THERMAL ACTIONS

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Summary

Changes in temperatures may cause additional deformations and stresses and may, in some cases, significantly affect ultimate and serviceability limit states of structures. Fundamental principles and rules described in this paper provide basic tools for specifications of temperature changes and for the evaluation of thermal actions effects in buildings, bridges and industrial structures.

1 INTRODUCTION

1.1 Background documents

This paper takes into account several background documents, mainly EN 1991-1-5 “Eurocode 1: Actions on structures – Part 1.5: General actions – Thermal Actions” [1], then the document [2], which is focused mainly on thermal actions on bridges (including references on number of papers, research reports and national standards), Chapter 4 of Handbook 3 of Leonardo da Vinci project CZ/02/B/F/PP-134007 [3] and also ISO documents [4,5,6], where procedures for buildings are provided. The presented text is primarily based, as for buildings, on the document [3], and on the background document [2] as for bridges.

1.2 General principles

Any given instantaneous thermal field $T(y, z)$ acting in a section can be decomposed into four separate components, as described in EN 1991-1-5 [1], is indicated in Figure 1:

- a uniform temperature component;
- a component varying linearly around the *z-z* axis (in direction of axis y);
- a component varying linearly around the *y-y* axis (in direction of axis z);
- a self-equilibrating non-linear temperature distribution.

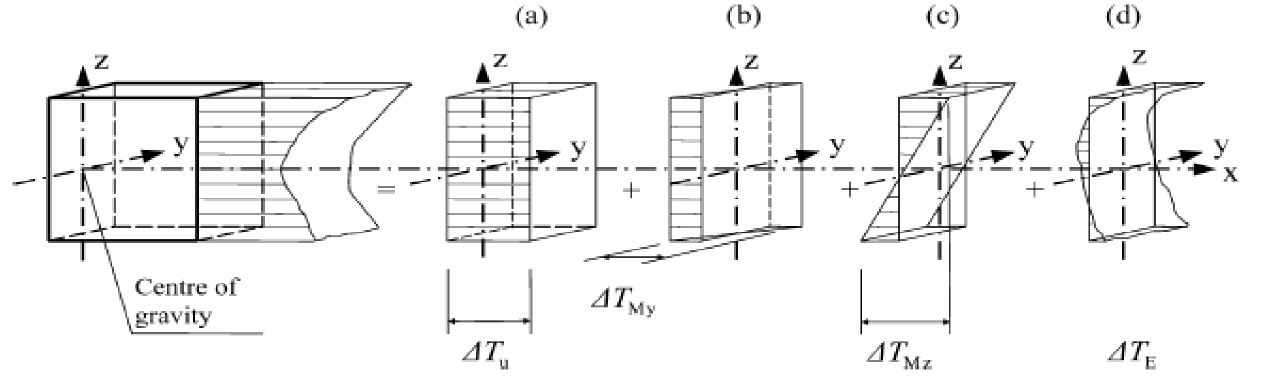

Figure 1. Diagrammatic representation of constituent temperature components.

Figure 1 indicates four components of temperature profile, which may be simplified in particular cases. Note that the notation used in Figure 1 may also be modified in specific cases.

Strictly speaking, the components of the thermal actions at any instant ‘t’ are given by the difference between the values at that instant and the corresponding values occurring at the initial state, i.e. at the time when the structure is restraint (completed). If the temperature in the initial state is not predictable, the average temperature during the construction period should be considered. According to the provisions of the document [1], the value of initial temperature T_0 may be specified in the National annex. If no information is available, T_0 = 10 °C may be taken. Such a value seems to be appropriate for most Member States.

2 EVALUATION OF THERMAL ACTIONS

Deformations and consequent stresses induced by the thermal actions introduced above and beyond those due to the value of the actions themselves, are dependent on the geometry of the element considered and the physical properties of the materials employed in its construction. Clearly, if the structure contains materials with different values of the linear thermal expansion coefficient, this must be adequately accounted for in calculations.

The magnitude of the thermal actions and their distribution throughout the single elements of the structure are a function of numerous parameters, some quite difficult to interpret numerically. There are wide-scale parameters correlated with the climate of the geographical location of the construction site and the consequent seasonal temperature variations. Then there are highly aleatory parameters, such as the presence of perturbations, which influence air temperatures and solar radiation, often with fluctuations on a daily scale or, in any event, over relatively short periods of time. Lastly, there are parameters strictly linked to the conditions of the particular building in question: the presence of other nearby structures that act as solar radiation screens, the building orientation, its total mass (and consequent thermal inertia), the properties of its finishing (i.e. the degree of their solar energy absorption and thermal isolation) and the characteristics of the interior heating, air conditioning and ventilation.

