Strength and stability of aluminium members according to EN 1999-1-1 – Eurocode 9

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Design values of loads and resistances

Design values of loads are given in Eurocode 0 and 1.

Eurocode 9 gives the design values of resistance at the ultimate limit state, e.g.

\[ M_{Rd} = \frac{M_{Rk}}{\gamma_M} = \frac{W_{el}f_0}{\gamma_{M1}} \]  

(class 3 cross section)

- \( M_{Rd} \) design value of bending moment resistance
- \( M_{Rk} \) characteristic value of bending moment resistance
- \( f_0 = R_{p0.2} \) characteristic value of 0.2 % proof strength
- \( \gamma_{M1} = 1.1 \) partial factor for general yielding
- \( W_{el} \) section modulus

For class 4 cross sections (slender sections, sections with large width/thickness ratio) \( W_{el} \) is replaced by \( W_{eff} \) for the effective cross section. However, if the deflection at the serviceability limit state is decisive then a simplified method may be used; see page 17.
In a section with reduced strength due to welding (heat affected zone, HAZ)

\[
M_{Rd} = \frac{W_{el}\rho_{u,\text{haz}}f_u}{\gamma_{M2}}
\]

(in a section with HAZ across the section)

- \(M_{Rd}\) design value of bending moment resistance
- \(f_u\) characteristic value of ultimate strength
- \(\gamma_{M2} = 1.25\) partial factor for failure
- \(\rho_{u,\text{haz}}\) reduction factor for the ultimate strength in HAZ
### Material properties

#### Table 3.2b - Characteristic values of 0.2% proof strength $f_0$ and ultimate tensile strength $f_u$ (unwelded and for HAZ), min elongation $A_{50}$, reduction factors $\rho_{o,haz}$ and $\rho_{u,haz}$ in HAZ, buckling class and exponent $n_p$ for wrought aluminium alloys - Extruded profiles, extruded tube, extruded, rod/bar and drawn tube

<table>
<thead>
<tr>
<th>Alloy EN-AW</th>
<th>Product form</th>
<th>Temper</th>
<th>Thickness $t$ (mm)</th>
<th>$f_0$</th>
<th>$f_u$</th>
<th>$A_{50}$</th>
<th>$f_{o,haz}$</th>
<th>$f_{u,haz}$</th>
<th>HAZ-factor</th>
<th>BC</th>
<th>$n_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EP,ET,ER/B</td>
<td>T4</td>
<td>$t \leq 25$</td>
<td>110</td>
<td>205</td>
<td>14</td>
<td>100</td>
<td>160</td>
<td>0.91</td>
<td>0.78</td>
<td>B</td>
<td>8</td>
</tr>
<tr>
<td>EP/O, EP/H</td>
<td>T5</td>
<td>$t \leq 5$</td>
<td>230</td>
<td>270</td>
<td>8</td>
<td>125</td>
<td>185</td>
<td>0.54</td>
<td>0.69</td>
<td>B</td>
<td>28</td>
</tr>
<tr>
<td>EP/O,EP/H</td>
<td>T6</td>
<td>$t \leq 5$</td>
<td>250</td>
<td>290</td>
<td>8</td>
<td>125</td>
<td>185</td>
<td>0.50</td>
<td>0.64</td>
<td>A</td>
<td>32</td>
</tr>
<tr>
<td>6082</td>
<td>ET</td>
<td>$5 &lt; t \leq 15$</td>
<td>260</td>
<td>310</td>
<td>10</td>
<td></td>
<td></td>
<td>0.48</td>
<td>0.60</td>
<td>A</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>T6</td>
<td>$t \leq 20$</td>
<td>250</td>
<td>295</td>
<td>8</td>
<td></td>
<td></td>
<td>0.50</td>
<td>0.63</td>
<td>A</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$20 &lt; t \leq 150$</td>
<td>260</td>
<td>310</td>
<td>8</td>
<td></td>
<td></td>
<td>0.48</td>
<td>0.60</td>
<td>A</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>DT</td>
<td>$t \leq 5$</td>
<td>255</td>
<td>310</td>
<td>8</td>
<td></td>
<td></td>
<td>0.49</td>
<td>0.60</td>
<td>A</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>T6</td>
<td>$5 &lt; t \leq 20$</td>
<td>240</td>
<td>310</td>
<td>10</td>
<td></td>
<td></td>
<td>0.52</td>
<td>0.60</td>
<td>A</td>
<td>17</td>
</tr>
</tbody>
</table>

**Example:** EN-AW 6082 T6, EP/O $t \leq 5$ mm

**Characteristic values**

- $f_0 = 250$ MPa
- $f_u = 290$ MPa
- $A_{50} = 8\%$
- $\rho_{o,haz} = 0.50$
- $\rho_{u,haz} = 0.64$
- BC = A

**Part of Table 3.2 b.**
Local buckling behaviour / cross section class 4

Except for massive sections and very stocky sections local buckling will occur in compressed parts at failure. However, the behaviour is different depending on the slenderness $\beta = b/t$ where $b$ is the width and $t$ is the thickness of the cross section part.

If $\beta > \beta_3$ where $\beta_3$ is roughly 6 for an outstand part and 22 for an internal part, then local buckling will occur before the compressive stress reach the 0.2 % proof stress $f_0$. Such a section part is called slender and the cross section is referred to as Class 4 cross section.

For very slender sections there is a post-buckling strength allowed for by using an effective cross section.
Cross section class 3, 2 and 1

If $\beta$ for the most slender part of the cross section is $\beta < \beta_3$ and $\beta > \beta_2$ where $\beta_2$ is roughly 4.5 (16), then the cross section belong to class 3, non slender section. Then buckling will occur for a stress equal to or somewhat larger than $f_o$ and some part of the cross section closer to the neutral axis (webs) may be larger than according to the theory of elasticity (linear stress distribution).

If $\beta$ for the most slender part is less than $\beta_2$ then also parts of the cross section close to the neutral axis will reach $f_o$ (class 2).

If $\beta_{max} < \beta_1 = 3$ (11) then rotation capacity is large enough for redistribution of bending moment using plastic global analysis (class 1).
The above given limits $\beta_3$, $\beta_2$ and $\beta_1$ are valid for material buckling class A and $f_o = 250$ N/mm$^2$. For buckling class B and welded sections the limits are smaller.

