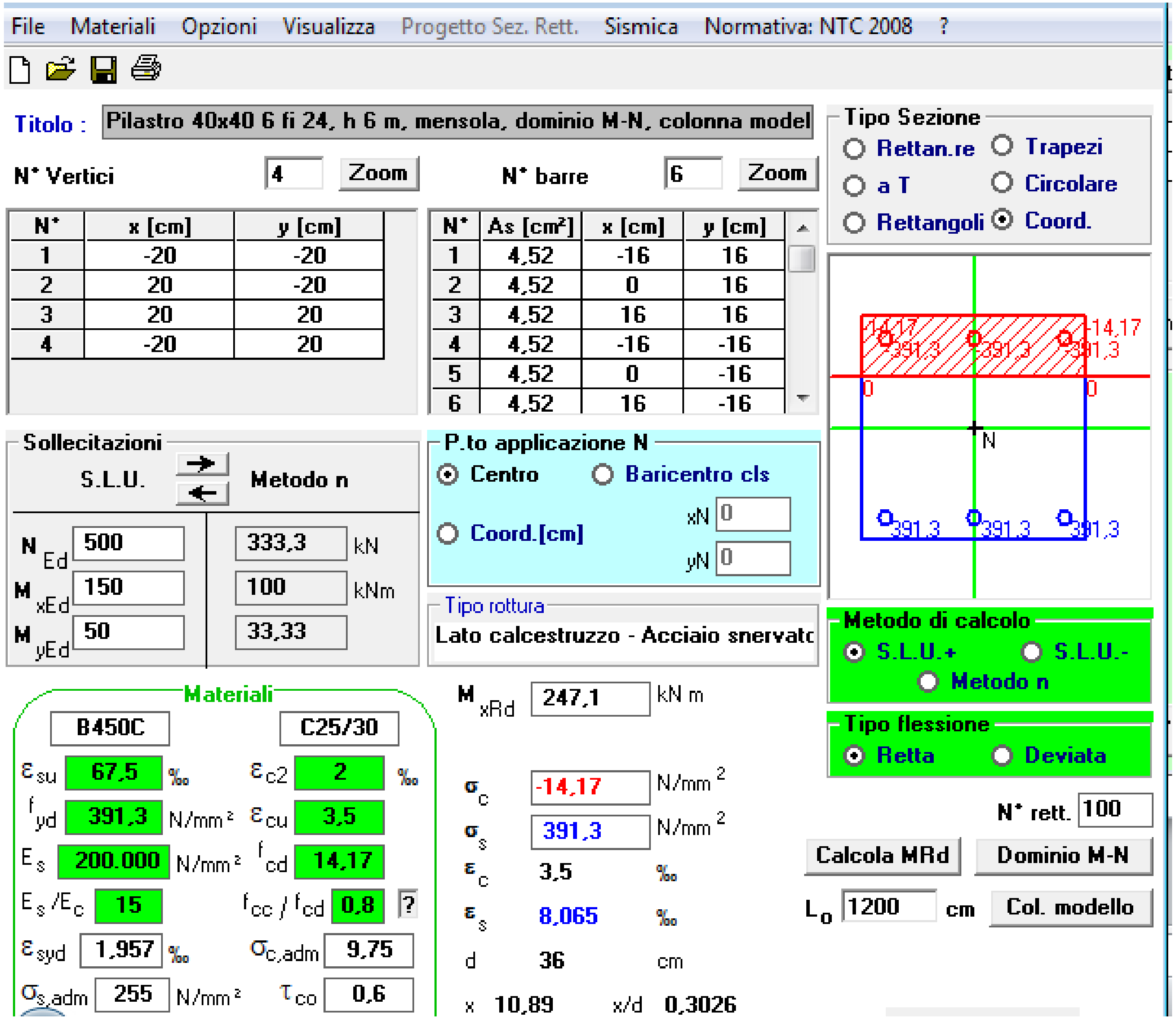
Conclusions

1. Column sizes and area easily identified

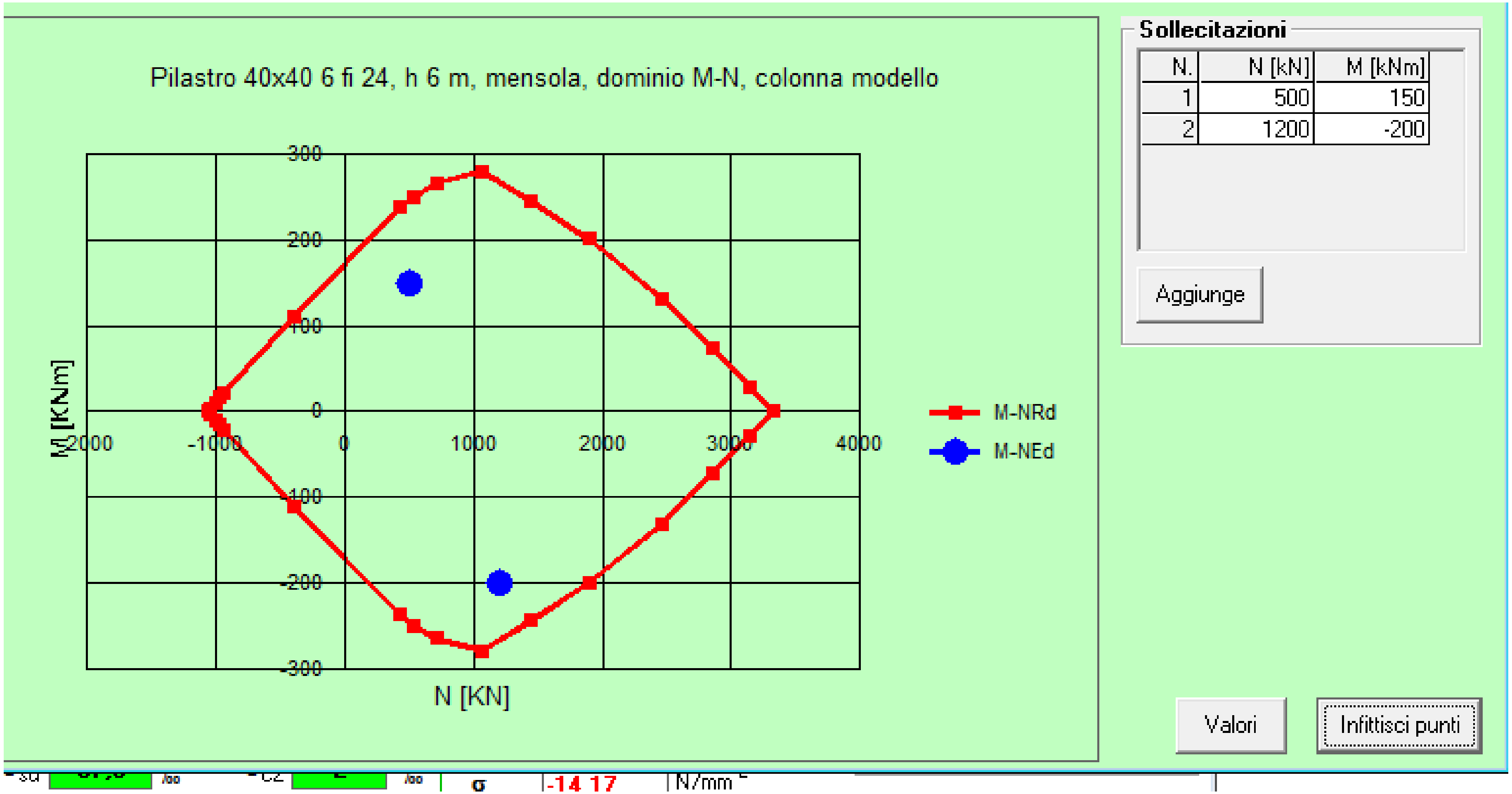
2. For global horizontal forces in x,y direction, minimum shear wall area may be determined on the basis of the variable truss method with truss inclination of 45°. (N,V) interaction should be taken into account