Thermal actions must be considered to be variable and indirect actions. Regulations furnish characteristic values whose probability of being exceeded is 0,02, which is equivalent to a return period of 50 years. The fundamental quantities on which thermal actions are based are the extreme air temperatures, that is, the maximum and minimum, in the shade at the building site. Such values are furnished by the National Meteorological Institute of each Member State. The shade air temperatures are measured by a device known as a “Stevenson screen”, which is simply a thermometer set in a white painted wooden box with louvres in the sides and door. The reason for shrouding the thermometer is to shield it from radiation by the sun, ground and surrounding objects during the day and finally protect it from precipitation, while, at the same time, the louvres allows air to pass freely about it.

Eurocode EN 1991-1-5 [1] does not include maps of extreme temperatures. Such task is left up to the National Meteorological Institutes. Indicative maps for some CEN countries were included in the preliminary standard ENV 1991-2-5 [8].

Figures 2 and 3 show maps of the isotherms of minimum and maximum shade air temperatures in the Czech Republic. The maps were developed by the Czech Hydro-Metrological Institute in accordance with the principles and rules specified in the Eurocode [1].

The maximum shade air temperature being exceeded by annual extremes with the probability of 0,02.

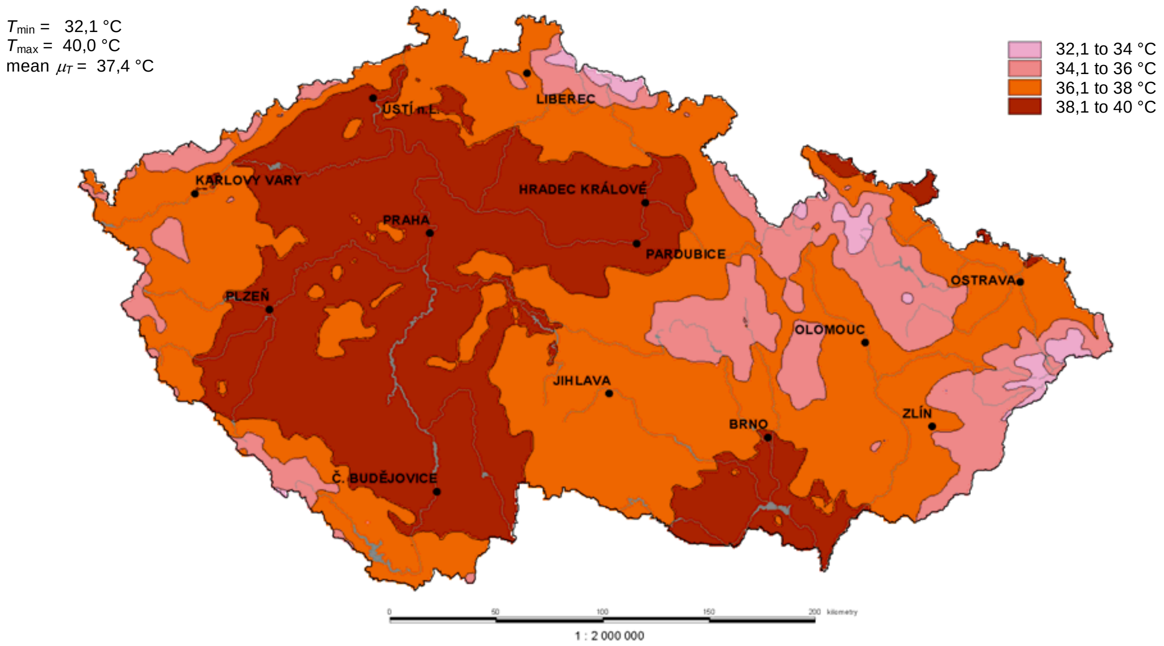

Figure 2. Map of maximum shade air temperatures in the Czech Republic.

The minimum shade air temperature being exceeded by annual extremes with the probability of 0,02.