<table>
<thead>
<tr>
<th>Buckling class</th>
<th>Internal part $\varepsilon$ = $\sqrt{250/f_o}$</th>
<th>Outstanding part</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, without weld</td>
<td>$\beta_1/\varepsilon$</td>
<td>$\beta_2/\varepsilon$</td>
</tr>
<tr>
<td>A, with weld</td>
<td>11</td>
<td>16</td>
</tr>
<tr>
<td>B, without weld</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>B, with weld</td>
<td>13</td>
<td>16,5</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>13,5</td>
</tr>
</tbody>
</table>

For the web of a symmetric beam in bending $\beta = 0,4b_w/t_w$
For outstanding cross section parts, $b$ is the width of the flat part outside the fillet. For internal parts $b$ is the flat part between the fillets, except for cold-formed sections and rounded outside corners.
For cross section parts with stress gradient \( \psi = \frac{\sigma_2}{\sigma_1} \) then
\[
\beta = \eta \frac{b_w}{t_w}
\]
where
\[
\eta = 0.70 + 0.30 \psi \quad \text{if} \ 1 > \psi > -1
\]
\[
\eta = \frac{0.80}{1 - \psi} \quad \text{if} \ \psi < -1
\]

If the part is less highly stressed than the most severely stressed fibres in the section, a modified expression may be used for \( \varepsilon \)
\[
\varepsilon = \sqrt{\frac{250}{f_o}} \cdot \frac{z_1}{z_2}
\]

**Example 2: Give cross section class**

<table>
<thead>
<tr>
<th>Loading</th>
<th>Cross section class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial compression</td>
<td>X</td>
</tr>
<tr>
<td>Bending</td>
<td>X</td>
</tr>
</tbody>
</table>
Axial force cross section resistance

For axial compression the cross section resistance (no flexural buckling) is the same for cross section **class 1, 2 and 3**

\[ N_{Rd} = A f_o / \gamma_{M1} \quad \text{where} \quad \gamma_{M1} = 1,1 = \text{partial factor for material} \]

For **class 4** cross section the cross section resistance is

\[ N_{Rd} = A_{\text{eff}} f_o / \gamma_{M1} \quad \text{where} \quad A_{\text{eff}} = \text{area of effective cross section} \]

This effective cross section is build up of section with effective thickness \( t_{\text{eff}} \) for the cross section parts that belong to class 4.

\[ t_{\text{eff}} = \rho_c t \quad \text{where} \quad \rho_c = \text{reduction factor for local buckling} \quad \rho_c = \frac{C_1}{(\beta / \varepsilon)} - \frac{C_2}{(\beta / \varepsilon)^2} \]

<table>
<thead>
<tr>
<th>Buckling class</th>
<th>Internal part</th>
<th>Outstand part</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( C_1 )</td>
<td>( C_2 )</td>
</tr>
<tr>
<td>A, without weld</td>
<td>32</td>
<td>220</td>
</tr>
<tr>
<td>A, with weld</td>
<td>29</td>
<td>198</td>
</tr>
<tr>
<td>B, without weld</td>
<td>29</td>
<td>198</td>
</tr>
<tr>
<td>B, with weld</td>
<td>25</td>
<td>150</td>
</tr>
</tbody>
</table>
**Bending moment resistance**

For bending moment the formulae for the resistance is depending on cross section class. For **class 2** cross section the resistance is given by

\[ M_{Rd,2} = M_{pl} = W_{pl} f_o / \gamma_{M1} \]

where \( W_{pl} \) = plastic section modulus \( W_{pl} = \sum A \cdot z \)

For **class 1** cross section the resistance may be somewhat larger but \( M_{pl} \) is a good approximation.

For **class 3** cross section the resistance is somewhere between \( M_{pl} \) and \( M_{el} \) where

\[ M_{el} = W_{el} f_o / \gamma_{M1} \]

with \( W_{el} \) = elastic section modulus \( W_{el} = I / e \)

The actual resistance if found by interpolation

\[ M_{Rd,3} = M_{el} + (M_{pl} - M_{el}) \frac{\beta_3 - \beta}{\beta_3 - \beta_2} \]

However, in most cases \( M_{el} \) could be used as a conservative approximation

For **class 4** cross section the resistance is

\[ M_{Rd,4} = W_{eff} f_o / \gamma_{M1} \]

where \( W_{eff} \) = section modulus for effective cross section
The effective cross section is different for axial force and bending moment.

No effective cross section is needed for the combined loading axial force and bending moment. The combination is solved using interaction formulae.
The effective cross section is based on the effective thickness of the cross section parts.

If the cross section is symmetric, then the effective cross section is also symmetric.

If the cross section is asymmetric, then there might be a shift in the neutral axis. For axially compressed extruded profiles this shift is ignored i.e. the axial force is taken as acting in the centre of the effective cross section. For cold-formed sections the shift should be allowed for by adding a bending moment \( \Delta M_{Ed} = N_{Ed} e_N \) where \( e_N \) is the shift in neutral axis for gross and effective cross section.

In principle only the flat parts between fillets need to be reduced, however, for simplicity, the whole flange or web may be reduced.
To find the effective cross section for bending moment is sometimes a tricky task and is not presented here in detail. Just a few comments:

• Local buckling may only occur on the compression side. For a member in bending, even if the cross section is symmetric, the effective section is asymmetric

• The neutral axis of the effective cross section is shifted closer to the tension side and the compressed part of the cross section is increased

• In principle an iteration procedure should be used, however, only two steps are necessary

E.g. for an I-section the first step is to calculate the effective thickness of the compression flange and calculate the neutral axis for that section. The second step is to calculate the effective thickness of the web based on this neutral axis. This is then the effective cross section.
Effective cross section for bending moment

From above we know the cross section class

<table>
<thead>
<tr>
<th>Loading</th>
<th>Cross section class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial compression</td>
<td>flange web</td>
</tr>
<tr>
<td>Bending</td>
<td>web flange</td>
</tr>
</tbody>
</table>

Compress, web

\[ \rho_c = \frac{C_1}{(\beta / \epsilon)} - \frac{C_2}{(\beta / \epsilon)^2} \]

\[ C_1 = 32, \quad C_2 = 220 \quad \beta = 90 / 4 = 22,5 \]

\[ \epsilon = 1, \quad \beta_3 = 22 \]

\[ \rho_c = \frac{32}{22,5} - \frac{220}{22,5^2} = 0,988 \]

\( \rho_c \) is very close to one.

Use gross cross section

Compression and bending, flange

\[ \beta = 70 / 14 = 5 \quad \beta_3 = 6, \quad \beta_2 = 4,5 \]

\[ M_{Rd,2} = W_{pl} f_o / \gamma_{M1} \]

\[ M_{Rd,3} = M_{el} + \left( M_{pl} - M_{el} \right) \frac{\beta_3 - \beta}{\beta_3 - \beta_2} \]

\[ M_{Rd,4} = W_{eff} f_o / \gamma_{M1} \]
Web slenderness

\[ M_{Rk} = \eta f_o W_{el} \]

\[ f_o W_{pl} \quad f_o W_{el} \quad f_o W_{eff} \]
The relatively low elastic modulus of aluminium (compared to steel) means that the deflection at the serviceability limit state is often decisive. Then conservative design at the ultimate limit state can often be accepted.

For class 1, 2 and 3 cross section the resistance according to the theory of elasticity could be used e.g.

\[ M_{Rd} = \frac{W_{el} f_o}{\gamma_{M1}} \]

corresponding to the horizontal line marked "steel" on the previous slide.