3. The “ellypsis of stifnesses” allows the visual control of spatial distribution of shear walls and cores in plan and identifies critical elements

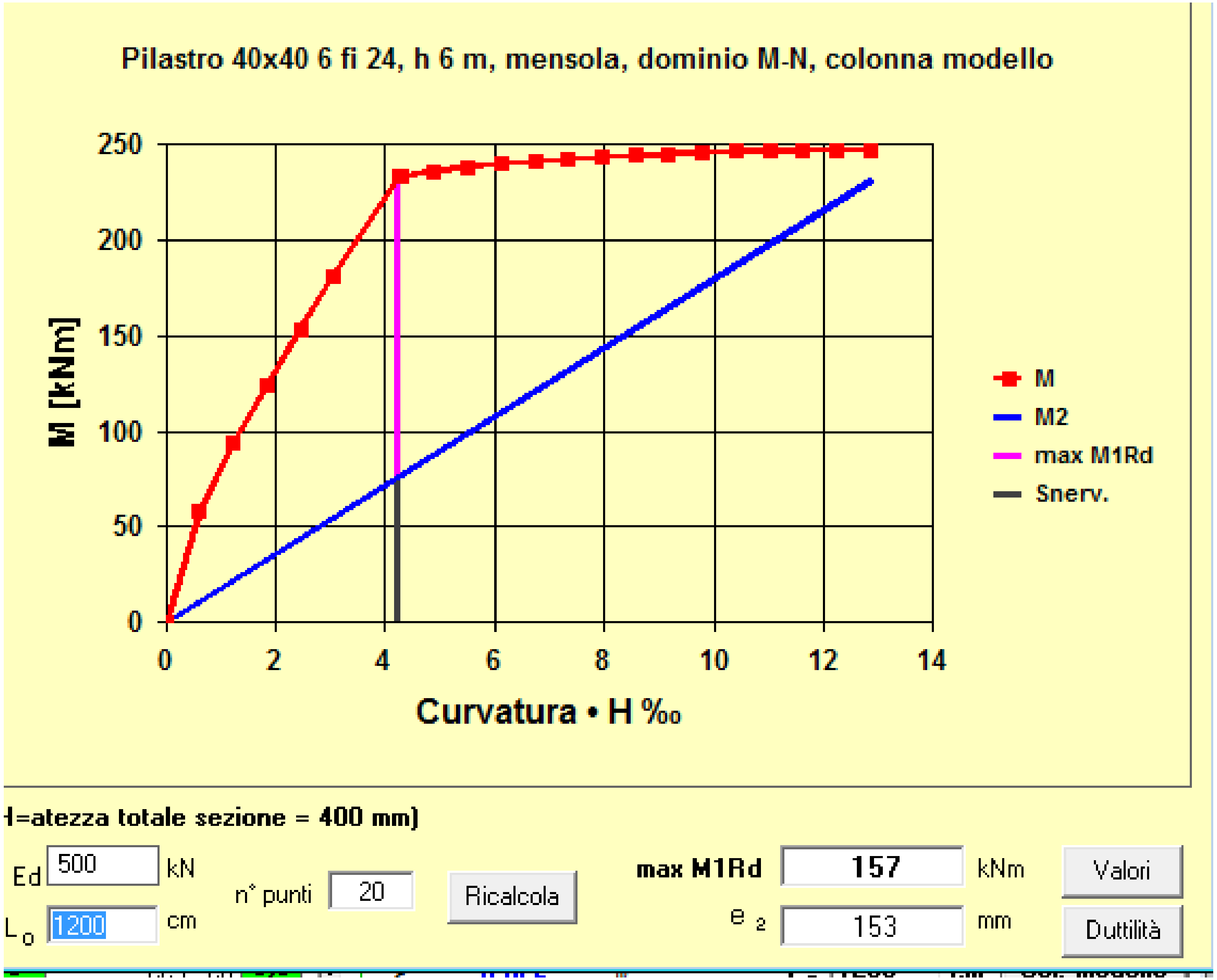
The engineers' tolbox



The engineers' tolbox



The engineers' tolbox



EC based design software - commercial

COMMERCIAL SOFTWARE

				Eurocodes									
Name	**SW House**	**Count.**	**Lan**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Tot Ecs**	**Link**
SCIA Engineer	Nemetschek	D	Y	x	x	x	x	x	x	x	x	8	www.scia-online.com
FRILO	Nemetschek	D	Y	x	x	x	x	x	x	x		7	www.frilo.com
BetonExpress, Fedra..	Runet	NO	Y	x	x		x	x	x	x		6	www.runet-software.com
STAAD	Bentley	USA/UK	Y	x	x	x		x	x	x		6	www.bentley.com
Dolmen Win	CDM Dolmen	IT	N	x	x			x	x	x		5	www.cdmdolmen.it
Fedra, Frame2D	Runet	NO	Y	x			x	x	x	x		5	www.runet-software.com
PowerConnect/Frame	BuildSoft	BE	Y	x	x	x	x			x		5	www.buildsoft.eu
AxisVM	AxisVM	H	Y	x			x		x	x		4	www.axisvm.eu
Midas	Midas	ROK	Y	x				x	x	x		4	www.cspfea.net/midas_gen.html
Robot	Autodesk	USA	Y	x	x		x				x	4	usa.autodesk.com
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Winstrand	Enexsys	IT	Y	x	x						x	3	www.enexsys.com
SAM Bridge design	Bestech	UK	Y	x	x	x						3	www.lrfdsoftware.com
3muri	S.T.A. Data	IT	Y	x			x			x		3	www.3muri.com
MatrixFrame	Matrix Software	NL	Y	x	x							2	www.matrix-software.com