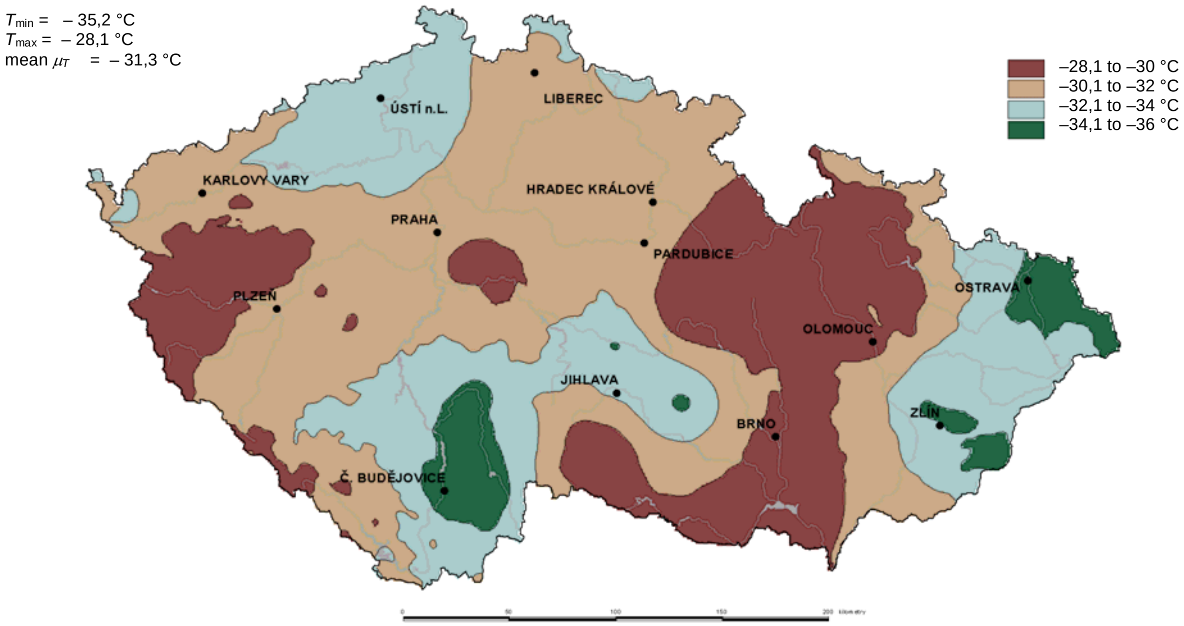

Figure 3. Map of minimum shade air temperatures in the Czech Republic.

In order to deal with shade air temperatures (maximum, $T_{\max,p}$, or minimum, $T_{\min,p}$) being exceeded by annual extremes with the probability other than 0,02, the following relationships (based on a type I of the extreme values distribution) can be used according to Annex A of EN 1991-1-5 [1]:

$$T_{\max,\mathrm{p}} = T_{\max} \cdot \{k_1 - k_2 \cdot \ln[-\ln(1-p)]\} \tag{1}$$

$$T_{\min,\mathrm{p}} = T_{\min} \cdot \{k_3 + k_4 \cdot \ln[-\ln(1-p)]\} \tag{2}$$

It moreover follows that:

$$k_1 = \frac{u \cdot c}{u \cdot c + 3.902} \tag{3}$$

$$k_2 = \frac{1}{u \cdot c + 3.902} \tag{4}$$

$$k_3 = \frac{u \cdot c}{u \cdot c - 3.902} \tag{5}$$

$$k_4 = \frac{1}{u \cdot c - 3.902} \tag{6}$$

where the parameters ‘u’ and ‘c’ are functions of the mean m and the standard deviation σ of the type I extreme value distribution [1, 2]:

for the maximum value
$$u = m - \frac{0.57722}{c} \tag{7}$$

$$c = \frac{1.2825}{\sigma} \tag{8}$$

for the minimum value
$$u = m + \frac{0.57722}{c} \tag{9}$$

$$c = \frac{1.2825}{\sigma} \tag{10}$$

If no specific data are available, the following values are recommended:

$$k_1 = 0.781 \tag{11}$$
$$k_2 = 0.056 \tag{12}$$
$$k_3 = 0.393 \tag{13}$$
$$k_4 = -\ 0.156 \tag{14}$$

For example, the following coefficients are specified in the Czech Republic on the basis of statistical evaluation of 45 years of measurements: $k_1 = 0{,}83$ $k_2 = 0{,}04$, $k_3 = 0{,}54$, $k_4 = -0{,}12$.

Annex A [1] includes also supplementary information concerning the adjustment of shade air temperatures taking into account the height of construction site above the sea level or local conditions, e.g. frost pockets. The values of shade air temperature may be adjusted for the height above sea level by subtracting 0,5 ^{o}C per 100 m height for the minimum shade air temperatures and 1,0 ^{o}C per 100 m height for the maximum shade air temperatures. This recommendation may be nationally modified on the basis of nationally evaluated data (e.g. the height of terrain is directly considered in some national maps and, therefore, in these cases the adjustment is not applied).

The effects of thermal actions shall be determined for every design situation deemed to be relevant either for the serviceability or ultimate limit states. Thermal actions may be neglected if the structures are not exposed to significant daily or seasonal temperature variations, or variations caused by technological temperature changes. All thermal actions can thus be attributed to either climatic effects or technological activities. Climatic effects shall be determined by considering variations in the shade air temperature and changes in solar radiation. The influence of activities carried out in the building interior (technological or industrial processes) must be evaluated according to its specific design characteristics and technological specifications.

3 TEMPERATURE COMPONENTS

In general, the following fundamental quantities are to be considered in the structural analysis:

- a uniform temperature component, ΔT_u, given by the difference between the mean temperature T of an element and its conventional initial temperature T_0;
- a component of linearly variable temperature, ΔT_M, given by the difference between the temperatures of the external and internal surfaces of a straight section;
- a temperature difference, ΔT_p, given by the difference between the mean temperatures of the structural parts in question.