For class 4 cross section the resistance could be given by

\[ M_{Rd} = \rho_c \cdot \frac{W_{el} f_o}{\gamma_{M1}} \]

where \( \rho_c \) is the reduction factor for local buckling for the cross section part with the largest value of \( \beta / \beta_3 \). This might be rather conservative but no effective cross section need to be determined.
• Small residual stresses in extruded profiles mean that the buckling curves are not depending on the shape of the cross section (as for steel)

• Buckling curve depends on material and longitudinal welding

• Material buckling class A or B depends on the $\sigma - \varepsilon$–diagram for small strains (proportional limit - 0,2-proof stress ratio, $f_p/f_o$)

• Buckling class is given in Table 3.2 a and b
Effective width - effective thickness

"real" stress distribution

stress distribution based on effective width effective thickness

gross cross section effective width effective thickness

\[ f_o \quad b \quad b_{\text{eff}}/2 \quad b_{\text{eff}}/2 \quad t_{\text{eff}} \]
Why effective thickness?

Simple calculations

You only need to reduce the thickness, not to define start and stop of effective widths - especially important for aluminium profiles.

Easier to allow for combination of local buckling and HAZ

Within the HAZs the lesser of the reduction for local buckling and HAZ softening is used.

Easy to combine with shear lag where effective width is used

The effects of plate buckling on shear lag may be taken into account by first reducing the flange width to an effective width, then reducing the thickness to an effective thickness for local buckling basing the slenderness $\beta$ on the effective width for shear lag. (National choice)
Heat Affected Zone, HAZ

Two reduction factors

\( \rho_{o,haz} \) for 0,2 % proof strength and
\( \rho_{u,haz} \) for ultimate strength

Example: Extruded profile, \( t < 5 \)

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Temper</th>
<th>0,2 % p. strength ( \rho_{o,haz} )</th>
<th>Ultimate strength ( \rho_{u,haz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6082</td>
<td>T4</td>
<td>0,91</td>
<td>0,78</td>
</tr>
<tr>
<td></td>
<td>T5</td>
<td>0,54</td>
<td>0,69</td>
</tr>
<tr>
<td></td>
<td>T6</td>
<td>0,50</td>
<td>0,64</td>
</tr>
<tr>
<td>7020</td>
<td>T6</td>
<td>0,71</td>
<td>0,80</td>
</tr>
</tbody>
</table>

Sheet, strip and plate, \( t < 5 \)

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Temper</th>
<th>0,2 % p. strength ( \rho_{o,haz} )</th>
<th>Ultimate strength ( \rho_{u,haz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3005</td>
<td>H14</td>
<td>0,37</td>
<td>0,64</td>
</tr>
<tr>
<td></td>
<td>H16</td>
<td>0,30</td>
<td>0,56</td>
</tr>
<tr>
<td>5754</td>
<td>H14</td>
<td>0,53</td>
<td>0,63</td>
</tr>
<tr>
<td>6082</td>
<td>T6</td>
<td>0,48</td>
<td>0,60</td>
</tr>
</tbody>
</table>
Width of heat affected zone

When $60^\circ C < T_1 < 120^\circ C$

multiply with

$$1 + \frac{(T_1 - 60)}{120} \quad 6\text{xxx alloy}$$

$$1 + \frac{(T_1 - 60)}{80} \quad 7\text{xxx alloy}$$

$T_1 =$ interpass cooling temperature when multipass welds are laid
For a longitudinally welded section the loss of strength in the heat affected zone HAZ should be allowed for. The cross section classification is made as for extruded sections, except that the limits $\beta_1$, $\beta_2$ and $\beta_3$ are somewhat smaller.

For the resistance a reduced thickness is used within the widths $b_{\text{haz}}$ of the HAZs

$$t_{\text{haz}} = \rho_{o,\text{haz}} t$$

where $\rho_{o,\text{haz}}$ is the reduction factor for the 0.2% proof stress.

If the cross section belong to class 4 the effective thickness is the lesser of $\rho_c t$ and $\rho_{o,\text{haz}} t$ within $b_{\text{haz}}$ and $\rho_c t$ besides HAZ.

Question 1: If a welded section is symmetric and belong to class 3 is then the reduced cross section due to HAZ asymmetric?

Question 2: If a welded section is symmetric and belong to class 4 is then the reduced cross section usually asymmetric?

<table>
<thead>
<tr>
<th>Qu.</th>
<th>yes</th>
<th>no</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>2</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>
For a member with a transverse cross weld the tension force resistance is the lesser of

a) The strength in the sections beside the weld and the HAZ
b) The strength in the HAZ
c) The strength of the weld

The strength of the sections besides the welds and the HAZs is based on the 0.2 % proof strength $f_o$ whereas the strength in the HAZs is the ultimate strength $\rho_{u, haz} f_u$ and in the weld $f_w$, but with larger partial factors $\gamma_{M2} = \gamma_{Mw} = 1.25$.

So, for a member in tension the resistance is the lesser of

a) $N_{o,Rd} = f_o A / \gamma_{M1}$

b) $N_{u,Rd} = \rho_{u, haz} f_u A / \gamma_{M2}$

c) $N_{w,Rd} = f_w A_w / \gamma_{Mw}$
Member with transverse welds or welded attachments

Question 1: Which is the lesser of the strength in HAZ and the weld for a tension member in EN-AW 6082-T6 with a but weld with $A_w = A$ made of filler metal 5356 ($\gamma_{M2} = \gamma_{Mw}$)

Table 8.8  \[ f_w = 210 \text{N/mm}^2 \]

Table 3.2b  \[ \rho_{u,haz} f_u = f_{u,haz} = 185 \text{N/mm}^2 \]

Question 2: What is the difference for a member with an attachment?

a)  \[ N_{o,Rd} = f_o A / \gamma_{M1} \]

b)  \[ N_{u,Rd} = \rho_{u,haz} f_u A / \gamma_{M2} \]

c)  \[ N_{w,Rd} = f_w A_w / \gamma_{Mw} \]  \[ \text{Formula c) is not applicable} \]
For a member with (bolt) holes the resistance is the lesser of
a) The strength in the sections beside the holes
b) The strength in the section with the holes

The net area $A_{net}$ shall be taken as the gross area less appropriate deductions for holes, see figure.