AmQuake	AmQuake	CZ	Y					x		x		2	www.amquake.eu
GSA Suite	Oasys	UK	Y	x	x							2	www.oasys-software.com
Jasp	IngegneriaNet	IT	N	x						x		2	www.ingegnerianet.it
Tekla Structures	Teckla	FIN	Y	x	x							2	www.tekla.com
EC6design	DTI - Danish Techn.	DK	Y					x				1	www.ec6design.com
GEO	LimitState	UK	Y						x			1	www.limitstate.com
RCCe11/21/41	Reinf. Con. Counc.	UK	Y	x								1	www.civl.port.ac.uk/rcc2000
RING	LimitState	UK	Y					x				1	www.limitstate.com
Stainless steel	Steel const. Inst.	GB	Y		x							1	www.steel-stainless.org/software/
Timbersizer	Trada	GB	Y				x					1	www.trada.co.uk

EC based design software - free

			Eurocodes									
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Prof. P Gelfi	IT	N	x	x					x		3	dicata.ing.unibs.it/gelfi
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IngegneriaNet	IT	N	x						x		2	www.ingegnerianet.it
Tracon	IT	x	x								1	www.cdmdolmen.it
APIS	UK	Y	x								1	www.apiscalcs.com
ArcelorMittal	L	Y			x						1	www.arcelormittal.com/sections
DTI - Danish Techn.	DK	Y					x				1	www.ec6design.com
Reinf. Con. Counc.	UK	y	x								1	www.civl.port.ac.uk/rcc2000
The steel const. Inst.	GB	y		x							1	www.steel-stainless.org/software
Trada	GB	y				x					1	www.trada.co.uk

Conceptual design – Columns

Thanks for your attention!

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