If the local effects of thermal actions are significant, they must also be considered in addition to the components ΔT_u, ΔT_M, ΔT_p.

The uniform component of temperature ΔT_u of a given structural element is calculated as the difference between the mean temperature T of the element due to climatic temperatures (in winter or summer season) and operational temperatures and the temperature T_0 at the initial instant:

$$\Delta T_u = T - T_0. \tag{15}$$

The first step is to determine the value of mean temperature T. It is calculated as the value of the mean winter or summer temperature of the structural element in question by adopting a specific profile that defines the temperature distribution throughout the element’s thickness. If the internal (T_{in}) and external (T_{out}) conditions are sufficiently similar, a simplified procedure can be adopted, and the mean temperature may be determined as

$$T = \frac{T_{out} + T_{in}}{2} \tag{16}$$

The internal (T_{in}) and external (T_{out}) temperatures are given in EN 1991-1-5 [1], as indicated in following Tables 1 to 3. Two different values of T_{out} are distinguished for the parts of the structure above and below the ground level. It should be noted that in the Table 2 the values of T_{out} for the summer season are a function of both the building orientation and the thermal absorption characteristics of its external surfaces. Obviously, the maximum values are reached on horizontal surfaces and those facing South or South-West, while the minima (which are equal to approximately one half the maximum values) are found on the surfaces facing North.

Table 1. Indicative temperature values for interiors

Season	*Temperature* T_{in}
Summer	T_1
Winter	T_2

Table 2. Indicative values of T_{out} for buildings above ground level.

Season	*Significant factor*		*Temperature* T_{out}
Summer	Surface absorption properties, colour-dependent	0,5 (very light surface)	$T_{max} + T_3$
		0,7 (light or coloured surface)	$T_{max} + T_4$
		0,9 (dark surface)	$T_{max} + T_5$
Winter			T_{min}

Table 3. Indicative values of T_{out} for underground parts of buildings.

Season	*Depth below ground level*	*Temperature* T_{out}
Summer	Less than 1 m	T_6
	More than 1 m	T_7
Winter	Less than 1m	T_8
	More than 1 m	T_9

The T_{in} and T_{out} values in Tables 1 to 3 are specified in °C. Moreover, with regard to such values:

– T_1 values are specified in the National Annex, though lacking more precise indications, a value of 20 °C is given in EN 1991-1-5 [1].

– T_2 values are also specified in the National Annex, though lacking more precise indications; a value of 25 °C is given in [1].

– The maximum and minimum values of the shade air temperatures, T_{max} and T_{min}, and the effects of solar radiations, T_3, T_4 and T_5, are to be specified in the National Annex.

For regions lying between latitudes 45°N and 55°N, the following values are recommended:

$T_3 = 0$ °C $T_4 = 2$ °C $T_5 = 4$ °C	For surfaces facing North-East

or

$T_3 = 18$ °C $T_4 = 30$ °C $T_5 = 42$ °C	For horizontal surfaces and those facing South-West

The values of T_6, T_7, T_8 and T_9 may be specified in the National Annex. For regions at latitudes between 45° N and 55° N, the following values are recommended:

$$T_6 = 8\ °\mathrm{C}$$

$$T_7 = 5\ °\text{C}$$

$$T_8 = -5\ °\text{C}$$

$$T_9 = -3\ °\text{C}$$

The initial temperature T_0 in equation (15) may be (when the time of completion is unknown) determined as the average of the summer and winter effective temperatures:

$$T_0 = \frac{T_s + T_w}{2} \tag{17}$$

Here T_s and T_w denote the summer and winter effective temperatures of a relevant structural member. If the time of completion is known beforehand, the effective temperature of the structure or its parts at the relevant stage of their completion shall be used. Note that in most cases T_0 may be considered within an interval from 10 to 15 °C (10 °C is indicated in Annex A [1]).

4 DETERMINATION OF TEMPERATURE PROFILES

The temperature profile should be generally determined in accordance with the thermal transmission theory. In the case of a simple sandwich element (e.g. slab, wall, shell) a temperature $T(x)$ at a distance x from the inner surface of the cross section may be determined assuming stationer thermal state using the following relationship

$$T(x) = T_\text{in} - \frac{T_\text{in} - T_\text{out}}{R_\text{tot}}(R_\text{si} + R(x)) \tag{18}$$

In this equation T_in denotes the air temperature of the inner environment, T_out the temperature of the outer environment, R_tot the total thermal resistance of the element including the resistance of both surfaces, R_si the internal surface resistance of the inner surface, and $R(x)$ the thermal resistance of the element from the inner surface up to the point x (see Figure 4). The resistance values R_tot, R_si, and $R(x)$ may be determined using the coefficient of heat transfer and coefficients of heat conductivity given in European standards EN ISO 6946 [4] and EN ISO 13370 [5] for the calculation of thermal resistance and thermal transmittance of buildings. An element compound of several layers made of similar materials may be considered as a single-layer element.