For a member in tension the resistance is the lesser of
a) $N_{o,Rd} = f_o A / \gamma_{M1}$

b) $N_{u,Rd} = 0.9 f_u A_{net} / \gamma_{M2}$  
   
   Note 0,9
Flexural, torsion-flexural and lateral-torsional buckling

Axial force

- (Flexural) buckling
- Torsional buckling
- Torsional-flexural buckling

Bending moment

- Lateral-torsional buckling

Axial force and bending moment

- Flexural buckling
- Lateral-torsional buckling
Flexural buckling

1. Critical load according to classic theory
   \[ N_{cr} = \frac{\pi^2 EI}{l_{cr}^2} \]
2. Yield load
   \[ N_y = A_{eff} f_o \]
3. Slenderness parameter
4. Buckling class and reduction factor from formulae or diagram
   \[ \lambda = \sqrt{\frac{N_y}{N_{cr}}} \]
5. Factor to allow for longitudinally or transverse welds
   \[ \kappa = 1 \text{ for members without welds} \]
6. Resistance
   \[ N_{b,Rd} = \kappa \chi N_y / \gamma_{M1} \]
Flexural buckling, members with longitudinal welds

For members with *longitudinal welds*

**Buckling class A**

\[
\kappa = 1 - \left(1 - \frac{A_1}{A}\right)10^{-\lambda} - \left(0,05 + 0,1 \frac{A_1}{A}\right)\lambda^{-1,3(1-\lambda)}
\]

where \( A_1 = A - A_{haz}(1 - \rho_{o,haz}) \)

\( A_{haz} = \text{area of HAZ} \)

**Buckling class B**

\[
\kappa = 1 \quad \text{if} \quad \lambda \leq 0,2
\]

\[
\kappa = 1 + 0,04(4\lambda)^{(0,5-\lambda)} - 0,22\lambda^{-1,4(1-\lambda)} \quad \text{if} \quad \lambda > 0,2
\]
Members with transverse welds at the ends

For members with cross welds the \( \kappa \) factor is depending on where the weld is placed along the member.

If the welds are at the ends then \( \kappa = 1 \) in the formula for flexural buckling (1). However, then a check is also needed of the section resistance at the ends where \( \kappa = \omega_o \). (2)

\[
\begin{align*}
N_{b,Rd} & \text{ is the lesser of 1 and 2} \\
(1) \quad N_{b,Rd} &= \chi N_y / \gamma_{M1} \\
(2) \quad N_{Rd} &= \omega_o N_y / \gamma_{M1} \\
\omega_o &= \frac{\rho_{u,\text{haz}} f_u / \gamma_{M2}}{f_o / \gamma_{M1}}
\end{align*}
\]

Utilization grade

\[
\begin{align*}
\frac{N_{Ed}}{\chi N_y / \gamma_{M1}} & \quad \frac{N_{Ed}}{\omega_o N_y / \gamma_{M1}}
\end{align*}
\]
Transverse welds at any section

If the weld is at a distance $x_s$ from one end then the resistance at that section is found for $\kappa = \omega_x$ (3). Furthermore the resistance for the member without weld should also be checked. (1)

If the weld is at the centre of the member then $\omega_x = \omega_o$.

$N_{b,Rd}$ is the lesser of 1 and 3

\begin{align*}
(1) \quad N_{b,Rd} &= \chi N_y / \gamma_{M1} \quad \text{and} \\
(3) \quad N_{b,Rd} &= \omega_x \chi N_y / \gamma_{M1}
\end{align*}

$$\omega_x = \frac{\omega_o}{\chi + (1 - \chi) \sin(\pi x_s / l_{cr})}$$

Utilization grade

$$\frac{N_{Ed}}{\omega_x \chi_{haz} N_y / \gamma_{M1}} \quad \frac{N_{Ed}}{\chi N_y / \gamma_{M1}}$$

Note that at the weld $\chi_{haz}$ is based on

$$\lambda_{haz} = \sqrt[\chi]{\omega_o} \quad (6.68a)$$
Torsional and torsional-flexural buckling

(1) For sections containing reinforced outstands such that mode 1 would be critical in terms of local buckling, the member should be regarded as "general" and $A_{\text{eff}}$ determined allowing for either or both local buckling and HAZ material.

2) For sections such as angles, tees and cruciforms, composed entirely of radiating outstands, local and torsional buckling are closely related. When determining $A_{\text{eff}}$, allowance should be made, where appropriate, for the presence of HAZ material but no reduction should be made for local buckling i.e. $\rho_c = 1$.

Formulae for critical load $N_{cr}$ are given in Annex I of Eurocode 9 part 1-1.

\[ \bar{\lambda} = \sqrt{\frac{A_{\text{eff}} \lambda_0}{N_{cr}}} \]

1 Cross section composed of radiating outstands, 2 General cross section
The buckling length should be taken as \( l_{cr} = kL \). The figure gives guidance for \( k \).

End conditions
1. Held in position and restrained in rotation at both ends
2. Held in position at both ends and restrained in rotation at one end
3. Held in position at both ends, but not restrained in rotation
4. Held in position at one end, and restrained in rotation at both ends
5. Held in position and restrained in rotation at one end, and partially restrained in rotation but not held in position at the other end
6. Held in position and restrained in rotation at one end, but not held in position or restrained at the other end
Lateral-torsional buckling of beams

Critical moment

\[ M_{cr} = \frac{\pi}{L} \sqrt{EI_y \left( GK_v + \frac{\pi^2 EK_w}{L^2} \right)} \]

Slenderness parameter

\[ \bar{\lambda}_{LT} = \sqrt{\frac{\alpha W_{el,y f_o}}{M_{cr}}} \]

Reduction factor \( \chi_{LT} \)

Resistance

\[ M_{b,LT} = \chi_{LT} \alpha W_{el,y f_o} / \gamma M_1 \]
Lateral-torsional buckling need not be checked in any of the following circumstances

a) Bending takes place about the minor principal axis (symmetric profiles)

b) Hollow sections with $h/b < 2$

c) Rotation is prevented

d) The compression flange is fully restrained against lateral movement throughout its length

e) The slenderness parameter $\bar{\lambda}_{LT}$ between points of effective lateral restraint is less than 0.4.
Bending and axial compression

1 Classification of cross-sections for members with combined bending and axial forces is made for the loading components separately. No classification is made for the combined state of stress.

major axis (y-axis) bending:

\[
\left( \frac{N_{Ed}}{\chi_y \omega_x N_{Rd}} \right)^{\xi_{yc}} + \left( \frac{M_{y,Ed}}{\omega_0 M_{y,Rd}} \right) \leq 1,00
\]

minor axis (z-axis) bending:

\[
\left( \frac{N_{Ed}}{\chi_z \omega_x N_{Rd}} \right)^{\eta_c} + \left( \frac{M_{z,Ed}}{\omega_0 M_{z,Rd}} \right)^{\xi_{zc}} \leq 1,00
\]

2 A cross-section can belong to different classes for axial force, major axis bending and minor axis bending. The combined state of stress is accounted for in the interaction expressions. These interaction expressions can be used for all classes of cross-section. The influence of local buckling and yielding on the resistance for combined loading is accounted for by the resistances in the denominators and the exponents, which are functions of the slenderness of the cross-section.