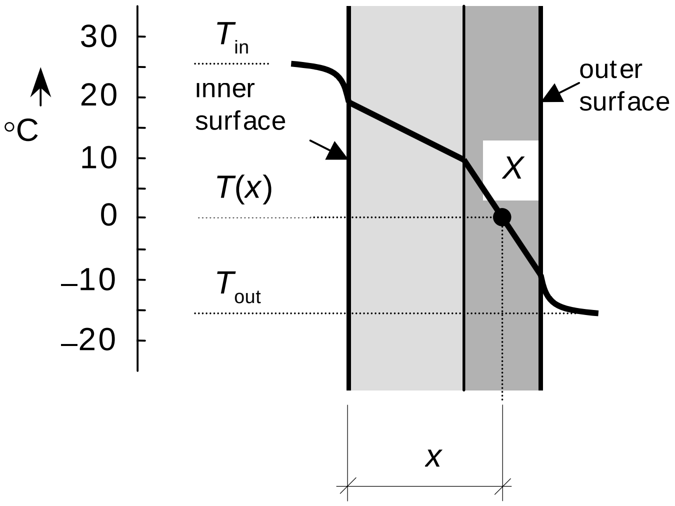

Figure 4. Thermal profile of a two-layer cladding element in winter.

The temperatures T_{in} and T_{out} should be specified on the basis of adequate national (regional) data (national values should be specified in National Annex). Further details of the temperature profile determination are given in Annex D of [1]. Software tools for determining the temperature profile are available (for example Mathcad free ebook [12]), an example worked up in Excel is presented in Annex 1.

5 THERMAL ACTIONS IN BRIDGES

5.1 Consideration of thermal actions

Three types of bridge superstructures are distinguished in EN 1991-1-5 [1]

– Type 1 Steel deck (steel box girder, steel truss or plate girder)
– Type 2 Composite deck (concrete deck on steel box, truss or plate girder)
– Type 3 Concrete deck (concrete slab, concrete beam, concrete box girder)

The uniform temperature component and the temperature difference components should be considered for determination of thermal actions in bridges.

The minimum and maximum uniform (effective) bridge temperatures $T_{e,min}$ ($T_{e,max}$) can be determined from the relationship given in Fig. 5 on the basis of isotherms of shade air temperatures T_{min} (T_{max}). The characteristic values of minimum and maximum shade air temperatures for a site location may be obtained e.g. from national maps of isotherms. These characteristic values represent shade air temperatures at mean sea level in open country being exceeded by annual extremes with the probability of 0,02.

The relationship given in Fig. 5 is based on a daily temperature range of 10 °C [1]. Such a range may be considered as appropriate for most Member States. Guidance for other ranges is given in document [2].

The maximum uniform temperature component $T_{e,max}$ and the minimum uniform temperature component $T_{e,min}$ for the three types of bridge decks may be determined from the following relationships based on Figure 5:

$$\left.\begin{array}{l}\text{Type 1 } T_{\mathrm{e,\,max}} = T_{\max} + 16\,^{\circ}\mathrm{C}\\ \text{Type 2 } T_{\mathrm{e,\,max}} = T_{\max} + 4{,}5\,^{\circ}\mathrm{C}\\ \text{Type 3 } T_{\mathrm{e,\,max}} = T_{\max} + 1{,}5\,^{\circ}\mathrm{C}\end{array}\right\}\text{for } 30\,^{\circ}\mathrm{C} \le T_{max} \le 50\,^{\circ}\mathrm{C} \quad \left.\begin{array}{l}T_{\mathrm{e,\,min}} = T_{\min} - 3\,^{\circ}\mathrm{C}\\ T_{\mathrm{e,\,min}} = T_{\min} + 4{,}5\,^{\circ}\mathrm{C}\\ T_{\mathrm{e,\,min}} = T_{\min} + 8\,^{\circ}\mathrm{C}\end{array}\right\}\text{ for } -50\,^{\circ}\mathrm{C} \le T_{min} \le 0\,^{\circ}\mathrm{C}$$

For construction works located in specific climatic regions as in e.g. frost pockets, additional information should be obtained and evaluated.

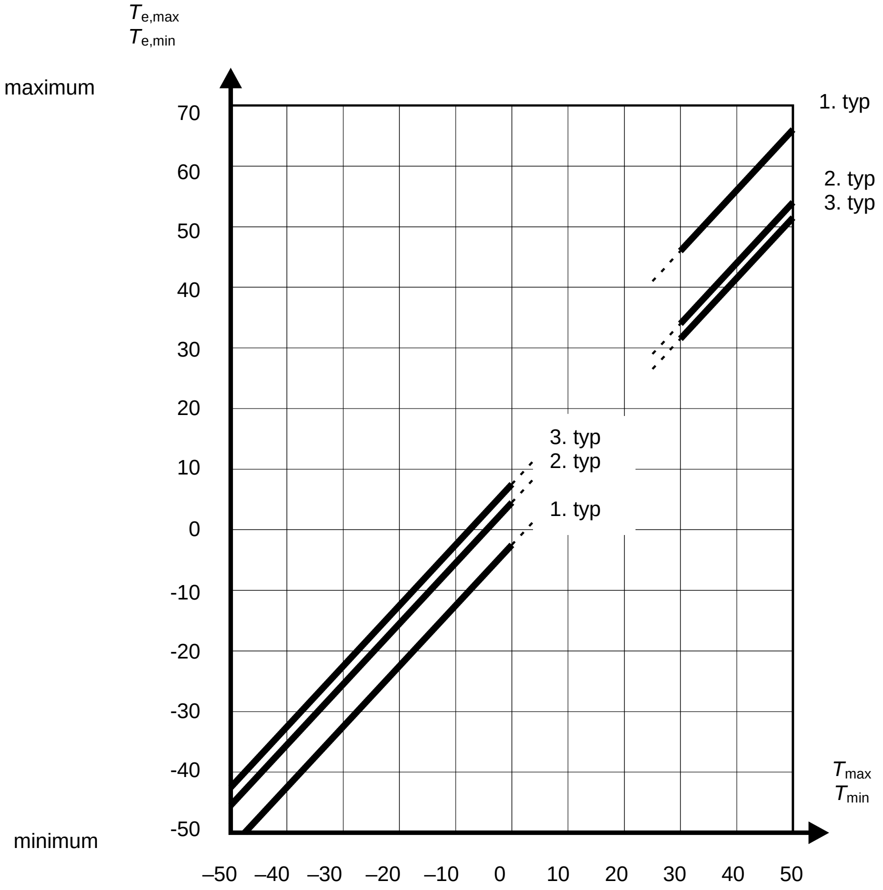

Figure 5 Correlation between minimum/maximum shade air temperature ($T_{\min}/T_{\max}$) and minimum/maximum uniform bridge temperature component ($T_{\mathrm{e.min}}/T_{\mathrm{e.max}}$)

5.2 Range of uniform temperatures

The characteristic value of the maximum contraction range of the uniform bridge temperature component, $\Delta T_{\mathrm{N,con}}$, is given as

$$\Delta\ T_{N,\mathrm{neg}} = T_0 - T_{\mathrm{e,min}} \tag{19}$$

and the characteristic value of the maximum expansion range of the uniform bridge temperature component, $\Delta T_{\mathrm{N,exp}}$, is given as

$$\Delta\ T_{N,\mathrm{pos}} = T_{\mathrm{e,max}} - T_0 \tag{20}$$

The total range of effective temperatures $\Delta\ T_N = T_{\mathrm{e,max}} - T_{\mathrm{e,min}}$.

5.3 Temperature difference components

For the vertical temperature difference component, two alternative approaches are provided in EN 1991-1-5 [1] which may be nationally selected: (1) linear, or (2) non linear temperature distribution.

The models applied in the linear approach are given in Table 4 for bridges based on a depth of surfacing of 50 mm. For other surfacing thicknesses, the coefficient k_{sur} should be applied [1].

The models of non-linear temperature distribution are illustrated in Chapter 6 for three types of bridge superstructures and numerical values indicated for different surfacing thickness and depth of a slab in Annex B [1].

Note that the linear approach was developed in Germany based on several research projects [2]. The non linear approach is based on the research carried on in the UK, accepted in BS 5400 and also adopted in [1].

Tab. 4 Characteristic values of linear temperature difference component for different types of bridge decks for road, foot and railway bridges.

Type of bridge deck	Top warmer than bottom	Bottom warmer than top
	$\Delta T_{M,\mathrm{heat}}$ (^{o}C)	$\Delta T_{M,\mathrm{cool}}$ (^{o}C)
Type 1: Steel deck	18	13
Type 2: Composite deck	15	18
Type 3: Concrete deck		
– concrete box girder	10	5
– concrete beam	15	8
– concrete slab	15	8

The temperature difference component should be commonly taken into account in the vertical direction. However, in some cases, e.g. when one side of a bridge deck is being more highly exposed to sunlight than the other one, a horizontal linear temperature difference component need to be considered (a value 5 ^{o}C is recommended in [1]).