3 Section check is included in the check of flexural and lateral-torsional buckling

All exponents may conservatively be given the value 0.8. Alternative expressions depend on shape factors \( \alpha_y \) or \( \alpha_z \) and reduction factors \( \chi_y \) or \( \chi_z \).
Comparison with Eurocode 3 for steel

Cross section class 3

Major axis bending, constant bending moment
Design section

Basic case

\[ M_2 < M_1 \]

Max(\( e + v \)) occur in the span if \( N \) is large and/or the slenderness of the member is large

\[ e_{\text{max}} = e_1 = \frac{M_1}{N} \]

\[ e_2 = \frac{M_2}{N} \]

Max(\( e + v \)) occur at the end if \( M_1 \) is large and/or the slenderness of the member is small

\[ \text{Max}(e + v) = e_1 \]

\[ N \cdot v = \text{second order bending moment} \]
Different end moments or transverse loads

\[
\left( \frac{N_{Ed}}{\chi_y \omega_x N_{Rd}} \right)^{\xi_{yc}} + \frac{M_{y,Ed}}{\omega_0 M_{y,Rd}} \leq 1.00
\]

\[
K + B \leq 1
\]

\[
\omega_0 = 1
\]

\[
\omega_x = \frac{1}{\chi + (1 - \chi) \sin \frac{\pi x}{l_{cr}}}
\]

\[
\frac{1}{\omega_x} \text{ varies according to a sine curve and so also the first term } K \text{ in the interaction formula}
\]

In principal all sections along the member need to be checked. However

\[
\max \left( \left( \frac{N_{Ed}}{\chi_y \omega_x N_{Rd}} \right)^{\xi_{yc}} + \frac{M_{y,Ed}}{\omega_0 M_{y,Rd}} \right)
\]

is found for

\[
\cos \left( \frac{x \pi}{l_c} \right) = \frac{(M_{Ed,1} - M_{Ed,2})}{M_{Rd}} \cdot \frac{N_{Rd}}{N_{Ed}} \cdot \frac{1}{\pi(1/\chi - 1)} \quad \text{but } x \geq 0
\]
In Eurocode 3 (steel) the method with equivalent constant bending moment is used. Then for different bending moment distribution different coefficient are needed. One example is given below.

\[
\frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} + k_{yy} \frac{M_{y,Ed}}{M_{y,Rk} / \gamma_{M1}} \leq 1
\]

\[
k_{yy} = \frac{C_{my}}{1 - \chi_y \frac{N_{Ed}}{N_{cr,y}}}
\]

Cross section class 3 and 4

For example for

\[
C_{yy} = 1 + \left( w_y - 1 \right) \left[ 2 - \frac{1,6}{w_y} C_{my}^2 \bar{x}_y \left( 1 + \bar{x}_y \right) \right]
\]

Cross section class 1 and 2

\[
w_y = \frac{W_{pl,y}}{W_{el,y}} \leq 1,5
\]
Arbitrary moment distribution

\[
\left( \frac{N_{Ed}}{\chi_y \omega_x N_{Rd}} \right)^{\zeta_y c} + \frac{M_{y,Ed}}{\omega_0 M_{y,Rd}} \leq 1.00
\]

\[
K + B \leq 1
\]

\[
\omega_0 = 1
\]

\[
\omega_x = \frac{1}{\chi + (1 - \chi) \sin \frac{\pi x}{l_{cr}}}
\]
Member with transverse weld

For members with transverse (local) weld two checks should be made

1. As if there were no weld

\[ \bar{\lambda} = \sqrt{\frac{N_y}{N_{cr}}} \rightarrow \chi \]

\[ \left( \frac{N_{Ed}}{\chi \omega x N_{Rd}} \right) \xi_{yc} + \frac{M_{y,Ed}}{\omega_0 M_{y,Rd}} \leq 1,00 \]

\[ \omega_0 = 1 \]

\[ \omega_x = \frac{1}{\chi + (1 - \chi) \sin \frac{\pi x}{l_{cr}}} \]

2. Check in the section with the weld

\[ \bar{\lambda}_{haz} = \bar{\lambda} \sqrt{\omega_o} \rightarrow \chi_{haz} \]

\[ \omega_{x,haz} = \frac{\omega_o}{\chi + (1 - \chi) \sin(\frac{\pi x}{l_{cr}})} \]

for \( \chi = \chi_{haz} \)

\[ \omega_o = \frac{f_u}{\rho_{u,haz} f_{u} / \gamma_{M2}} \]

\[ \omega_o = \frac{f_{o}}{\gamma_{M1}} \]

\[ \frac{N_{Ed}}{\omega_x,\chi N_y / \gamma_{M1}} \]

\[ \frac{N_{Ed}}{\omega_{x,haz} N_y / \gamma_{M1}} \]

\[ \frac{M_{Ed}(\chi)}{M_y / \gamma_{M1}} \]

\[ \frac{M_{Ed}(\chi_s)}{\omega_0 M_y / \gamma_{M1}} \]
Lateral-torsional buckling

Check for flexural buckling and

\[
\left( \frac{N_{Ed}}{\chi_z \omega_x N_{Rd}} \right)^{\gamma_c} + \left( \frac{M_{y,Ed}}{\chi_{LT} \omega_x \chi_{LT} M_{y,Rd}} \right)^{\gamma_c} + \left( \frac{M_{z,Ed}}{\omega_0 M_{z,Rd}} \right)^{\gamma_{zc}} \leq 1.00
\]

As for flexural buckling all exponents may conservatively be given the value 0.8. Alternative expressions depend on shape factors \( \alpha_y \) or \( \alpha_z \) and reduction factors \( \chi_y \) or \( \chi_z \).

The shape factors are:

For class (1 and) 2 cross sections \( \alpha = \frac{W_{pl}}{W_{el}} \)

For class 3 cross sections \( \alpha = \text{between } \frac{W_{pl}}{W_{el}} \text{ and } 1 \)

For class 4 cross sections \( \alpha = \frac{W_{eff}}{W_{el}} \)
Lateral-torsional buckling

If there are no lateral bending moment $M_{z,Ed} = 0$ then

$$\left( \frac{N_{Ed}}{\chi_z \omega_x N_{Rd}} \right) \eta_c + \left( \frac{M_{y,Ed}}{\chi_{LT} \omega_x,LT M_{y,Rd}} \right) \gamma_c \leq 1$$

$\eta_c = 0.8$ or $\eta_0 \chi_z$ where $\eta_0 = 1$ or $\alpha_z^2 \alpha_y^2$ but $1 \leq \eta_0 \leq 2$

$\gamma_c = \gamma_0$ where $\gamma_0 = 1$ or $\alpha_z^2$ but $1 \leq \gamma_0 \leq 1.56$

$\omega_x$ and $\omega_{x,LT}$ are coefficients which allow for HAZ across the member and/or of the moment distribution along the member. If there are no cross welds and constant moment then both $\omega$ are $= 1$ else

$$\omega_x = \frac{\omega_0}{\chi_z + (1 - \chi_z) \sin \frac{\pi x}{l_{cr}}}$$

$$\omega_{x,LT} = \frac{\omega_0}{\chi_{LT} + (1 - \chi_{LT}) \sin \frac{\pi x}{l_{cr}}}$$
Three methods are possible:

a) “Equivalent buckling length method”

\[ h \]