In some cases, it may be necessary to take into account both the temperature difference $\Delta T_{\mathrm{M,heat}}$ (or $\Delta T_{\mathrm{M,cool}}$) and the maximum range of uniform bridge temperature component $\Delta T_{\mathrm{N,exp\_}}$(or $\Delta T_{\mathrm{N,con}}$) given as:

$$\Delta T_{M,\mathrm{heat}}\ (\text{or } \Delta T_{M,\mathrm{cool}})\ +\ \omega_N\ \ \Delta T_{N,\mathrm{heat}}\ (\text{or } \Delta T_{N,\mathrm{cool}})\quad \text{or} \tag{21}$$

$$\omega_M \Delta\ T_M + \Delta\ T_N \tag{22}$$

where the most adverse effect should be chosen

$\omega_N = 0{,}35$ and $\omega_M = 0{,}75$.

In structures where differences in the uniform temperature component between different element types may cause adverse load effects, these effects should be taken into account.

6 THERMAL ACTIONS IN INDUSTRIAL STRUCTURES

Some industrial structures, e.g. chimneys, pipelines and silos, are influenced by hot gases, liquids or materials, and should be designed for climatic and also operating process temperatures.

The temperatures profiles in normal operational conditions as well as in accidental situations during operational failures need to be considered.

For determination of thermal actions, in addition to the previously described temperature components also a stepped temperature component is recommended to be applied in one quarter of the cross-sectional circumference. This stepped component should be considered simultaneously with wind actions.

The maximum and minimum uniform temperature components should be taken as those of the maximum and minimum shade air temperatures provided e.g. in the national maps of isotherms.

The linear temperature difference component between the inner and outer faces of the wall for concrete pipelines is recommended as 15 °C in EN 1991-1-5 [1]. For chimneys, reference is made to EN 13084-1.

The rules for simultaneity of temperature components are given in [1] taking into account thermal actions due to climatic effects and those due to process effects.

7 LINEAR EXPANSION COEFFICIENTS

The values of the materials' linear expansion coefficients are fundamental to performing structural analyses to determine the effects of thermal actions. For the materials usually applied in buildings, the coefficient α_T (taken from the table in Annex C) are as follows.

Table 5 Coefficients of linear expansion.

Material	α_T (x 10^{-6} x °C^{-1})
Aluminium, aluminium alloys	24
Stainless Steel	16
Structural steel	12
Concrete (except as specified below)	10
Concrete with light aggregates	7
Masonry	6-10
Timber, along to grain	5
Timber, across to grain	30-70

Finally, in composite structures and reinforced concrete, the linear thermal expansion coefficient of steel can be assumed to be equal to that of concrete.

REFERENCES

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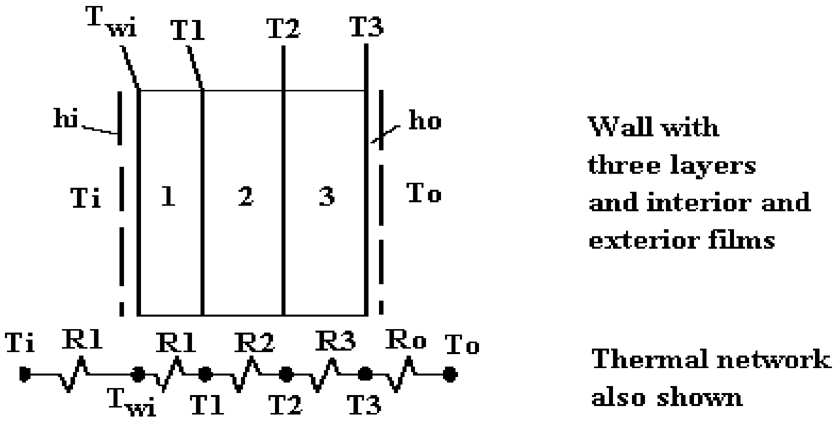
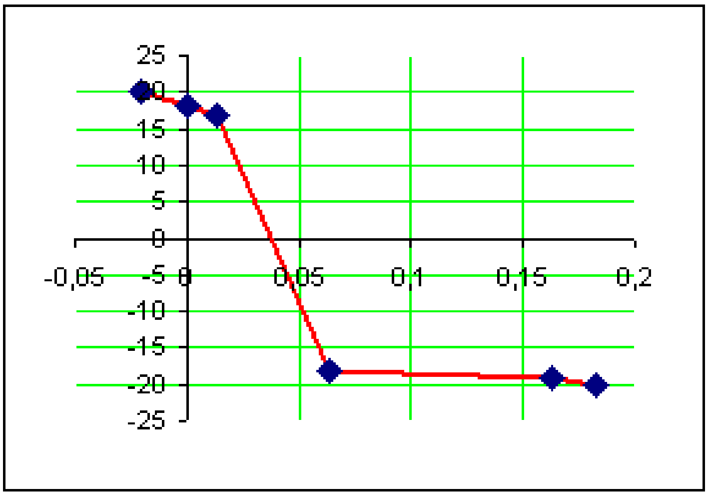
Annex 1 An example of analysis of temperature distribution within a structural member

The following example demonstrates calculation of the thermal resistance and temperature distribution within a wall assuming one-dimensional steady-state heat transfer. Note that in some cases different parts of the wall may have different layers, such as wood studs providing structural support.
To determine a correct wall R-value in such cases, we need to calculate the correct value through each heat flow path and determine the overall R-value based on the relative area of each path.



Input temperatures		Ti=	20	To=	-20	Heat flow Q=	17,323
		Transfer coef.	Thermal conduct.	Thickness	Resistance	Temperatures	
Layer	Material	W/m^2/°C	W/m/°C	m		°C	
	Inside					20	
0	Surface	9			0,111	18,075	
1	Gypsum		0,16	0,013	0,081	16,668	
2	Insulation		0,025	0,05	2,000	-17,979	
3	Brick		1,5	0,1	0,067	-19,134	
4	Outside	20			0,050	-20,000	
		The total resistance of wall Rtot =			2,309		

Graph	x	temp
	-0,02	20,000
	0	18,075
	0,013	16,668
	0,063	-17,979
	0,163	-19,134
	0,183	-20,000



Heat flow q_i through a wall layer i of thickness L_i, surface area A and thermal conductivity k_i

is given by

$q_i = \frac{T_{\text{in}} - T_{\text{out}}}{R_i}$ with $R_i = \frac{L_i}{k_i A}$ for resistance of wall layer *i*

Heat flow q_{w} by convection or radiation described in approximate terms by a heat transfer coefficient *h* (convective, radiative, or combined):

$q_{WI} = \frac{T_{\text{in}} - T_{\text{out}}}{R_{\text{I}}}$ with $R_{\text{I}} = \frac{1}{h_{\text{I}} A}$ for interior film resistance

$q_{\text{WO}} = \frac{T_{\text{in}} - T_{\text{out}}}{R_{\text{O}}}$ with $R_{\text{O}} = \frac{1}{h_{\text{O}} A}$ for exterior film resistance

$R_{\text{tot}} = R_{\text{I}} + \sum R_i + R_{\text{O}}$ total resistance

$q_{tot} = \frac{T_{\text{in}} - T_{\text{out}}}{R_{\text{tot}}}$ total heat flow from inside (temperature T_{in}) to outside (temperature T_{out})

$T_{\text{i}} = T_{\text{in}} - q_{\text{tot}} R_{\text{i}}$ temperature in layer *i*

In the example below, worked out in Mathcad, the same wall with three layers (gypsum board, insulation and brick) is considered.

Input parameters: $\text{degC} \equiv 1$

$\text{hi} := 9 \cdot \frac{\text{watt}}{\text{m}^2 \cdot \text{degC}}$ interior heat transfer (film) coefficient

$\text{ho} := 20 \cdot \frac{\text{watt}}{\text{m}^2 \cdot \text{degC}}$ exterior heat transfer coefficient

$A := 1.0 \cdot \text{m}^2$ heat transfer area (surface)

L_{i} = thickness of layer i

k_{i} = thermal conductivity of layer i

$L_1 := 0.013 \cdot \text{m}$ $\quad k_1 := 0.16 \cdot \frac{\text{watt}}{\text{m} \cdot \text{degC}}$

$L_2 := 0.05 \cdot \text{m}$ $\quad k_2 := 0.025 \cdot \frac{\text{watt}}{\text{m} \cdot \text{degC}}$

$L_3 := 0.10 \cdot \text{m}$ $\quad k_3 := 1.5 \cdot \frac{\text{watt}}{\text{m} \cdot \text{degC}}$

$N := 3$ N = number of layers (i denotes layer) $i := 1..N$

$RI := \frac{1}{hi \cdot A}$ RI = interior film resistance

$R_i := \frac{L_i}{k_i \cdot A}$ R_i = resistance of wall layer i $RO := \frac{1}{ho \cdot A}$ RO = exterior film resistance

$Rtot := RI + \sum_i R_i + RO$ Rtot = total resistance of wall

$$Rtot = 2.309 \frac{degC}{watt}$$

Calculation of heat flow Q from inside (temperature TI) to outside (temperature TO):

$TO := -20 \cdot degC$ $TI := 20 \cdot degC$

$m := 2..N$ $Q := \frac{TI - TO}{Rtot}$

$Twi := TI - Q \cdot RI$ Twi = wall room side surface temperature

$T_1 := Twi - Q \cdot R_1$ $T_m := T_{m-1} - Q \cdot R_m$

$Q = 17.323\,watt$ $Twi = 18.075\,degC$

$$i = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad \frac{T_i}{degC} = \begin{pmatrix} 16.668 \\ -17.979 \\ -19.134 \end{pmatrix}$$