(b) \[ L \]

(c) \[ q_{Ed} \]

(h) \[ H_{Ed} \]

(l) \[ I_0 \]

(b) \[ x \]

(c) \[ M_1 \]

(d) \[ \phi \]

(e) \[ L_{cr} \]

(f) \[ M_{II} \]

(g) \[ A - A \]

\[ l \]

\[ q_{Ed} \]

\[ H_{Ed} \]

\[ I_0 \]

\[ M_{II} \]

\( \bar{\lambda} = \sqrt{\frac{N_{Rk}}{N_{cr}}} \)
The equivalent column method

(a) System and load
(b) Equivalent column length
(c) First order bending moment

\[ M = \] The second order bending moment is allowed for by the critical buckling length.
The equivalent sway method

(d) System, load and initial sway imperfection
(e) Initial local bow imperfection and buckling length for flexural buckling
(f) Second order moment including moment from sway imperfection
(g) Initial local bow and buckling length for lateral-torsional buckling
The effect of initial sway imperfection and bow imperfection may be replaced by systems of equivalent horizontal forces introduced for each columns.

\[
\frac{q_{eqv} L^2}{8} = N_{Ed} e_{0,d} \quad \text{gives} \quad q_{eqv} = \frac{8 N_{Ed} e_{0,d}}{L^2}
\]
Initial sway imperfection

\[ \phi = \phi_0 \cdot \alpha_h \cdot \alpha_m \]

\[ \phi_0 = \frac{1}{200} \]

\[ \alpha_h = \frac{2}{\sqrt{h}} \quad \text{but} \quad \frac{2}{3} \leq \alpha_h \leq 1,0 \]

\[ \alpha_m = \sqrt{0,5 \left(1 + \frac{1}{m}\right)} \]

- \( h = \) height in m meters
- \( m = \) number of column in a row including only those columns which carry a vertical load \( N_{Ed} > 50 \% \) of the average value for the columns

Equivalent horizontal forces
Alternative method

In principle

\[ \bar{\lambda} = \sqrt{\frac{N_{Rk}}{N_{cr}}} \]  \[ \chi \rightarrow e_{0,d} \]
Elastic or plastic global analysis

Elastic global analysis may be used in all cases.

Plastic global analysis

1. Plastic global analysis may be used only where the structure has sufficient rotation capacity at the actual location of the plastic hinge, whether this is in the members or in the joints. Where a plastic hinge occurs in a member, the member cross sections should be double symmetric or single symmetric with a plane of symmetry in the same plane as the rotation of the plastic hinge and it should satisfy the requirements for cross section class 1.

2. Where a plastic hinge occurs in a joint the joint should either have sufficient strength to ensure the hinge remains in the member or should be able to sustain the plastic resistance for a sufficient rotation.

3. Only certain alloys have the required ductility to allow sufficient rotation capacity.

4. Plastic global analysis should not be used for beams with transverse welds on the tension side of the member at the plastic hinge locations.

5. For plastic global analysis of beams recommendations are given in Annex H.

6. Plastic global analysis should only be used where the stability of members can be assured.
Aluminium profiles are often asymmetric resulting in torsion. Example:

**Shear centre**

The beam is twisted around the shear centre.

The deflection due to twisting may be larger than the deflection due to bending.

The load also deflects laterally, in this case to the left because the lateral deflection due to twist is larger than due to bending.

1. Divide the load in the direction of the principal axes
2. Calculate the deflection in those directions
3. Calculate the vertical deflection
How to avoid torsion?

- **a.** Add stiffeners
- **b.** Change cross section so that the load acts through the shear centre
- **c.** Use hollow sections

\[ C_v = \text{torsion stiffness (relative)} \]
St Venants torsion resistance

For members subjected to torsion for which distortional deformations and warping torsion may be disregarded (St Venants torsion) the design value of the torsional moment at each cross-section shall satisfy

\[ T_{Ed} \leq T_{Rd} \quad \text{where} \quad T_{Rd} = \frac{W_{T,pl} f_0}{(\sqrt{3} \gamma_{M1})} \]

St Venants torsion

Fillets increase torsion stiffness and strength considerably; see Annex J

Warping torsion
Warping torsion resistance

For members subjected to torsion for which distortional deformations may be disregarded but not warping torsion (Vlasov torsion) the total torsional moment at any cross-section should be considered as the sum of two internal effects:

The following stresses due to torsion should be taken into account:

- the shear stresses $\tau_{t,Ed}$ due to St. Venant torsion moment $T_{t,Ed}$
- the direct stresses $\sigma_{w,Ed}$ due to the bimoment $B_{Ed}$ and shear stresses $\tau_{w,Ed}$ due to warping torsion moment $T_{w,Ed}$.

Check the von Mises yield criterion

$$\left(\frac{\sigma_{x,Ed}}{f_o / \gamma_{M1}}\right)^2 + \left(\frac{\sigma_{z,Ed}}{f_o / \gamma_{M1}}\right)^2 - \left(\frac{\sigma_{x,Ed}}{f_o / \gamma_{M1}}\right)\left(\frac{\sigma_{z,Ed}}{f_o / \gamma_{M1}}\right) + 3\left(\frac{\tau_{Ed}}{f_o / \gamma_{M1}}\right)^2 \leq C$$

where $C = 1,2$

If the resultant force is acting through the shear centre there is no torsional moment due to that loading.

Formulae for the shear centre for some frequent cross-sections. see Annex J
Other structures covered in part 1-1

Built-up columns with lacings and battening [Eurocode 3]

Un-stiffened and stiffened plates under in-plane loadings [2]
Plate girders

Bending

\[ t_{w,ef} = \rho_{w} t_{w} \]

\[ t_{f,ef} = \rho_{f} t_{f} \]

Shear

\[ V_{w} \]

\[ V_{f} \]

\[ M_{Ed} \]

\[ V_{w} + V_{f} + M_{Ed} \]

Patch loading

\[ V_{1,Ed}, V_{2,Ed} \]

\[ V_{1,Ed} + V_{2,Ed} = F_{Ed} \]

Corrugated web

\[ a_1 \]

\[ a_2 \]

References

BS 8118 [4]
Höglund [2, 8]

Höglund [5]
Others [6]

Lagerkvist [6j]
Tryland [6l]

Höglund [5]
Benson [6a]
Ullman [12]
About 30 worked examples based on the ENV version of Eurocode 9 are available on the TALAT CD-ROM, also available at:


where also TALAT Lecture 2301 Design of Members and 2302 Design of Joints can be found

About 50 examples based on EN 1999 and updated lectures will be available on the EAA homepage. A list of the examples are given at the end of this presentation
Main references


See first of all [2]


English Translation in: Höglund, T., *Steel structures, Design according to the Swedish Regulations for Steel Structures, BSK*. Dept. of Steel Structures, Royal Inst. of Technology, Stockholm 1988


Eurocode 9, strength and stability

Thank you for your attention!
Eurocode 9, worked examples
Torsten Höglund

1. Mathcad formulations
2. Serviceability limit state
3. Axial tension
4. Bending moment
5. Axial force
6. Shear force
7. Concentrated force
8. Torsion
9. Axial force and bending moment
10. Nonlinear stress distribution
11. Trapezoidal sheeting
12. Shells
Mathcad formulations

The calculations in the following examples are set out in detail. In most cases, the designer can make simplifications when he/she has learned by experience which checks are not usually critical.

The examples are worked out in the mathematics program Mathcad, version 8. Some of the operators and notations used in the examples are explained below.

\[ x := 50.6 \text{ mm} \]
\[ y = 2.5 \text{ mm} \]
\[ x + y = 53.1 \text{ mm} \]
\[ a = b \]
\[ 0.5 \]
\[ c := (1 \ 3 \ 2) \]
\[ \xrightarrow{(c \cdot d)} \]
\[ d := (2 \ 4 \ 3) \quad a := (c \cdot d) \]
\[ \text{Vectorise operator, i.e. perform arithmetical operation on each element of a vector or matrix} \]
\[ g := \begin{pmatrix} 1 & 8 & 2 \\ 3 & 4 & 7 \\ 5 & 6 & 9 \end{pmatrix} \]
\[ \text{Matrix} \]
\[ c^T \]
\[ \text{Transpose, i.e. rows and columns are interchanged} \]
\[ \xrightarrow{c^T} \]
\[ g^T = \begin{pmatrix} 1 & 3 & 5 \\ 8 & 4 & 6 \\ 2 & 7 & 9 \end{pmatrix} \]
\[ \text{Subscript} \ i \]
\[ A_{ef} \]
\[ A_i \]

Example:
\[ g_{1,2} = 7 \]

Example:
\[ \text{Augmentation of matrices} \]
\[ \text{Part of matrix} \ (a=\text{matrix}, 0 \text{ och } 1 \text{ define rows, } 1 \text{ and } 2 \text{ define columns}) \]
\[ \text{Normally, in a matrix, the first row is numbered 0 and the first column is numbered 0} \]
\[ g = \begin{pmatrix} 1 & 8 & 2 \\ 3 & 4 & 7 \\ 5 & 6 & 9 \end{pmatrix} \]
\[ \text{submatrix} (g,0,1,1,2) = \begin{pmatrix} 8 & 2 \\ 4 & 7 \end{pmatrix} \]
\[ \text{augment} (f,g) \]
\[ \text{Column} \]
\[ \text{Example:} \]
\[ \text{Example:} \]
\[ \text{Notation} \ (ef \text{ is not a subscript but part of variable notation}) \]
\[ \text{Subscript } i \]
\[ \text{Example:} \]
# Serviceability limit state

<table>
<thead>
<tr>
<th>No.</th>
<th>Cross section etc.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td><img src="image1" alt="Cross section" /></td>
<td>Deflection of class 4 cross section girder made of two extrusions and one plate. Distributed load.</td>
</tr>
<tr>
<td>2.2</td>
<td><img src="image2" alt="Cross section" /></td>
<td>Simple method to check resistance and deflection of class 4 cross section girder. Distributed load.</td>
</tr>
<tr>
<td>2.3</td>
<td><img src="image3" alt="Cross section" /></td>
<td>Deflection of asymmetric extruded profile due to bending and torsion of concentrated load. Check of stresses included.</td>
</tr>
</tbody>
</table>
## Axial tension

<table>
<thead>
<tr>
<th>No.</th>
<th>Cross section etc.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>$f_o$, $f_u$, $f_{o,haz}$, $f_{u,haz}$ etc</td>
<td>Characteristic values of material properties</td>
</tr>
<tr>
<td>3.2</td>
<td><img src="image" alt="Diagram a)" /></td>
<td>a) Axial tension force resistance of a plane plate</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Diagram b)" /></td>
<td>b) Axial tension force resistance of a plate with MIG butt weld across the plate</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Diagram c)" /></td>
<td>c) Axial tension force resistance of a plate with longitudinal butt weld</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Diagram d)" /></td>
<td>d) Axial tension force resistance of a plate with welded attachment across the plate</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Diagram e)" /></td>
<td>e) Axial tension force resistance of a plate with bolt holes</td>
</tr>
</tbody>
</table>
## Bending moment

<table>
<thead>
<tr>
<th>No.</th>
<th>Cross section etc.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td><img src="image" alt="Cross section" /></td>
<td>Bending moment resistance of cross section with closed cross section parts and outstands</td>
</tr>
<tr>
<td>4.2</td>
<td><img src="image" alt="Cross section" /></td>
<td>Bending moment resistance of extruded hollow cross section</td>
</tr>
<tr>
<td>4.3</td>
<td><img src="image" alt="Cross section" /></td>
<td>Bending moment resistance of welded hollow cross section with outstands. Class 2 cross section</td>
</tr>
<tr>
<td>4.4</td>
<td><img src="image" alt="Cross section" /></td>
<td>Bending moment resistance of welded hollow cross section with outstands. Class 4 cross section</td>
</tr>
</tbody>
</table>
## Axial compression 1

<table>
<thead>
<tr>
<th>No.</th>
<th>Cross section etc.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td><img src="image1" alt="Square Hollow Cross Section" /></td>
<td>Axial compression force resistance of square hollow cross section (local and flexural buckling)</td>
</tr>
<tr>
<td>5.2</td>
<td><img src="image2" alt="Symmetric Hollow Extrusion" /></td>
<td>Axial compression force resistance of symmetric hollow extrusion (local and flexural buckling)</td>
</tr>
<tr>
<td>5.3</td>
<td><img src="image3" alt="Cross Section with Radiating Outstands" /></td>
<td>Axial compression force resistance of cross section with radiating outstands (torsional buckling)</td>
</tr>
<tr>
<td>5.4</td>
<td><img src="image4" alt="Channel Cross Section" /></td>
<td>Axial compression force resistance of channel cross section (distortional buckling)</td>
</tr>
</tbody>
</table>
### Axial compression 2

<table>
<thead>
<tr>
<th>No.</th>
<th>Cross section etc.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5</td>
<td><img src="image" alt="Axial force resistance of laced column" /></td>
<td>Axial force resistance of laced column</td>
</tr>
</tbody>
</table>
| 5.6 | ![Axial force resistance of orthotropic plate with](image) | Axial force resistance of orthotropic plate with  
  a) open stiffeners  
  b) trapezoidal stiffeners  
  c) closed stiffeners |
| 5.7 | ![Axial force resistance of orthotropic double-skin plate](image) | Axial force resistance of orthotropic double-skin plate  
  a) profiles joined with grooves and tongues  
  b) truss cross section  
  c) frame cross section |
### Shear force 1

<table>
<thead>
<tr>
<th>No.</th>
<th>Cross section etc.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td><img src="image1.png" alt="Image" /></td>
<td>Shear force resistance of a plate girder with no intermediate stiffeners incl. contribution from the flanges</td>
</tr>
<tr>
<td>6.2</td>
<td><img src="image2.png" alt="Image" /></td>
<td>Shear force resistance of a web with a) flexible intermediate stiffeners b) rigid intermediate stiffeners</td>
</tr>
<tr>
<td>6.3</td>
<td><img src="image3.png" alt="Image" /></td>
<td>Shear force resistance of a web with transverse and longitudinal stiffeners</td>
</tr>
<tr>
<td>6.4</td>
<td><img src="image4.png" alt="Image" /></td>
<td>Shear force resistance of a plate girder with corrugated web</td>
</tr>
<tr>
<td>No.</td>
<td>Cross section etc.</td>
<td>Description</td>
</tr>
<tr>
<td>------</td>
<td>------------------------------------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>6.5</td>
<td>a) <img src="image1" alt="Diagram" /></td>
<td>Shear force resistance of orthotropic plate with</td>
</tr>
<tr>
<td></td>
<td>b) <img src="image2" alt="Diagram" /></td>
<td>a) open stiffeners</td>
</tr>
<tr>
<td></td>
<td>c) <img src="image3" alt="Diagram" /></td>
<td>b) trapezoidal stiffeners</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c) closed stiffeners</td>
</tr>
<tr>
<td>6.6</td>
<td><img src="image4" alt="Diagram" /></td>
<td>Shear force resistance of orthotropic double-skin plate</td>
</tr>
<tr>
<td></td>
<td>a) <img src="image5" alt="Diagram" /></td>
<td>a) profiles joined with grooves and tongues</td>
</tr>
<tr>
<td></td>
<td>b) <img src="image6" alt="Diagram" /></td>
<td>b) truss cross section</td>
</tr>
<tr>
<td></td>
<td>c) <img src="image7" alt="Diagram" /></td>
<td>c) frame cross section</td>
</tr>
</tbody>
</table>
## Concentrated force and interaction $M+V$

<table>
<thead>
<tr>
<th>No.</th>
<th>Cross section etc.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td><img src="image1.png" alt="Diagram" /></td>
<td>Concentrated force resistance of beam and plate girder (patch loading).</td>
</tr>
<tr>
<td>7.2</td>
<td><img src="image2.png" alt="Diagram" /></td>
<td>Interaction between shear force and bending moment for a plate girder at the support region.</td>
</tr>
<tr>
<td>7.3</td>
<td><img src="image3.png" alt="Diagram" /></td>
<td>Plate girder in shear, bending from concentrated forces. Rigid end post, no intermediate stiffeners.</td>
</tr>
</tbody>
</table>
## Torsion

<table>
<thead>
<tr>
<th>No.</th>
<th>Cross section etc.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1</td>
<td><img src="image1.png" alt="Cross section" /></td>
<td>Cross section shape to avoid torsion</td>
</tr>
<tr>
<td>8.2</td>
<td><img src="image2.png" alt="Hollow cross section" /></td>
<td>Torsion constant for a hollow cross section</td>
</tr>
<tr>
<td>8.3</td>
<td><img src="image3.png" alt="Deck profile" /></td>
<td>Torsion constant for a deck profile</td>
</tr>
<tr>
<td>8.4</td>
<td><img src="image4.png" alt="Thin-walled section" /></td>
<td>Torsion and bending of thin-walled section</td>
</tr>
</tbody>
</table>
Axial force and bending moment

<table>
<thead>
<tr>
<th>No.</th>
<th>Cross section etc.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.1</td>
<td><img src="image" alt="Tensile force and bending moment" /></td>
<td>Tensile force and bending moment</td>
</tr>
<tr>
<td>9.2</td>
<td><img src="image" alt="Beam-column with rectangular hollow section" /></td>
<td>Beam-column with rectangular hollow section</td>
</tr>
<tr>
<td>9.3</td>
<td><img src="image" alt="Beam-column with eccentric load" /></td>
<td>Beam-column with eccentric load</td>
</tr>
<tr>
<td>9.4</td>
<td><img src="image" alt="Eccentrically loaded beam-column with cross weld" /></td>
<td>Eccentrically loaded beam-column with cross weld</td>
</tr>
<tr>
<td>9.5</td>
<td><img src="image" alt="Axial force resistance of cantilever column" /></td>
<td>Axial force resistance of cantilever column fixed to ground with bolted foot plate or fixed into a concrete block</td>
</tr>
</tbody>
</table>
### Nonlinear stress distribution

<table>
<thead>
<tr>
<th>No.</th>
<th>Cross section etc.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1</td>
<td><img src="image" alt="Cross section" /></td>
<td>Transverse bending of asymmetric flanges</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Diagram" /></td>
<td>Warping and bending of thin-walled member in torsion and bending. See 8.4 and also 2.2</td>
</tr>
</tbody>
</table>
# Trapezoidal sheeting

<table>
<thead>
<tr>
<th>No.</th>
<th>Cross section etc.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.1</td>
<td></td>
<td>Trapezoidal sheeting without stiffeners</td>
</tr>
<tr>
<td>11.2</td>
<td></td>
<td>Trapezoidal sheeting with one stiffener in the webs</td>
</tr>
<tr>
<td>11.3</td>
<td></td>
<td>Trapezoidal sheeting with one stiffener in the top flanges and one stiffener in the webs</td>
</tr>
<tr>
<td>11.4</td>
<td></td>
<td>Trapezoidal sheeting with two stiffeners in the flanges and two stiffeners in the webs</td>
</tr>
<tr>
<td>No.</td>
<td>Cross section etc.</td>
<td>Description</td>
</tr>
<tr>
<td>-----</td>
<td>--------------------</td>
<td>-------------</td>
</tr>
</tbody>
</table>
| 12.1 | ![Image](image1.png) | Cylindrical shell in  
  a) Meridional (axial) compression and bending  
  b) Meridional (axial) compression with coexistent internal pressure |
| 12.2 | ![Image](image2.png) | Cylindrical shell in circumferential compression |
| 12.3 | ![Image](image3.png) | Cylindrical shell in shear |
| 12.4 | ![Image](image4.png) | Cylindrical shell with stepwise wall thickness in circumferential compression |
## Stiffened shells and shells with torus parts

<table>
<thead>
<tr>
<th>No.</th>
<th>Cross section etc.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5</td>
<td><img src="image1.png" alt="Image" /></td>
<td>Horizontally corrugated wall treated as an orthotropic shell. Axial compression and external pressure</td>
</tr>
<tr>
<td>12.6</td>
<td><img src="image2.png" alt="Image" /></td>
<td>Welded torispherical shell under external pressure</td>
</tr>
<tr>
<td>12.7</td>
<td><img src="image3.png" alt="Image" /></td>
<td>Welded toriconical shell under external pressure</td>
</tr>
</tbody>
</table>